

**Study on two-echelon supply chain models
with variable demand under different
shipment policies**

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FACULTY OF SCIENCES

DOCTORAL THESIS

**Study on two-echelon supply chain models
with variable demand under different
shipment policies**

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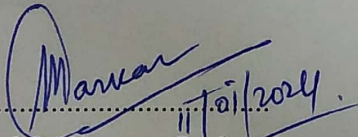
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CERTIFICATE FROM THE SUPERVISORS

This is to certify that the thesis entitled “**Study on two-echelon supply chain models with variable demand under different shipment policies**” submitted by Smt. **Anamika Dash** who got her name registered on 2nd February, 2018 (INDEX NO : 16/18/Maths./25) for the award of Ph. D. (Science) degree of Jadavpur University, is absolutely based upon her own work under the supervision of **Prof. Ashis Kumar Sarkar** and **Prof. Bibhas Chandra Giri** and that neither this thesis nor any part of it has been submitted for either any degree/diploma or any other academic award anywhere before.


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Dedicated to
My beloved family

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Contents

1	Introduction	1
1.1	Supply chain and supply chain management	1
1.1.1	Historical evolution of supply chain management	2
1.1.2	Supply chain structure	3
1.1.3	Objectives of supply chain management	4
1.2	Green supply chain management	5
1.2.1	Objectives of green supply chain management	6
1.2.2	Hindrance of green supply chain	7
1.3	Some factors considered in SCM	8
1.3.1	<i>Demand</i>	8
1.3.1.1	<i>Price</i>	8
1.3.1.2	<i>Green policy</i>	8
1.3.1.3	<i>Promotion or sales effort</i>	9
1.3.1.4	<i>Advertisement</i>	9
1.3.2	Batch shipment	10
1.3.3	Lead time	11
1.3.4	Imperfect production	11
1.3.5	Coordination	12
1.4	Scope of the study	12
1.5	Organization of the study	14
2	Literature Review	19
2.1	Batch shipment policy	19
2.2	Stochastic lead time	21
2.3	Demand	22
2.3.1	Price dependent demand	22
2.3.2	Sales effort dependent demand	24
2.3.3	Green-sensitive demand	25
2.3.4	Advertisement-sensitive demand	26

2.4	Imperfect production	27
2.5	Supply chain coordination	29
2.6	Single-manufacturer multi-retailer model	31
2.7	Greening investment	33
2.8	Carbon emissions	34
2.9	Sample inspection	35
2.10	Set up cost reduction	36
3	Coordination and defect management strategy for a two-level supply chain under price and sales effort sensitive demand	39
3.1	Introduction	39
3.2	Problem Definition and Assumptions	40
3.3	Notation	43
3.4	Model Development	45
3.4.1	Decentralized Model	49
3.4.2	Centralized Model	53
3.4.3	Coordinated Model	55
3.4.3.1	<i>Trade credit mechanism</i>	55
3.4.3.2	<i>Price discount contract</i>	59
3.5	Numerical Analysis	61
3.6	Sensitivity Analysis	67
3.7	Concluding Remarks	71
4	Two-echelon supply chain model with price and green sensitive demand under equal and unequal batch shipment policy	81
4.1	Introduction	81
4.2	Problem Definition and Assumptions	82
4.3	Notations	83
4.4	Model Development	84
4.4.1	Decentralized Model	87
4.4.2	Coordinated Model with Cost Sharing Contract	88
4.4.3	Integrated Model	90
4.5	Numerical Study	91
4.6	Sensitivity Analysis	93
4.7	Concluding Remarks	95
5	Optimal batch shipment policy for an imperfect production system under price, advertisement and green-sensitive demand	97
5.1	Introduction	97
5.2	Problem Definition and Assumptions	99
5.3	Notations	100
5.4	Model Formulation	101
5.4.1	Decentralized policy	104
5.4.2	Centralized policy	109
5.4.3	Cost Sharing Contract	111
5.5	Numerical Analysis	114

5.6	Sensitivity Analysis	116
5.7	Concluding Remarks	126
6	Coordination of a sustainable supply chain under unequal-sized shipments, sample inspection and set-up cost reduction	129
6.1	Introduction	129
6.2	Problem Definition and Assumptions	132
6.3	Notations	134
6.4	Model Development	136
6.4.1	Decentralized Model	139
6.4.2	Centralized Model	142
6.4.3	Coordinated Model	145
6.4.3.1	<i>Trade credit mechanism</i>	146
6.4.3.2	<i>Wholesale price discount contract</i>	149
6.5	Numerical Analysis	152
6.6	Sensitivity Analysis	160
6.7	Managerial Insights	166
6.8	Concluding Remarks	168
7	A single-manufacturer multi-retailer integrated inventory model with price dependent demand and stochastic lead time	171
7.1	Introduction	171
7.2	Problem Definition	172
7.3	Notations and Assumptions	173
7.4	Model Formulation	175
7.4.1	Decentralized Model	178
7.4.2	Centralized Model	182
7.5	Numerical Analysis	187
7.6	Sensitivity Analysis	189
7.7	Concluding Remarks	191
8	Coordination of a single-manufacturer multi-retailer supply chain with price and green sensitive demand under stochastic lead time	193
8.1	Introduction	193
8.2	Problem Definition	194
8.3	Notations and Assumptions	195
8.4	Model Formulation	196
8.4.1	Decentralized Model	200
8.4.2	Centralized Model	205
8.4.3	Coordinated Model	208
8.5	Numerical Analysis	210
8.5.1	A comparative study	214
8.6	Sensitivity Analysis	215
8.7	Concluding Remarks	219
9	Conclusion and future research prospects	223

Bibliography	225
List of Publications	249

List of Figures

1.1	Supply chain structure	4
1.2	Green supply chain management	6
3.1	Flow of inventory in the system	41
3.2	Accumulation and depletion of the manufacturer's inventory	46
3.3	Inventory of the retailer	48
3.4	Supply chain profit versus retailer's interest rate (I_r) and credit period (ξ)	63
3.5	Supply chain profit versus manufacturer's interest rate (I_m) and credit period (ξ)	64
3.6	Average profit of the manufacturer and the retailer versus ϕ	66
3.7	Average supply chain profit versus sales effort cost coefficient g	68
3.8	Average supply chain profit versus rework cost R_w	70
3.9	Average supply chain profit versus carbon emission cost c_v	70
3.10	Average supply chain profit versus transportation cost t_v	71
4.1	Inventory of the manufacturer and retailer system	85
4.2	Inventory of the retailer	86
4.3	retailer's average profit vs. β	94
4.4	manufacturer's average profit vs. β	94
4.5	retailer's average profit vs. k	94
4.6	manufacturer's average profit vs. k	94
5.1	Inventory of the retailer	102
5.2	The manufacturer's inventory accumulation and depletion	104
5.3	Average expected profit of the manufacturer and the retailer vs. k	117
5.4	Average expected profit of the supply chain vs. k	118
5.5	Average expected profit of the manufacturer and the retailer vs. α	119
5.6	Average expected profit of the supply chain vs. α	120
5.7	Average expected profit of the manufacturer and the retailer vs. γ	121

5.8	Average expected profit of the supply chain vs. γ	122
5.9	Average expected profit of the manufacturer and the retailer vs. β	123
5.10	Average expected profit of the supply chain vs. β	124
5.11	Average expected profit of the manufacturer and the retailer vs. I	125
5.12	Average expected profit of the supply chain vs. I	125
6.1	Schematic diagram of inventory flow in the supply chain	132
6.2	Accumulation and depletion of the manufacturer's inventory	137
6.3	Inventory structure of the retailer	138
6.4	Concavity of the average supply chain profit versus n	153
6.5	Supply chain profit versus retailer's interest rate (ψ_r) and credit period (τ)	153
6.6	Supply chain profit versus manufacturer's interest rate (ψ_m) and credit period (τ)	154
6.7	Average expected supply chain profit for different values of I_b in sampling inspection vs. full inspection	157
6.8	Average expected profit of the manufacturer and the retailer versus χ	159
6.9	Average expected supply chain profit vs. price elasticity coefficient (β)	161
6.10	Average expected supply chain profit vs. promotional effort sensitivity coefficient (γ)	162
6.11	Average expected supply chain profit vs. greening effort sensitivity coefficient (η)	162
6.12	Average expected supply chain profit for different values of price discount ratio (z)	163
6.13	Average expected supply chain profit in (a) decentralized, (b) centralized and (c) coordinated model for different values of I_b in sampling inspection vs. full inspection	164
6.14	Average expected supply chain profit in (a) decentralized, (b) centralized and (c) coordinated model vs. δ	165
6.15	Average expected supply chain profit in (a) decentralized, (b) centralized and (c) coordinated model vs. I_u for sample inspection and no inspection	166
7.1	Logistics diagram of the proposed single-manufacturer multi-retailer supply chain model	173
7.2	Inventory of i -th retailer under stochastic lead time	176
7.3	Joint inventory of the manufacturer and the retailers	180
7.4	Concavity of the average expected profit of the supply chain w.r.t. n	187
7.5	Concavity of the average expected profit function of the first retailer	188
7.6	Concavity of the average expected profit function of the second retailer	188
7.7	Average expected profit of the supply chain vs. c_1	189
7.8	Average expected profit of the supply chain vs. c_2	189
7.9	Average expected profit of the supply chain vs. F	191
8.1	Inventory of i -th retailer under stochastic lead time	197
8.2	Joint inventory of the manufacturer and the retailers	200

8.3	Concavity of the average expected profit function of the first retailer	211
8.4	Concavity of the average expected profit function of the second retailer	211
8.5	Price discount rate vs. average expected profit	212
8.6	Comparison with existing literature	215
8.7	Average expected profit vs. a_2	215
8.8	Average expected profit vs. β_2	217
8.9	Product's greening level vs. α_2	217
8.10	Product's greening level vs. I	218
8.11	Average expected profit vs. F	218

List of Tables

3.1	Optimal results for different models	65
3.2	Optimal results for different models	67
3.3	Optimal results for different models	68
3.4	Optimal results for different models	69
4.1	Optimal results for different models	92
4.2	Optimal results for different values of I, α, β and k	93
5.1	Optimal results for different models	115
5.2	Optimal results for different models (without shortage)	116
5.3	Sensitivity with respect to k	117
5.4	Sensitivity with respect to α	119
5.5	Sensitivity with respect to γ	121
5.6	Sensitivity with respect to β	122
5.7	Sensitivity with respect to I	124
6.1	Optimal results for different models	155
6.2	Optimal results for different models when $M_\rho = 0$	156
6.3	Optimal results for different models when $\delta = 1$	157
6.4	Optimal results for different models when $\delta = 0$	158
7.1	Optimal results for different models	188
7.2	Optimal results for different values of β_1, β_2, a_1 and a_2	190
8.1	Optimal results for different models	211
8.2	Optimal results for different models	213
8.3	Sensitivity analysis of the parameters a_1, β_1, α_1 and F	216

CHAPTER 1

Introduction

1.1 Supply chain and supply chain management

“Supply Chain is like nature, it is all around us.” - Dave Waters

A supply chain (SC) refers to the network of all the people, organizations, resources, operations, and technological advancements involved in the production and distribution of goods. An entire supply chain incorporates everything from the distribution of raw materials from the supplier to the producer to the final delivery to the customer. Chopra and Meindl, 2001 provided the following formal definition of supply chain:

“A supply chain consists of all stages involved, directly or indirectly, in fulfilling a customer request. The supply chain not only includes the manufacturers and suppliers, but also transporters, warehouses, retailers, and customers themselves.” - Chopra and Meindl, 2001

An efficient supply chain encompasses viable strategies that manage the whole production flow of a product or service efficaciously in order to enhance profitability through optimized pricing strategies and assure customer satisfaction. Supply chain management (SCM) is the art of managing delivery of the ideal product to the ideal customer at the ideal time, at the right location, at a reasonable price. Undoubtedly,

in this new era of industrialization, supply chain management has a significant impact on the business world and paves an endless exploration opportunity. There are numerous leading companies worldwide like Apple, Zara, etc. who have benefited from adopting supply chain management strategies, policies, and planning (Hofmann, 2013).

Supply chain management is one of the world's fastest growing and well-regarded management disciplines. The need for supply chain management has risen four-fold as globalization has reached every corner of the world. Because of the extremely competitive environment and cutthroat competition, resource optimization is a major concern. Customer service as well as product satisfaction is now of utmost importance. Both pricing and the quality of service are of paramount significance to consumers. In this sense, supply chain management has evolved into a global competitive market demand in order to sustain profitability while assuring targeted service levels by focusing on numerous interfaces and functions such as procurement, manufacturing, inventory management, and distribution, etc.

1.1.1 Historical evolution of supply chain management

"Supply chain management is the process of planning, implementing, and controlling the operations of the supply chain with the purpose to satisfy customer requirements as efficiently as possible. It spans all movement and storage of raw materials, work-in-process inventory, and finished goods from point-of-origin to point-of-consumption." - **Keith Oliver**

Although supply chains have been in existence since ancient times, commencing with the first produced and marketed item or service, the term "supply chain management" was first coined out by Keith Oliver during an interview for the Financial Times in 1982. Before the Industrial Revolution in Europe and the United States, the great majority of supply chains were regionally constrained and of a local nature. Things began to change with the industrial revolution. Global supply networks began to take shape and numerous organizations started opening up their doors gradually. After World War II, industrial executives started to consider upgrading manual operations, investigating the application of mechanization, and illustrating

the advantages of analytics in logistics for military. Also, with the increased usage of railroads, transporting goods across greater distances became quicker, simpler, and more affordable. In 1925, there was a significant advancement in supply chain storage when warehouses began to utilize pallets. In the middle of 1960s, computerization began to get more popular and in 1967, the first computerized inventory management and information forecasting system was developed by IBM. In 1975, the first real-time warehouse management system was developed by JC Penney, which made it simpler to manage orders, inventory, and delivery and also led to increased efficiency. Later in the 1990s, the advent of enterprise resource planning systems further spurred the logistics boom. Computer technology has greatly advanced the utilization of supply and logistics since the 1980s. Moreover, the skyrocketing use of internet has yielded a higher level of sophistication in the supply chain management system. Rapid advancements in communication technologies and electronics have boosted logistics planning and execution technology and surged ahead the supply chain management to unprecedented heights. The supply chain of today is not any more a linear entity. It is rather a convoluted amalgam of several disparate networks. Consumers are at the core of these networks, who anticipate having their orders to be delivered - "when they want them, the way they want them". Additionally, it is undeniable that a new advanced generation of logistics planning and supply chain technology based upon central planning and distributed cooperation is now within reach. So research has become exceptional importance in the advancement of conventional supply chain and logistics domains, including warehouse operations, manufacturing logistics and transportation.

1.1.2 Supply chain structure

SCM refers to the handling of the flow of goods and services and encompasses all activities involved in the conversion of raw materials into finished products. It entails proactively simplifying a company's supply-side processes in order to enhance customer value and achieve a competitive edge over its competitors. The structure of a conventional supply chain is depicted in Figure 1.1. The stages involved in a traditional supply chain are: raw material/component supplier, manufacturer, wholesaler/distributor, retailer, and customer/end user. The supply chain is linked by the flow of goods, information, and currency through these various nodes. In this

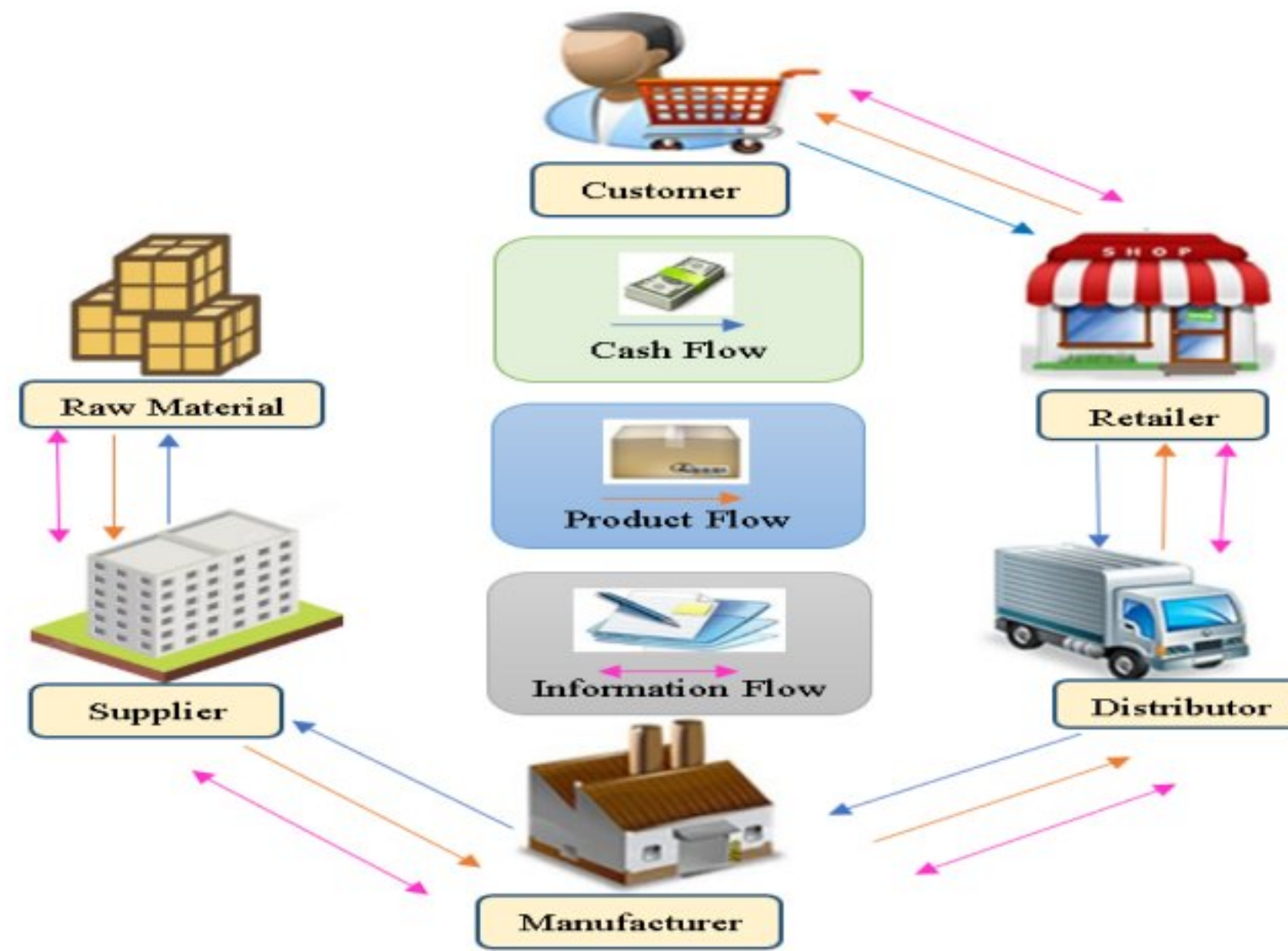


Figure 1.1: Supply chain structure

typical supply chain, the physical flow of finished materials is in the forward direction (from supplier to customer) whereas information and cash flow is in reverse direction (from consumer to supplier). It refers to an integrated production system wherein the supplier provides the manufacturer with raw materials or semi-finished items, which are subsequently produced or assembled into finished products. The finished products are then supplied to the wholesaler, the retailer, and eventually to the customers. Not every node in the supply network needs to be present. The appropriate supply chain design is determined by both the demands of the customer and the activities needed by various nodes to meet those needs.

1.1.3 Objectives of supply chain management

The supply chain management is treated as the backbone of modern businesses. The key goals of SCM are:

- **Improvement of efficiency-** Efficiency is one of the most important goals of SCM. Waste reduction and efficiency are inextricably linked. Wastes may manifest in many different ways, including wastage of resources, waste of money, wasted labour-hours, waste of delivery times, and so on. Waste minimization is a crucial aspect of SCM.

- **Quality improvement-** Another important objective is to make sure that the product is of the greatest quality possible. To gain competitive advantages in the marketplace while lowering operational expenditures simultaneously, supply chain quality management is essential.
- **Transportation and logistics optimization-** SCM enables a smooth process flow while eliminating the negative impacts of poor scheduling and coordination, and it optimizes logistics and transportation activities like ordering, transportation and shipping of goods.
- **Cost reduction-** Another key objective of SCM is to reduce all types of operational expenses for a business such as product purchase, production, and delivery costs.
- **Improving customer satisfaction-** Enhancing customer satisfaction is a vital objective of SCM. Efficient SCM affects two major components of customer satisfaction: pricing and distribution. It enables businesses to beat their competitors in terms of selling prices and profitability by delivering items on time and at the anticipated price.
- **Developing better coordination-** SCM strives to enhance coordination among various stakeholders in the business. It helps to establish an organization with remarkable synergies and promotes information exchange among all stakeholders.

1.2 Green supply chain management

In recent times, green supply chain is an emerging area that integrates and explores environmental concerns into the conventional supply chains. It is due to the growing awareness of the need of addressing environmental issues in addition to traditional economic metrics. Natural resource depletion, water and air pollution, climate change, deforestation, global warming and ozone depletion have all become a grave concern for the present world. Numerous well-known businesses have now committed to implementing green business strategies in light of these issues. The fundamental tenet of the "green idea" is to improve environmental sustainability,

although businesses sometimes refer to it as "killing two enemies with one stone." Because, green supply chains not only mitigate the environmental degradation it also spurs economic growth, generate competitive advantages in the form of higher customer satisfaction, a positive brand image, and a strong reputation, as well as improved opportunities for exporting their goods. Some aspects of green supply chain management (GSCM) are reflected in Figure 1.2.



Figure 1.2: Green supply chain management

1.2.1 Objectives of green supply chain management

- To reduce detrimental environmental effects and working towards more ethical and sustainable business practices while providing products and services to customers.
- Utilize renewable energy sources to their fullest potential while lowering carbon emissions generated by business activities.
- Utilizing resources effectively to save expenses and provide a more efficient manner of conducting company.

- ✎ To achieve an equilibrium between environmental protection and economic growth while enhancing ecological sustainability by lowering the toxic material use, generation of waste, and emissions.
- ✎ To raise customer awareness by highlighting the benefits of green practices and emphasizing how they may be implemented in businesses to alleviate the environmental destruction.

1.2.2 Hindrance of green supply chain

To address environmental concerns, manufacturing industries have recently started to incorporate the green practices into their supply chain management. But, adopting and implementing green business practises is tricky due to the presence of numerous impediments. Due to the many aspects of green supply chain management, identifying obstacles to its implementation is challenging. The following are some of the main impediments to the adoption of green supply chain management:

- ✎ **Inadequate IT implementation** - An effective IT system is essential to support the GSCM throughout the product life cycle. This can be extremely beneficial for product development initiatives involving environmental design, reclamation, and reuse. Efficiently managing green SC requires managing information flows related to both upstream and downstream of material and all resources.
- ✎ **Financial constraint** - Excessive investment in adopting green practices such as green manufacturing, green packaging, green designing, etc. can pose significant barriers to GSCM implementation.
- ✎ **Lack of customer awareness** - Customers' lack of information about the benefits of green products and environmental concern is a key hurdle to GSCM adoption.
- ✎ **Inadequate government intervention** - Insufficient government actions, as well as lack of special incentives, such as tax deductions for GSCM practitioners, are important obstacles to green supply chain adoption.

1.3 Some factors considered in SCM

1.3.1 Demand

With the rapid advancement of technologies and industrialization, things are changing at an alarming pace and over the last couple of decades, consumer demand has risen to prominence in the efficient management of the supply chain. Demand planning is a strategy used in supply chain management that forecasts or predicts product demand to assure delivery and customer satisfaction. Its main objective is striking the right balance between carrying enough inventory to fulfil consumer demand without holding an excess inventory. Some of the crucial factors having a substantial impact on market demand are as follows:

1.3.1.1 Price

The most impactful factor affecting market demand is the price of the product. Customers on a tight budget may not afford high-priced products and it forces them to look for an alternative. On the other hand, a low-priced product may attract customers and yields a higher sales volume. Nowadays, competition-prone business scenario compels companies to push down product prices. Companies set prices based on their competitor's pricing strategies. Such rivalry reduces the market price of the product. It's not just good for consumers – when goods become affordable to more people, it stimulates organizations to produce and strengthens the economy in general. Apart from reducing prices, they also have to maintain the quality of their products. But in doing so, they may not always be able to meet their expenses and face financial losses. Therefore, determining the pricing strategy is not an easy task for the decision managers to run the business smoothly.

1.3.1.2 Green policy

Another major concern emerging these days is the environmental degradation. If not addressed immediately, it could lead to the extinction of mankind. The robust impacts of rapid industrialization and urbanization has caused several environmental issues. Natural resources are being depleted drastically and hazardous

substances are entering our environment at an alarming rate. So, it is the high time to protect our environment and conserve natural resources. This fact is increasingly well understood and people are becoming environmentally conscious day by day. Industrialists are adopting stringent means to reduce their environmental impact and government is also intervening to propel people for choosing eco-friendly ways. Different rules and regulations have been implemented towards environmental protection. The government is also offering various subsidies and tax exemption to companies for adopting eco-friendly business practices. Many leading companies worldwide have taken drastic measures such as greenhouse gas emission reduction, green manufacturing, green packaging, recycling, waste reduction, etc. to reduce pollution. Wal-Mart, Honda, Dell, Coca-Cola, Starbucks, Toyota, and Hewlett Packard are some of the front-runners among these. According to several consumer polls, attitudes toward green products are growing more prevalent among consumers every day.

1.3.1.3 *Promotion or sales effort*

In tough economic times and recessions, sales effort and promotional strategies have become a key for companies to establish themselves as a viable brand in the market. Sometimes small trade owners reduce their advertising and promotion expenditure for saving money. Unfortunately, this often opposed of things they must do to remain competitive. Forward-thinking corporate executives have begun to realize that it maintains public image of their brand and prevents competitors from acquiring a foothold in the market sectors they have secured. Companies can substantially improve their market position through advertising and promotions. These increase customer loyalty and make the product relevant when the competitors appear.

1.3.1.4 *Advertisement*

Advertising is a highly effective commercial strategy and a sophisticated form of communication. It is an extremely potent phenomena that has an impact on members' perspectives on economic, social, cultural, and behavioural factors. Advertising, together with the product and its packaging, pricing, distribution, and personnel selling, is deduced to be a marketing tactic. To cope with the very rapid

technological developments and marketing advancements in today's competitive marketplaces, advertising has become a popular management strategy. It plays an important role in the unveiling and the development of a product or service and persuades customers to purchase the goods. Advertising aids the business in fostering a favorable societal image of its brands and goods. It makes people acquainted with the product and strives to boost customer preference for that certain product over its rivals. Overall, among marketing weapons, the advertisement may yield long-term benefits since an effective advertisement sticks in people's thoughts for a long time and has broader consequences. Pampers, Gillette, L'Oreal, Ford, Amazon, Sony, Samsung and NIKE are some global brands among the heavy spenders on advertising.

1.3.2 Batch shipment

Hundreds of consumer orders a day are received by enterprises and delivery networks. Each order must be dispatched to consumers within the time frame specified at the time the order was placed to ensure smooth running of business and retain customer satisfaction. Batch optimization allows firms to process orders in accordance with their specific business demands with high accuracy, little human intervention, and minimal time. To ensure timely delivery and prevent out-of-stock bottlenecks, items can be delivered to the retailer's end in a single lot only after being processed. Shipping in a single lot certainly helps to decrease the number of shipments and the related expenses; nevertheless, for large sized lots, it may result in significant inventory carrying costs. This often leads to excess inventory and retailers incur additional holding costs for it. Another method of transferring products is to divide a whole lot into many batches and ship from the manufacturer to the retailer whenever a batch is finished. But, whenever the shipping expenses are high, business managers may opt to transfer several goods in a single shipment to decrease the total number of batch shipments and the associated transportation costs. So balancing inventory holding and transportation costs is challenging. According to the literature, batch shipments may be helpful to lower inventory holding costs as the consuming phase begins sooner as compared to the scenario when only whole lots are dispatched. Additionally, allowing batch shipments can result in bigger lots and fewer setups, which cuts down on setup times.

1.3.3 Lead time

Delivering the right items to the right location, at the correct time, in the appropriate quantity, and under the right circumstances are the key objectives of logistics. In this regard, the lead time between the initiation of an order and the final delivery is extremely important for both customer satisfaction and inventory levels. In the bustling world of today, customers desire goods or services as quickly as possible and with a minimal effort. Thus, longer lead times can have a negative impact on customer perception and result in lower sales volume. Therefore, companies should reduce lead times in order to deliver products or services to the customer at the anticipated time. However, there is a risk of understocking or overstocking while doing so. Whereas overstocking might place a burden on the budget, understocking can result in a loss of prospective revenue. All of this makes it difficult to carry out production effectively. Factors that may affect lead times include a scarcity of raw materials, equipment malfunction, transportation problems, a personnel shortage, natural calamities, and human errors. In a word, although difficult, reducing and efficiently managing lead time and optimizing inventory level boost productivity, increasing production and profitability.

1.3.4 Imperfect production

For the smooth running of the business, ensuring the quality of the product and verdict the optimal production procedure has become a salient object so that it gratifies all the customer's needs. But because of the various components involved in the manufacturing system, such as labour, equipment, and technology, it is typical in real-world production environment for each manufactured item not to be flawless. It is observed that, every production system initially lies in an in-control state and the manufactured goods are of an acceptable quality as every component connected to the manufacturing system is new and fresh. In general, long production-runtime raises the likelihood of malfunctioning such as equipment failure and labours' impativeness and accelerates the decline of product quality. As a consequence, the production system encounters an out-of-control situation and generates a mix of flawless and flawed goods. The rate of imperfectness is measured relative to the total number of produced items, and it can be constant or variable, as well as classified

as crisp, probabilistic, or fuzzy depending on its nature. Perfect quality items are always compatible for sale, however defective quality products must be reworked at a cost in order to turn into a new flawless product. Some defective items can be sold at a discount, while the worst leftover defective goods are entirely rejected to avoid loss of goodwill and uphold customer satisfaction.

1.3.5 Coordination

The evolution of supply chain networks in the twenty-first century was influenced by changes in the corporate environment. Substantial success factors have been posited as a consequence of globalization and the emergence of multinational corporations, joint ventures, technological advances, strategic coalitions, and commercial partnerships. Due to its negative impacts on production costs, replenishment times, product availability, product price, transportation and labour costs, and other factors, devoid of coordination has a profound adverse effect on supply chain performance. Lack of coordination occurs whenever distinct stages/entities of the supply chain focus on achieving conflicting objectives or when information is delayed and distorted while moving through the different stages. Due to the intricacies and competing objectives involved, supply chain coordination greatly improves performance compared to dealing with individual supply chain participants. Coordination may aid in the management of interdependencies and the reduction of uncertainty.

1.4 Scope of the study

The intense competition prevailing in the today's global marketplace has compelled the organizations to implement a robust system that efficiently absorbs variations dictated by current market conditions. This line of improvement seeks to streamline operations involved in providing the end consumer with the desired product or service at a reasonable price while retaining excellent quality. In addition, the inevitable shortage of natural resources and growing environmental awareness have driven companies to reduce environmental impact and adopt more sustainable business processes than ever before. In this regard, emphasis is placed on several important

factors affecting efficient strategic operations to achieve overall higher profitability and productivity. This doctoral study addresses such crucial issues like imperfect production, planned backorder, various demand patterns, stochastic lead time, defect management, batch shipment, sample inspection, multi-retailer business scenario, synergy among the supply chain members and so on. The following are some worth mentioning scope of this thesis:

- ✎ This thesis introduces a strategic defect management policy, paving the way for a sustainable future. Under this policy, business managers can grasp profits by redesigning or reworking the defective products rather disposing them off. It inevitably turns into a sustainable move that greatly contributes in waste reduction and lowers carbon-emission costs.
- ✎ To get a greater insight into various market scenarios, it analyzes the influence of several critical aspects of market demand such as retail price, greening level, sales and promotional activities, advertising frequency, and so on.
- ✎ On the way to establishing organizational success, facilitating the supply commensurate with the demand may be treated as a major concerning area. In this regard, this thesis considers all types of batch shipments policies namely, equal sized batch shipment, unequal sized batch shipment and combined equal and unequal batch shipment policies.
- ✎ To retain customer satisfaction and strengthen business competitiveness, emphasis has been given on the impact of stochastic lead time.
- ✎ To get closer to reality, this study addresses imperfect production process and inspection errors. Moreover, it implements a sub-lot inspection policy that becomes inexpensive and time saving compared to the full inspection.
- ✎ In order to develop and enable synergy among different supply chain entities, i.e., manufacturers and retailers, it employs various coordination mechanisms, such as cost sharing contract, wholesale price discount contract, and trade credit mechanism.
- ✎ This study also takes into account cost cutting initiatives like establishing a set-up cost reduction policy.

- ✎ Consideration of multi-retailer business scenario provides a pragmatic basis for this study.

1.5 Organization of the study

The thesis incorporates nine chapters. The overview of each chapter is abbreviated below to provide the reader with an indication of the organization of the thesis.

The thesis begins with **Chapter 1: Introduction**. As an introductory chapter, it gives a succinct summary of supply chain management. This chapter also covers several terminologies and fundamental ideas of supply chain management pertinent to the thesis.

Chapter 2 presents a brief review of existing literature spanning across the research domains - imperfect production, lead time, demand, batch shipment policy and supply chain coordination.

Chapter 3 presents a single-manufacturer single-retailer supply chain model with an imperfect production system under unequal-sized shipment policy. After receiving each delivery, the retailer performs an inspection process that is erroneous and results in misclassification errors (Type-I and Type-II errors). Defective items are returned to the manufacturer, who conducts a rigorous screening to classify the defective items into repairable, design-flawed and non-repairable ones. The manufacturer incurs carbon emissions cost due to transportation and disposal of scrap items to get one step closer to sustainability. Customers at the retailer are influenced by retail price and sales effort. Optimal shipment policy, pricing and inventory decisions are determined in both the decentralized (based on the Stackelberg leader-follower gaming approach) and the centralized scenarios. A trade credit mechanism and wholesale price discount contract are employed and a fair profit-sharing strategy is implemented based on the supply chain member's bargaining power. The conditions for participating in these coordination mechanisms are examined from the point of view of each supply chain member. Finally, a numerical experiment along with sensitivity analysis is carried out to illustrate the applicability of the proposed coordination mechanisms and defect management policy.

Chapter 4 considers a two-stage supply chain model consisting of a single manufacturer and a single retailer with a combination of equal and unequal sized batch shipment policy. All unequal sized batches are in a geometric progression with a fixed common ratio. Consumer's demand is sensitive to the retail price and greening improvement level of the item. At the retailer, planned shortage is allowed in each shipment cycle and is completely backlogged. Assuming that the retailer has higher power than the manufacturer, we develop non-coordinated decentralized model and coordinated model with cost sharing contract, and the integrated centralized model. The numerical results show that the cost sharing contract gives an impressive increment in the profits of both the manufacturer and the retailer over their profits in the non-coordinated model. A sensitivity analysis is performed to explore how the changes in the key model-parameters effect the optimal profits of the manufacturer and the retailer and the whole supply chain.

Chapter 5 considers an imperfect production-inventory system which consists of a single manufacturer and a single retailer. The manufacturer delivers the order quantity to the retailer in some unequal-sized batches. To separate the defective items, the retailer performs an error-free screening process after receiving each delivery from the manufacturer. Shortage in retailer inventory is allowed and completely backlogged. The customer demand is influenced by the retail price, advertisement frequency and greening level of the product. The centralized model and the decentralized model based on a Stackelberg gaming approach are developed to determine optimal pricing, advertising and inventory decisions. A cost sharing contract between the manufacturer and the retailer is implemented, which enhances the environmental performance, advertisement frequency and profitability of the supply chain significantly. The proposed model is illustrated with a numerical example. Sensitivity analysis for some key parameters is carried out and several managerial insights are also highlighted.

Chapter 6 studies a single-manufacturer single-retailer green supply chain model with an unequal sized batch shipment policy. Upon arrival of every shipment, the retailer conducts a sample inspection on a few randomly selected items. The manufacturer adopts a set-up cost reduction strategy. Consumer demand is influenced by retail price and retailer's promotional and greening efforts. Mathematical models

are developed and investigated under both decentralized and centralized structures. Thereafter, two incentive coordination schemes, namely, a trade credit contract and a wholesale price contract are proposed. The necessary conditions for involvement in these coordination mechanisms are derived from each member's perspective. Numerical examples demonstrate that the proposed coordination mechanisms efficiently improve the economic as well as environmental performance of the entire supply chain. Sensitivity of some key model parameters is analyzed.

Chapter 7 considers a two-level supply chain which is composed of a single manufacturer and multiple retailers. The ordered quantities of the retailers are delivered in some equal sized batches from the manufacturer. Customer demand is linearly dependent on the retail price of the product. Lead time is random and it follows a normal distribution. The proposed models are developed for both the centralized and the decentralized scenarios. In the decentralized model, a Stackelberg gaming approach is used to find the optimal solution. The developed models are illustrated by a numerical example. It is found that integration of the supply chain players gives an impressive increment in profit of the whole supply chain. Sensitivity analysis is also carried out to explore the impacts of key parameters on the expected average profit of the supply chain system.

Chapter 8 investigates a two-level supply chain comprising a single manufacturer and multiple retailers in which the manufacturer produces a single product and delivers it to the retailers in equal-sized batches. Each retailer faces a price and green-sensitive market demand. The lead time is assumed to be a random variable that follows a normal distribution. Shortages for retailer inventory are allowed to occur and are completely backlogged. The centralized model and a decentralized model based on the leader-follower Stackelberg gaming approach are developed. A price discount mechanism between the manufacturer and retailers is proposed. For the acceptance of this contract, the upper and lower limits of the price discount rate are established. Numerical outcomes exhibit that the price discount mechanism effectively coordinates the supply chain and enhances both environmental and economical performances. A sensitivity analysis with respect to some key parameters is performed, and certain managerial insights are emphasized.

Chapter 9 provides a brief outline of the key findings from the study conducted for this doctoral research, and suggests some potential areas for further investigation.

CHAPTER 2

Literature Review

This chapter provides a brief review of the literature on two-echelon supply chain models focusing on some key issues pertinent to this thesis and highlights a few limitations of earlier investigations.

2.1 Batch shipment policy

In a two-stage supply chain management, an interesting decision making problem arises at the retailer's end to determine economic ordering quantity and optimal number of shipments in which the manufacturer supplies the retailer's ordered quantity. Goyal and Szendrovits, 1986 were the early researchers who presented a constant lot size model in which the products are produced through a sequence of production stages and parts of the lot are transferred from one stage to another in some equal or unequal sized batches. Banerjee, 1986 considered a joint economic lot-size model with finite production rate and lot-for-lot shipment policy. By relaxing Banerjee, 1986's lot-for-lot assumption, a more general JELS model was proposed by Goyal, 1988. Hill, 1999 developed a global optimal production and shipment policy for a single manufacturer-single retailer integrated model. Hoque and Goyal, 2000

extended the idea of Goyal and Szendrovits, 1986 and developed an optimal policy for an integrated single-manufacturer single-retailer model in which a single product is shipped to the retailer from the manufacturer in some equal and unequal sized batches with capacity constraint of transportation. Hill and Omar, 2006 considered an single-manufacturer single-retailer integrated model and derived optimal production and shipment policy under the circumstances when the holding costs for the retailer is more expensive than the manufacturer. Hoque, 2008 synchronized a single-manufacturer multi-retailer integrated supply chain model transferring the lot in some equal and unequal sized batches. He developed three models, two of which transfer equal-sized batches and the third one transfers unequal batches of the product, and derived solution procedure of them. Hoque, 2013 presented a manufacturer-retailer integrated model assuming equal and/or unequal sized batch shipment policy and stochastic lead time. Giri and Sharma, 2014 developed an integrated manufacturer-retailer model under imperfect production process and unequal sized batch shipment policy. They observed from numerical study that their proposed model outperformed Huang, 2004 model. Beck and Glock, 2016 suggested two popular approaches to solve economic lot scheduling problem extending them by including both equal and unequal-sized batch shipments. Giri et al., 2017 re-established the observation of Bylka and Górný, 2015 for a vendor management inventory model with unequal batch shipments and consignment stock policy. Beck et al., 2017 studied a multi-manufacturer single-retailer JELS model under two different batch shipment policies - one where the manufacturer delivers the batches just after their completion and one where the manufacturer places his next shipment whenever the inventory level at the retailer becomes zero and compared these two policies to the equal sized batch shipment situation. Yu and Chen, 2018 investigated an integrated production-inventory model under an equal batch shipment policy incorporating quality improvement investment and warranty policy for acceptable defective items. Liu et al., 2019 proposed a single-manufacturer single-retailer supply chain model with deteriorating items and provided optimal production and shipment policies to decrease joint total cost of the system. Hemapriya and Uthayakumar, 2020 introduced a neoteric approach to geometric batch shipment policy in a two-level integrated supply chain model and analyzed the impact of several factors such as controllable lead time, setup cost reduction, lost sales due to stock-out and freight cost under service-level constraint. Latha et al., 2021 derived

optimal strategies in a two-echelon joint economic lot size model with unequal batch shipment policy, backorder price discount, and ordering cost reduction investment. Khanra et al., 2022 considered unequal sized batch shipment policy in an integrated three-layer supply chain model consisting of a supplier, a manufacturer, and a retailer. Juman et al., 2023 determined the optimal production shipment strategy in a multi-vendor multi-buyer model with an unequal sized batch shipment policy.

2.2 Stochastic lead time

Lead time has attracted the interest of many researchers in the areas of inventory management and production control. It plays a crucial role in business management. Lead time can be shortened by paying an additional cost. Many researchers (Ouyang et al., 2007; Ho, 2009; Glock, 2012; Vijayashree and Uthayakumar, 2017) investigated several production-inventory models with lead time reduction. Li et al., 2011 considered controllable lead time in their models. To address the shortcomings of deterministic lead time, researchers devised supply chain models that take into account the stochastic nature of lead time. Vijayashree and Uthayakumar, 2015 provided an effective iterative algorithm to minimize the cost of an integrated inventory model under controllable lead time. Sajadieh et al., 2009 developed a single vendor single buyer supply chain model with stochastic lead time following exponential distribution and deterministic demand, and exhibited a significant cost reduction in integrated system than decentralized ones. Later, Hoque, 2013 extended this model by considering combined equal and unequal batch shipments and showed that the model with normal distribution is more profitable than the model with exponential distribution for lead time. Lin, 2016 derived production strategy and investment policy for reducing lead time variability in a vendor-buyer supply chain model with stochastic lead time. Kim and Sarkar, 2017 developed a two-level supply chain model considering several important factors such as stochastic lead time demand, quality improvement, set up cost reduction, trade-credit policy. Kim et al., 2018 investigated the impact of set up cost reduction strategy in an inventory model considering stochastic lead time, back-order price discount, and reorder point. Giri and Masanta, 2019 derived optimal production and shipment policy for a closed-loop supply chain model with stochastic lead time, and observed that learning in

production and remanufacturing leads to a significant cost reduction for the supply chain. Wangsa and Wee, 2019 studied an integrated inventory model with inspection error, stochastic lead time and freight cost. Then concluded that type I error is more significant than type II error and the distance between the buyer and vendor impacts the lead time and total cost effectively. Giri and Masanta, 2020 developed a closed-loop supply chain model with learning in production, price and quality sensitive demand under stochastic lead time, and elaborated the positive impact of learning in production process on the optimal decisions. Sarkar and Giri, 2020a investigated an integrated vendor-buyer model considering time value of money with partially backlogged shortage under stochastic lead time where the lead time is variable but dependent on the order size of the buyer and production rate at the vendor. Safarnezhad et al., 2021 derived optimal ordering, pricing and inspection policies in a vendor-buyer supply chain model with price dependent demand and stochastic lead time. Hoque, 2021 developed a single-manufacturer multi-retailer supply chain model under stochastic lead time where the manufacturer delivers the lots to the retailers either only with equal batch sizes or only with unequal batch shipments. Geranios et al., 2022 analyzed the impact of stochastic lead time in a serial supply chain with multiple nodes and external demand.

2.3 Demand

2.3.1 Price dependent demand

Determination of the best pricing strategy and inventory decisions that influence the customer demand has been focused by many researchers and practitioners. A seminal research work in this direction was done initially by Whitin, 1955. Later, many researchers (Yang et al., 2009; Khanra et al., 2010; Maragatham and Lakshmidevi, 2014) investigated joint pricing and inventory problems. Teng et al., 2005 derived an algorithm to find optimal price and ordering policy of the retailer when the manufacturer allows delay in payments. Ho et al., 2008 derived an iterative algorithm to obtain optimal strategy for an integrated supplier-retailer inventory model with price sensitive demand. Chen and Kang, 2010, and Chung and Liao, 2011 considered integrated inventory models with price sensitive demand. Kim et al., 2011

derived a joint pricing and ordering policies in a single-manufacturer single-retailer supply chain with price-dependent consumer demand. Ahmadi Rad et al., 2014 investigated the effect of coordination on a two-stage supply chain model when the customer's demand is sensitive to products selling price. Sohrabi et al., 2016 investigated a supplier selection problem in the vendor managed inventory (VMI) scenario under price dependent customer demand. Alfares and Ghaithan, 2016 developed an inventory model considering storage time-dependent holding cost, quantity discount and price-dependent demand. Maragatham and Palani, 2017 determined an optimal replenishment policy aimed at minimization of the total cost under time dependent ordering and holding costs and deterioration rate with price sensitive demand. Giri et al., 2018a implemented a profit sharing contract during trade credit period between the retailer and the manufacturer under price-sensitive market demand. Rastogi and Singh, 2018 derived optimal retail price and optimal time interval for an economic order quantity (EOQ) model considering non-instantaneous deterioration of items and price-sensitive consumer demand. Pervin et al., 2019 developed a two-level supply chain model with deteriorating multiple items under trade-credit policy and price- and stock-dependent demand. Biswas and Giri, 2019 developed an integrated inventory model and derived optimal decisions in order to optimize the system profitability for deteriorating item and price-dependent demand rate. Pervin et al., 2019 incorporated inflation in a economic production quantity (EPQ) inventory model for deteriorating items with preservation technology investment under price- and stock-dependent demand. Sarkar et al., 2020 proposed a price discount policy where the vendor offers a price discount depending on the buyer's ordered quantity in a price-sensitive market. Mashud et al., 2020 studied a deteriorating inventory model with selling price dependent demand under trade credit policy and preservation technology investment. Das et al., 2020 applied preservation technology in an inventory model for non-instantaneous deteriorating items, price sensitive demand and partially backlogged shortage. Mashud et al., 2021 implemented both the advance payment and delay payment in a supply chain model with the trade credit policy under price- and advertisement-sensitive customer demand. Duary et al., 2022 considered a mixed type trade credit policy (both advance and delay in payments) and a price discount offer in an inventory model for deteriorating items with price, time, and advertisement sensitive demand. Akhtar et al., 2023 proposed a hybrid DESGO algorithm to determine optimal pricing and

inventory decisions in a non-instantaneous deteriorating inventory model with time and price dependent demand.

2.3.2 Sales effort dependent demand

Among the factors that influence the market demand, sales effort is the impactful one. Researchers and practitioners have implemented various strategies such as sales effort to enhance the customer demand. Wei and Xiong, 2015 investigated a two-stage supply chain model for fashion products with price and sales effort sensitive demand. Giri and Roy, 2016 derived optimal pricing strategies in a single-manufacturer multi-retailer dual channel supply chain model under price and sales effort dependent demand. Chen et al., 2017 examined the effect of uncertainty of price elasticity and base market demand on the pricing and sales effort decisions in a single-manufacturer single-retailer supply chain model with price and sales effort dependent demand. Sane Zerang et al., 2018 studied a three-echelon closed-loop supply chain model with price and sales effort sensitive market demand under different channel leaderships. Rastogi and Singh, 2018 presented a solution procedure to find optimal production period and shortage period for deteriorating items with price dependent demand in an inflationary environment. Saha et al., 2019 dealt with a three-echelon supply chain model with price and sales effort dependent consumer demand under sales effort cost sharing contract. Rabbani et al., 2020 proposed an option-revenue sharing contract in a single-manufacturer single-retailer supply chain with consumer demand sensitive to the retail price and sales effort. Heydari and Asl-Najafi, 2021 implemented a revised sales rebate contract in a supplier-retailer supply chain model with sales effort sensitive demand. Li et al., 2021 studied a stylized optimization model considering price and sales effort sensitive demand, and investigated how these optimal decisions are affected by the firm's risk attitude. Heydari et al., 2022 investigated a two-level green supply chain model with two substitutable green and non-green products where the retailer executes a green sales effort to attract customers towards green products. Thereafter, they coordinated the sales effort and ordering strategies by applying revenue-cost sharing and buyback contract. Wang et al., 2022 studied a three-echelon closed-loop supply chain model considering corporate social responsibility (CSR) under price and sales effort sensitive demand. They examined how the channel power structures affect

the profitability of each participant and showed that no matter who the leader is, profitability of that member increases with the increment of CSR level. Hu et al., 2023 investigated joint dynamic pricing and sales effort strategies in a dual channel e-commerce platform for seasonal products and proposed a revenue-cost sharing contract to coordinate the channel.

2.3.3 Green-sensitive demand

Researchers and practitioners are currently focusing on issues including the reduction of harmful effects of production on the environment. During the last several years, green supply chain has become a hot topic of research for operations management researchers. Srivastava, 2007 provided a depth review of green supply chain literature. Abdallah et al., 2010 developed a carbon sensitive supply chain model which helps to minimize carbon emissions with green procurement. Wahab et al., 2011 considered a coordinated two-level supply chain model for imperfect items taking into account fixed and variable carbon emission costs. Ghosh and Shah, 2012 studied the impact of customer's sensitivity towards green products in various supply chain structures. Jaber et al., 2013 developed a two-level manufacturer-retailer model in which the manufacturer and retailer jointly minimize the GHG emission costs and the total inventory cost. Chen et al., 2013 derived conditions under which a significant reduction of carbon emission is possible without increasing the cost significantly. Swami and Shah, 2013 studied a vertical supply chain consisting of a single manufacturer and a single retailer where the members put an effort for greening their operations, and the customer demand at the retailer's end is price and green sensitive. Zaroni et al., 2014 investigated a two-level joint economic lot size model with customer demand sensitive to price and environmental quality, and concluded that investing in improving a product's environmental performance is more beneficial, and implementing an integrated policy can increase both environmental and economic performance. Ghosh and Shah, 2015 explored the positive impact of a cost sharing contract on the optimal decisions of a green supply chain to enhance the profit level and produce items with higher greening quality. Li et al., 2016 initiated e-commerce in green supply chain management and proposed a coordination mechanism for decentralized dual channel green supply chain. Basiri and Heydari,

2017 investigated coordination issues in a green supply chain with a non-green traditional product and a substitutable green product under price, greening level and sales effort dependent demand. Giri et al., 2018b analyzed a closed-loop supply chain model and exhibited that the model with price, green and warranty period dependent demand provides better response on the optimal decisions than the model with price and warranty period dependent demand under a revenue sharing contract. Dey and Saha, 2018 focused on three procurement scenarios of a two-period green supply chain where the consumer demand is price and green-sensitive. Heydari et al., 2019 developed a three-tier dual channel supply chain model with price and green sensitive demand that is not only economically beneficial but also reduces the selling price in both channels. Panja and Mondal, 2020 investigated a two-layer green supply chain model and proposed a revenue sharing contract where customer demand depends on the selling price, the greening degree of the product and also the credit period offered to the customers by the retailer. Heydari et al., 2021 proposed a hybrid coordination scheme of cost sharing contract and revenue sharing contract in a two-level green supply chain with price and green sensitive demand. Paul et al., 2022 studied a green inventory model under a carbon tax policy to determine the optimal replenishment cycle and greening level of the product. They also analyzed the impact of greening level on the purchasing and selling price of the product. Sepehri and Gholamian, 2023 investigated the impacts of shortages in a sustainable inventory model with price and emission sensitive demand considering quality improvement and inspection process concurrently.

2.3.4 Advertisement-sensitive demand

Advertisements convince customers to purchase the product and raises brand value. Recently, deriving optimal advertising strategies in different supply chain model scenarios have drawn considerable attention of the researchers. SeyedEsfahani et al., 2011 considered vertical cooperative advertising strategy in a single-manufacturer single-retailer supply chain model with price and advertisement sensitive consumer demand. They also discussed practicability of bargaining game in sharing extra joint profit between the retailer and the manufacturer. Sadigh et al., 2012 studied

a manufacturer-retailer supply chain model for multiple products under two different Stackelberg gaming structures with the customer demand dependent on the price and advertisement expenditure. Chanda and Kumar, 2017 investigated optimal pricing and advertising strategy in an EOQ model for technology products and customer demand influenced by retail price and advertisement expenditure in a fuzzy environment. Rad et al., 2018 developed a vendor-buyer supply chain model with price and advertisement sensitive demand under imperfect production, and showed that as the sensitivity of the price and advertisement increases, coordination becomes more and more beneficial. Mishra, 2018 presented a production-inventory model for deteriorating items under price and advertisement cost dependent demand, and showed that a higher demand rate results in more ordering quantity per cycle and enhances total profit. Shaikh et al., 2019 developed a two-level inventory model with price and advertisement sensitive demand under partially backlogged shortages where product deterioration rate follows a Weibull distribution. Shaikh and Cárdenas-Barrón, 2020 studied an inventory model with price and advertisement sensitive demand for non-instantaneous deteriorating products under the trade credit policy. Mandal and Pal, 2021 dealt with an EPQ model under unreliable production system where the production of defective items depends on the reliability of the machinery system and customer demand is dependent on selling price and advertisement. Gautam et al., 2022 proposed an efficient rework strategy to handle the imperfect items and derived optimal inventory decisions in an inventory model with price and advertisement sensitive demand. Chaudhari et al., 2023 investigated the impact of down-cash-credit method on the business strategies for perishable products under carbon tax regulations where the customer demand depends on price, stock and advertisement.

2.4 Imperfect production

Despite having sophisticated machineries, many production systems produce defective items due to several unavoidable reasons such as mishandling, human error, poor management, etc. Many researchers have investigated the effect of imperfect production on optimal inventory strategies. In practice, each production lot may contain a significant number of imperfect items because of various reasons such as

insufficient work instruction, poor maintenance, mechanical error, etc. This fact led many researchers to study the EPQ model relaxing the traditional assumption of perfect production system. Taft, 1918 was the first to study in this field. Inspired by his work, later many researchers have carried out significant amount of works on the impact of imperfect production system in the classical inventory model. Giri and Sharma, 2014 developed a production-inventory model consisting single manufacturer and single retailer with imperfect production system and determined optimal production and shipment policy to minimize total cost of the system. Li and Chen, 2015 studied a single-manufacturer single-retailer production-inventory model with imperfect quality items and unequal shipment policy. They showed that, the proper multi-batch handling strategy influences the total cost, and unequal batch shipment policy provides significant cost reduction than equal shipment policy. Lai et al., 2015 derived optimal ordering and shipment policies in a two-level supply chain model with imperfect production system and defect rate dependent quality improvement investment. Chang et al., 2016 determined optimal ordering strategy in an EOQ model considering inspection errors and imperfect items under a trade credit mechanism. Nobil and Taleizadeh, 2016 considered a multi-product EPQ inventory model with imperfect production process under rework process and auction. Khanna et al., 2017 incorporated inspection error, imperfect rework process and full price refund on sales returns in traditional imperfect environment. Giri and Sharma, 2017 investigated the effect of inspection errors and ordering cost reduction investment in an integrated inventory model with imperfect production and combined equal and unequal batch shipment policy. Liao et al., 2018 studied an EOQ model for imperfect items with limited storage capacity and permissible delay in payment. Mandal and Giri, 2019 studied an imperfect production-inventory model with two warehouses and stock dependent demand considering a quantity discount offer. Saga et al., 2019 investigated energy impact and carbon emission in an imperfect production process with service-level constraint and inspection errors. Kamna et al., 2021 addressed carbon emission and energy usage during imperfect production in a sustainable supply chain with volume agility. Gautam et al., 2022 proposed a proficient rework strategy to handle the defective items and derived optimal inventory strategies in an imperfect production inventory model with price and advertisement sensitive demand. Dolai et al., 2023 analyzed impacts of advertisement on credit period and learning effect on the inspection process in a green supply

chain model with imperfect production process. Sepehri and Gholamian, 2023 incorporated both environmental concerns and quality improvement and studied the impact of shortages in an inventory model with imperfect production process.

2.5 Supply chain coordination

Coordination between the supply chain members has become indispensable to survive in the competitive business world. Several researchers have performed numerous works in order to find appropriate coordination schemes for the supply chain players to achieve maximum profitability. A significant amount of works has been carried out by the researchers in order to coordinate a supply chain with a suitable contract such as risk sharing contract (Norrman, 2008), two-part tariff contract (Swami and Shah, 2013), cost sharing contract (Zhao et al., 2014; Yang et al., 2017; Zhu et al., 2018), revenue sharing contract (Zhang and Feng, 2014; Peng et al., 2018), price discount contract (Xu et al., 2018; Sarkar et al., 2020), trade credit contract (Heydari et al., 2017), and so on. Li et al., 2011 investigated the impact of a price discount mechanism in a single-vendor single-buyer supply chain model with service level constraint and controllable lead time. Tsao and Sheen, 2012 discussed the strategy of coordination of a manufacturer and a retailer through a cost sharing contract. Saha and Goyal, 2015 demonstrated the effectiveness of three coordination mechanisms such as joint rebate contract, cost sharing contract and wholesale price discount contract in a two-level supply chain model with price and stock dependent demand. Li and Liu, 2015 considered three different coordination schemes such as cost sharing contract, wholesale price contract and two-part tariff contract in a two-echelon supply chain model under price and sales effort dependent demand. Zhou et al., 2015 analyzed how carbon-emission reduction cost sharing contract influences the order quantity, profits of supply chain members and the whole supply chain. Zhou et al., 2016 considered a two-level low-carbon supply chain model and investigated how does emission reduction and advertising cost sharing contracts impact the optimal decisions. Giri and Sarker, 2016 proposed a wholesale price discount contract to coordinate a two-level supply chain model consisting of two competitive retailers and a single manufacturer with production disruptions. Although they concluded that

the effectiveness of this scheme could not be ensured whenever retailers are asymmetric and engaged in price and service competition simultaneously. Lee et al., 2016 examined a vendor-managed inventory system under limited storage capacity, and showed that a stockout-cost sharing contract provides the same performance as the integrated supplier-customer system if and only if reservation cost equals to the minimum total cost of integrated supply chain. Aljazzar et al., 2017 coordinated a three-level supply chain model with discount dependent demand, permissible delay in payments and price discount contracts. Bai et al., 2017 proposed a contract of sharing promotional cost and revenue to coordinate a two-level sustainable supply chain system. Xu et al., 2017 studied a two-echelon supply chain model under cap-and-trade regulation and implemented both wholesale price and cost sharing contract aimed at reducing carbon emissions. Hosseini-Motlagh et al., 2018 developed a compensation based wholesale price discount contract in a green supply chain model for two substitutable products with quality and warranty period dependent demand. Xu et al., 2018 investigated the role of a price discount contract in coordinating a dual-channel supply chain under carbon emission capacity regulation, with consumer demand in both online and offline channels influenced by the product's selling price. Nouri et al., 2018 proposed a compensation-based wholesale price contract between the manufacturer and the retailer where the customer demand is stochastic and dependent on innovation and promotional efforts. Taleizadeh et al., 2019 studied three different incentive contracts viz. credit payment, market segmentation and a quantity discount policy in a two-level pricing-inventory model. Chakraborty et al., 2019 studied that a cost sharing contract between a retailer and two competitive manufacturers is beneficial than the wholesale price contract to improve the quality of the product and their profitability. Ebrahimi et al., 2019 coordinated promotional effort and inventory decisions in a two-level supply chain model through a delay in payment contract. Ebrahimzadeh-Afruzi and Aliahmadi, 2020 implemented a trade credit mechanism in a two-echelon supply chain model for deteriorating items with time-varying customer demand sensitive to the retail price and quality of the product. Sarkar et al., 2020 suggested a price discount coordination mechanism in a two-level supply chain with price sensitive customer demand to encourage the supply chain players to take part in joint decision-making strategy. Zu et al., 2021 analyzed the effectiveness of two coordination contracts viz. the consignment contract and the wholesale price contract for carbon emissions

reduction in a single-manufacturer single-retailer supply chain model. Das et al., 2021 analyzed a two-level supply chain model with price dependent demand, and proposed a wholesale price discount contract which effectively coordinates the ordering and pricing strategies of the supply chain members. Nouri et al., 2021 developed a price discount coordination scheme to coordinate a two-level supply chain model with a promotional effort sensitive demand under periodic review replenishment policy. Gholamian and Ebrahimzadeh-Afruzi, 2021 proposed a credit payment incentive and a wholesale price discount contract in order to coordinate a single-supplier single-retailer supply chain model with price dependent demand. Yang et al., 2021 explored the optimal cooperation strategy between an upstream supplier and two competing manufacturers considering a wholesale price contract and manufacturers' technology investment. In order to reduce products' carbon emissions, Zu et al., 2021 analyzed a single-manufacturer single-retailer supply chain model under two different mechanisms viz. wholesale price contract and consignment contract. Hou et al., 2022 made a comparison analysis of the wholesale price contract and revenue-sharing contract in a supply chain model with customer demand depending on retail price and manufacturer rebate. Zhang et al., 2022 incorporated consumer reference price effect in a green supply chain model and performed a comparative analysis between wholesale price contract and cost-sharing contract. Qiu et al., 2022 implemented three coordination strategies such as wholesale price contract, cost-sharing contract and two-part tariff contract in order to coordinate a single-manufacturer single-retailer supply chain model with price and consumer reference quality under an O2O environment. Zhang et al., 2023 incorporated a wholesale price contract and a cost sharing contract in a dyadic green supply chain model and examined that which coordination mechanism is more effective to improve the greenness of the product and boost the market demand.

2.6 Single-manufacturer multi-retailer model

In today's competitive business world, it is hard to find that one manufacturer sells a product to a single retailer. It is more practical to focus on the scenarios where the manufacturer supplies the product to several retailers or trading for multiple items. To come closer to the reality, focusing on multi-retailer models has become a great

topic of interest for the researchers (Darwish and Odah, 2010, Krichen et al., 2011). Jha and Shanker, 2013 considered a single-vendor multi-buyer supply chain model under controllable lead time and applied the Lagrangian multiplier technique to evaluate optimal results under service level constraints. Poorbagheri and Akhavan Niaki, 2014 studied a VMI model composed of a single vendor and multiple retailers under stochastic demand. Giri and Roy, 2015 developed a single-vendor multi-buyer inventory model considering controllable lead time to maximize the profit of the whole supply chain. Giri and Roy, 2015 considered a supply chain model consisting of a single manufacturer and multiple retailers with price sensitive customer demand. They found that lead time reduction by paying extra crashing cost does not affect the retail price significantly but enhances the entire system profit. Chen and Sarker, 2017 investigated a single-manufacturer multi-retailer production-inventory model for deteriorating items with price sensitive demand under just-in-time delivery environment. They solved the model using particle swarm optimization (PSO) and quantum-behaved PSO (QBPSO) techniques. Majumder et al., 2018 studied a single-vendor multi-buyer supply chain model with variable production rate and controllable lead time reduction where the production cost at the vendor is a function of the production rate. Chan et al., 2018 proposed a coordination mechanism in a single-vendor multi-buyer supply chain model with stochastic demand, and synchronized the manufacturer's production cycle and retailers' ordering cycle. Uthayakumar and Kumar, 2019 studied a single-vendor multi-buyer integrated model with stochastic demand and controllable lead time. Ben-Daya et al., 2019 developed a single manufacturer multi-retailer closed-loop supply chain model with an environment-friendly approach of remanufacturing the used products under consignment stock policy. Esmaeili and Nasrabadi, 2021 presented a single-vendor multi-retailer supply chain model for deteriorating items with trade credit and inflationary conditions, where the demand is price sensitive. Najafnejhad et al., 2021 used an imperialist competitive algorithm to solve a single-vendor multi-retailer inventory model under vendor managed inventory policy considering upper limits of inventories as decision variables. Nandra et al., 2021 studied a single-vendor multi-buyer model that took into account variable production cost, imperfect items and environmental factors. Malleeswaran and Uthayakumar, 2022 introduced a discrete investment for ordering cost reduction in a single-manufacturer

multi-retailer EPQ model with green and environmental sensitive consumer demand and reworking system under carbon emissions policies.

2.7 Greening investment

With the rapid progress of industrialization, environmentalists are expressing great concerns regarding its environmental repercussions. In this regard, organizations are showing a great awareness and adopting a greener path to reduce their environmental impacts. Numerous research works have been done in this direction which reveal that such green initiatives not only glorify the image of companies in front of the customers but contribute to create a sustainable future. Ghosh and Shah, 2015 proposed a cost sharing contract to motivate the supply chain members to adopt green initiatives and studied its impact on the optimal pricing and greening strategies and the optimal profitability of a two-level supply chain model. Li et al., 2016 made a comparative analysis between single channel and dual-channel, and achieved the optimal strategies for pricing and greening in a two-level green supply chain model. Basiri and Heydari, 2017 studied a two-stage green supply chain model considering two types of substitutable products, traditional non-green and green products. They proposed a collaborative scheme which generates profit level of the supply chain very close to the centralized structure. Song and Gao, 2018 analyzed how two different revenue-sharing contracts such as a retailer-led contract and a revenue sharing contract based on the bargaining power of the members, affect the optimal greening level and overall profitability of a green supply chain model. Nielsen et al., 2019 studied the impact of government incentive schemes and different game structures in a green supply chain model with two-period procurement setting. Yan et al., 2021 considered a green supply chain model for two cross market firms with price, green, and promotional effort sensitive demand and derived optimal strategies in order to maximize their profitability, and analyzed the impacts of cooperative promotion on the supply chain. Heydari et al., 2021 considered a two-level green supply chain model with a price and green sensitive demand and introduced a hybrid coordination scheme combining greening cost sharing contract and revenue sharing contract. Yang et al., 2022 investigated a green supply chain model where the manufacturer and the retailer invest in green manufacturing

and green marketing efforts, respectively. Das et al., 2023 proposed a cost-sharing contract and a profit-sharing contract to coordinate a two-level dual-channel supply chain model under price and green sensitive customer demand and derived the optimal pricing and greening strategies.

2.8 Carbon emissions

Carbon emissions reduction as well as maintaining environmental cleanliness has now become a challenging but essential task for most of the firms. Researchers have paid close attention and done numerous research works in order to find optimal business strategies enhancing environmental performance. Sarkar et al., 2015 incorporated carbon emission cost in a single-vendor single-buyer supply chain model with unequal batch shipment policy. Wang et al., 2016 studied a two-level supply chain model with retail price and carbon emission reduction rate dependent demand under cost sharing and wholesale price contracts. Ghosh et al., 2017 implemented a strict carbon cap policy in a two-echelon supply chain, and determined optimal ordering and shipment policies. Tiwari et al., 2018 considered an integrated inventory model for imperfect deteriorating items and aimed to minimize the total cost and carbon emission cost reduction. Tiwari et al., 2019 demonstrated a fuzzy green supply chain model for non-instantaneous deteriorating products with carbon emissions and price sensitive demand under a trade credit mechanism. Rout et al., 2020 developed an integrated sustainable supply chain model with various carbon emission regulatory policies considering carbon emission during production process, transportation, warehousing and discarding of scrap items. Qu et al., 2021 studied the impacts of the cap-and-trade regulation on the carbon emissions reduction and warranty policy of the product in a low-carbon supply chain. Jauhari et al., 2021 considered a two-level closed-loop supply chain model with hybrid production system, stochastic demand and take-back incentives under a carbon tax regulation, and minimized the joint total cost of the system aiming at carbon emissions reduction. Shi and Liu, 2022 investigated how the profitability of each supply chain member as well as the entire supply chain, selling price of the product, and carbon emission reduction level are affected by the risk aversion degree of the manufacturer

and carbon tax rate. In addition, they implemented a carbon emission reduction investment sharing contract that effectively enhances the overall performance of the supply chain. Taleizadeh et al., 2022 considered a sustainable EOQ model under price and carbon-emission sensitive demand with partial backordering and partial delay in payment. Halat et al., 2023 studied pricing and inventory competition between two green supply chains considering carbon emissions regulations.

2.9 Sample inspection

Incorporating sample inspection mechanism in supply chain models is experiencing a great deal of attention in the extant literature. Wu et al., 2007 assessed the impact of sample inspection policy and lead time reduction strategies on an integrated vendor-buyer model. Al-Salamah, 2011 proposed a single acceptance sampling scheme in an EOQ inventory model with imperfect production system where destructive testing is applied to identify the defective items and the inspection process is erroneous. Moussawi-Haidar et al., 2013 proposed a special type of sub-lot inspection policy in an integrated EOQ model in which a sample is inspected instead of screening the entire lot and if the number of defective items in the sample does not exceed a pre-defined threshold value, then no more inspection is conducted. If not, the entire lot goes through a 100% screening. Cheikhrouhou et al., 2018 developed an economic order quantity model with sample inspection policy and suggested that if the retailer immediately returned the defective products to the manufacturer with an additional transportation cost, it would be more profitable instead of waiting for the next lot to arrive. Öztürk, 2020 incorporated a sub-lot inspection policy in an EOQ model with misclassification errors and investigated the model in two different situations such as defective items being reworked and the case when the defective items are sold and replaced by new perfect items bought at a higher price. Nakhaeinejad, 2021 developed a classical EOQ inventory model considering a sampling inspection policy where a lot is accepted if the sample inspection results in zero defects. Otherwise, it is sent for a 100% inspection. Tuffaha and AlDurgam, 2021 studied the effects of sample inspection in a two-echelon supply chain model, including process targeting and production lot-sizing. Bose and Guha, 2021 and Sarkar et al., 2022b proposed a trade-credit coordination scheme in a two-level joint

economic lot size model with a sub-lot sample inspection policy under price and greening effort sensitive customer demand. Wu et al., 2023 developed a multiple sampling strategy in which sample inspection is independent at each step and the average sample quantity is minimised by meeting the requisite quality levels and accepting risks.

2.10 Set up cost reduction

Cost reduction strategies have become the most affordable and indispensable ways to ensure flexible manufacturing and better customer satisfaction. In this context, one of these effective policies is the set-up cost reduction technique which plays an important role among all the costs associated with the business structure. Researchers are also investigating pensively to determine optimal cost reduction strategies. Huang et al., 2011 derived the optimal inventory and investment strategies in an integrated production inventory model considering set up cost reduction and compound Poisson process lead time demand. Sarkar and Majumder, 2013 considered a set up cost reduction investment policy in an integrated vendor-buyer model and focused on two different cases such as lead time demand following a normal distribution and next applied a distribution-free approach. Majumder et al., 2017 investigated a vendor-buyer inventory model involving set up cost reduction strategy and quality improvement of the product. Gautam and Khanna, 2018 considered a discrete investment aimed at reducing set-up costs in a sustainable supply chain model with imperfect production.. Malik and Sarkar, 2018 proposed quality improvement and set up cost reduction investment strategies in a multi-product inventory model incorporating continuous review system in a fuzzy environment. Dey et al., 2019a studied an integrated inventory model taking into account quality improvement, discrete investment for set up cost reduction, controllable lead time and environmental issues. Guchhait et al., 2020 examined the impact of set up cost reduction investment and process quality improvement strategies on the optimal production run time of an imperfect production system. Tiwari et al., 2020 and Karthick and Uthayakumar, 2021 developed a two-level, multi-item sustainable inventory model with variable production rate considering investments for set up

cost and carbon emission reduction. Das Roy and Sana, [2021](#) integrated the regular manufacturing system with a process of remanufacturing in a multi-level green supply chain model while an additional capital investment is made by the producer for set up cost reduction. Sarkar et al., [2022a](#) considered an initial set up cost reduction strategy by some discrete type investment in a three-echelon green supply chain model for biodegradable products.

CHAPTER 3

Coordination and defect management strategy for a two-level supply chain under price and sales effort sensitive demand

3.1 Introduction

The world is on the verge of catastrophe due to pollution and other regrettable anti-environmental activities. So, this is the high time to take initiative in reducing environmental degradation. In this regard, supply chain members have been enterprising to reduce their pernicious impact on the environment along with achieving high economic level and maintaining customer satisfaction. Carbon emissions from transportation contribute significantly to global warming. Moreover, waste disposal is one of the major reasons behind environmental degradation as it releases toxic substances and greenhouse gases. Therefore, implementing sustainable strategies for transportation and waste disposal has become an inevitable task for the supply chain players in mitigating environmental impacts. In the real business scenario, production of defective items is an undeniable phenomena. So, efficient management of these defective items will be tactful for waste reduction. Companies can reap the financial benefits of extracting reusable products from these defective products. This chapter* introduces an efficient defect management strategy in a two-level

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supply chain model with price and sales effort dependent market demand. Demand forecasting is one of the crucial issues for smooth running of a supply chain. There are several factors that significantly affect the market demand. Selling price of the product is one of them. Higher selling price can drop the demand or force customers to buy less. In addition to the selling price, retailer's sales effort plays an important role in influencing customer demand. Retailers can enhance customer demand in many different ways viz. hiring more skillful salespeople, increasing advertisement frequency, attractive and large shelf space, etc. Even though this sales effort attracts more customers and makes the product more procurable, it also increases retailer's cost. Thus, to maintain its own profit level as well as customer satisfaction, it becomes a challenging task to the retailer to determine appropriate pricing and sales effort level. It is also of great importance to make these strategies in a coordinated manner to achieve optimal environmental and economic efficiency. In this context, suitable coordination schemes can incentivize the supply chain players to adopt collaborative decision making policies. Now considering all of these significant concerns collectively, the following are some pertinent queries that could come up:

- Does a strategic defect management policy play a significant role in reducing carbon emissions?
- How will it be beneficial to the supply chain players both financially and environmentally?
- Is there any coordination mechanism capable of coordinating the supply chain and persuading all the supply chain members to adopt a strategic defect management policy and global optimal strategy?

To address the questions posed above, in this chapter we investigate a two-echelon supply chain model with imperfect production, strategic defect management policy, and implement an appropriate coordination scheme for channel coordination. Our main goal is to find the optimal defect management, pricing and ordering strategies for the supply chain members.

3.2 Problem Definition and Assumptions

The proposed model considers of a single-manufacturer single-retailer supply chain

where the manufacturer delivers the order quantity in some unequal sized batch shipments. Since the production process is imperfect, the manufacturer transfers the products to the retailer without any quality assurance. So, it is conceivable that each batch contains a certain proportion of defective items. In this aspect, performing an inspection process before selling the product to the end-customers becomes indispensable for the retailer to maintain the customer service level. But, various inescapable reasons like human errors, mishandling, weak maintenance etc. make the inspection process erroneous and results in Type-I and Type-II errors at the retailer's end. Type-I error causes direct economical loss to the retailer as the perfect items are

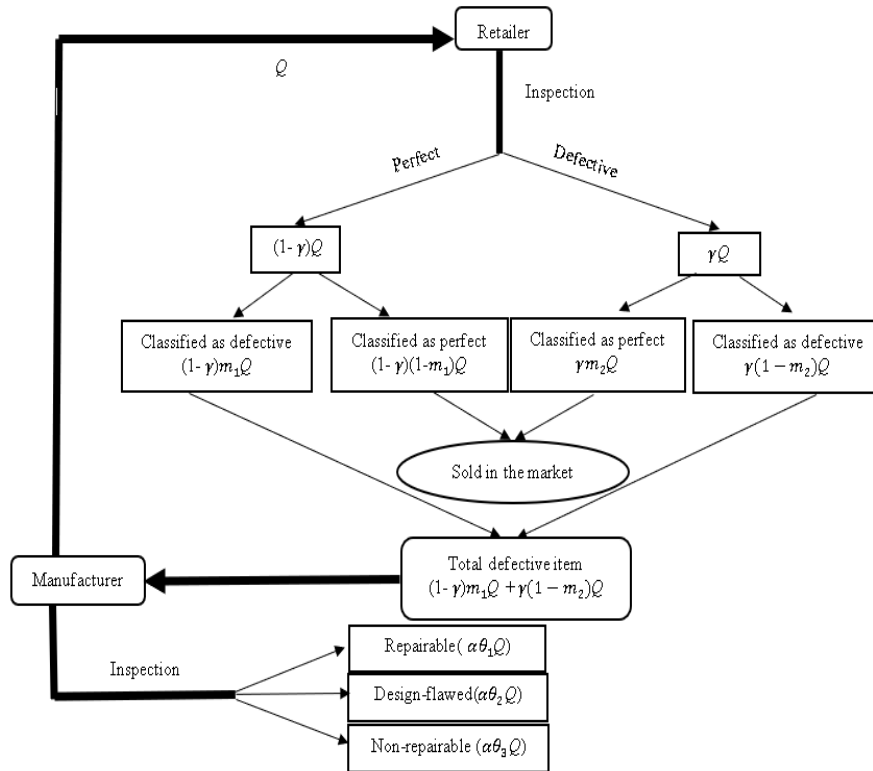


Figure 3.1: Flow of inventory in the system

classified as defective mistakenly and sent back to the manufacturer. In Type-II error, defective items are classified as non-defective and are sold out to the market. It hampers the retailer's relationship with the customer as well as causes financial loss with sales return. All the defective items obtained from inspection are carried in the inventory until the next shipment arrives and then are reverted to the manufacturer. The manufacturer incurs a warranty cost for each defective item. After receiving the returned lot from the retailer, the manufacturer likes to invest on these

defective items to earn more revenue instead of disposing or selling these defective products at a lower price. The manufacturer performs an error-free rigorous inspection process by a very efficient team to categorize the imperfect items as repairable, design-flawed and non-repairable. After design modification, these items are considered as brand new products and sold at the same wholesale price. Repairable items are reworked and then sold at a lower price in a secondary market. Left-over non-repairable items are disposed. The manufacturer bears carbon-emission cost for the disposal of scrap items and each shipment to the retailer. Inventory flow between the supply chain players is illustrated in Figure 3.1. The following assumptions are made to develop the proposed model:

1. The supply chain consists of a single manufacturer and a single retailer who deal for a single product.
2. The manufacturer delivers the order quantity of the retailer in some unequal sized batch shipments and the batch sizes follow a geometric progression with common ratio λ ($1 < \lambda \leq \frac{R}{D}$). The successive deliveries received by the retailer are $q, \lambda q, \lambda^2 q, \dots, \lambda^{n-1} q$ (Giri & Sharma, 2014).
3. The production process at the manufacturer is imperfect and the manufacturer does not perform inspection before delivering the products to the retailer. So, each batch contains a certain proportion of defective items (Cheikhrouhou et al., 2018). The production rate (R) at the manufacturer is higher than the demand rate (D) at the retailer.
4. We assume the customer demand (D) at the retailer is linearly dependent on the retail price of the product and the sales effort level. We take $D = a - \beta p + \mu f$, where $\beta, \mu > 0$ and $a + \mu f > \beta p$ (Li & Liu, 2015).
5. The inspection process at the retailer is erroneous. There is a possibility to classify a non-defective item as defective (Type-I error) and a defective item as non-defective (Type-II error) during the inspection process (Cheikhrouhou et al., 2018; Manna et al., 2020). To meet the end customer demand only with the non-defective items, we assume the screening rate (x) at the retailer much higher than the demand rate (D) (Gautam et al., 2019).
6. Shortages are allowed at the retailer's end and are considered to be completely backlogged i.e., the customers are ready to wait during the shortage period.

7. To avoid shortages during screening process, we assume that $\alpha < \left(1 - \frac{D}{x}\right)$ (Hsu and Hsu, 2013; Gautam et al., 2019).
8. To reduce carbon-emissions, after completion of the inspection process, the retailer holds the accumulated defective items till the next shipment arrives, and he returns them to the manufacturer in the same vehicle (Gautam et al., 2019). So the retailer does not bear any transportation cost or carbon-emission cost to send back the defective items.
9. The manufacturer pays the warranty cost for each returned imperfect item to retain his goodwill (Rad et al., 2018).
10. For various products such as computer and electronic gadgets, defective items can be reworked and sold them to earn further revenue rather disposing them off. Here, on receiving the returned defective items, the manufacturer conducts a strong investigation to categorize the products into three types such as design-flawed (sold after design modification), repairable (sold in the secondary market after reworking) and non-repairable (disposed).
11. A variable transportation cost incurs at the manufacturer who also bears fixed and variable carbon-emission costs for upstream and downstream transports.
12. The retailer invests on boosting the market demand through sales efforts like advertisement, promotion, etc. The sales effort cost of the retailer is a convex and quadratic function of the sales effort level(f). We take it as gf^2 (Li and Liu, 2015; Heydari and Asl-Najafi, 2021).

3.3 Notation

The following notations are used for developing the proposed model:

R	: Production rate (units/unit time)
w	: Manufacturer's unit wholesale price (\$/unit)
A_m	: Set-up cost of the manufacturer (\$/set-up)
h_m	: Holding cost of the manufacturer (\$/unit/unit time)

θ_1	: Fraction of repairable items
θ_2	: Fraction of design-flawed items
θ_3	: Fraction of scrap items ($\theta_1 + \theta_2 + \theta_3 = 1$)
c_1	: Unit selling price of reworked items (\$/unit)
c_2	: Unit selling price of design-flawed items (\$/unit)
c_3	: Unit salvage price (\$/unit)
c_w	: Unit warranty cost for defective items (\$/unit)
c_m	: Unit production cost (\$/unit)
c_v	: Unit variable carbon-emission cost (\$/unit)
c_e	: Unit fixed carbon-emission cost (\$/unit)
T_v	: Unit variable transportation cost (\$/unit)
s_2	: Unit inspection cost of the manufacturer(\$/unit)
R_w	: Unit rework cost (\$/unit)
R_s	: Unit design cost (\$/unit)
R_d	: Unit disposal cost (\$/unit)
ϕ	: Price discount ratio offered by the manufacturer
ξ	: Credit period offered by the manufacturer to the retailer (time)
I_m	: Interest rate of the manufacturer
T	: Cycle length (time)
D	: Demand rate (units/unit time)
a	: Basic market demand (units/unit time)
β	: Price elasticity of customer demand
μ	: Sales effort sensitivity of the customer demand
Q	: Total order quantity (= nq) (units)
x	: Screening rate (units/unit time)
λ	: Rate of increasing batch size ($1 < \lambda \leq \frac{R}{D}$)
γ	: Percentage of defective items
m_1	: Percentage of classifying perfect items as defective
m_2	: Percentage of classifying defective items as perfect
g	: Sales effort cost parameter (\$)
α	: Total items classified as defective (units/cycle)
A_r	: Ordering cost of the retailer (\$/order)
h_{r1}	: Holding cost of perfect items of the retailer (\$/unit/unit time)
$h_{r2}(h_{r1} \geq h_{r2})$: Holding cost of defective of the retailer (\$/unit/unit time)
c_b	: Shortage cost at the retailer (\$/unit/unit time)
c_r	: Cost of rejecting perfect items (Type-I error)(\$/unit)
$c_a(c_a > c_r)$: Cost of accepting defective items(Type-II error) (\$/unit)

s_1	: Inspection cost of the retailer (\$/unit)
t_{i1}	: Inventory depletion time after i -th shipment
t_{i2}	: Shortage building time after i -th shipment
t_{i3}	: Shortage fulfillment time after i -th shipment
t_i	: Inspection time after i -th shipment
$\psi(0 \leq \psi \leq 1)$: Bargaining power of the retailer over the manufacturer
I_r	: Interest rate of the retailer
AP_m	: Average profit of the manufacturer (\$/ unit time)
AP_r	: Average profit of the retailer (\$/unit time)
AP_s	: Average profit of the whole supply chain (\$/unit time)
$(.)^d$: Decision variable in decentralized policy
$(.)^c$: Decision variable in centralized policy
$(.)^{cp}$: Decision variable in coordinated system
<hr/>	
Decision variables	
<hr/>	
n	: Number of batch shipments (positive integer)
q	: Smallest batch size (units)
p	: Unit retail price (\$/unit)
b	: Shortage quantity (units)
f	: Sales effort level
<hr/>	

3.4 Model Development

We suppose that the retailer places an order of size Q and the manufacturer delivers it in n unequal-sized batch shipments. The batch sizes are $q, \lambda q, \lambda^2 q, \dots, \lambda^{n-1} q$. Therefore, $Q = \left(\frac{\lambda^n - 1}{\lambda - 1}\right)q = kq$, where $k = \left(\frac{\lambda^n - 1}{\lambda - 1}\right)$.

A. Manufacturer's profit

The inventory level of the manufacturer is depicted in Figure 3.2. The cost components of the manufacturer are as follows:

Set-up cost of the manufacturer = A_m .

Production cost = $c_m kq$.

Inventory holding cost of the manufacturer obtained from the Figure 3.2

$$= h_m \left[\frac{q^2(1-\alpha)(\lambda^n - 1)(\lambda^{n-1} - 1)\lambda}{D(\lambda + 1)(\lambda - 1)^2} + \frac{q^2(\lambda^n - 1)(2\lambda - \lambda^n - 1)}{2R(\lambda - 1)^2} \right] \text{ (See Appendix A.1 in 3.7).}$$

Warranty cost paid by the manufacturer for defective items reverted by the retailer
= $c_w \alpha k q$.

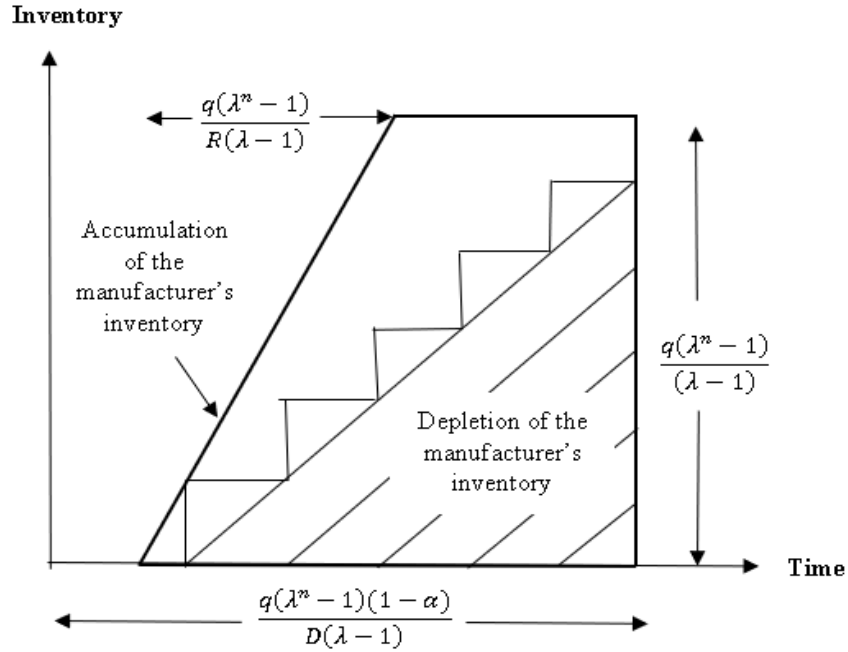


Figure 3.2: Accumulation and depletion of the manufacturer's inventory

Screening cost to classify the returned lot = $s_2 \alpha k q$.

Rework cost of the manufacturer to prepare the repairable items for sale in a secondary market = $R_w \theta_1 \alpha k q$.

Design cost borne for design rectification = $R_s \theta_2 \alpha k q$.

Disposal cost for non-repairable items = $R_d \theta_3 \alpha k q$.

Variable carbon emission cost due to transshipment between the retailer and the manufacturer, and disposal of the waste products = $c_v k q + c_v \alpha k q + c_e \theta_3 \alpha k q$.

Variable transportation cost = $T_v k q + T_v \alpha k q$.

Total cost of the manufacturer

$$\begin{aligned}
 &= A_m + c_m k q + c_w \alpha k q + s_2 \alpha k q + R_w \theta_1 \alpha k q + R_s \theta_2 \alpha k q + R_d \theta_3 \alpha k q + c_v k q + c_v \alpha k q \\
 &+ c_e \theta_3 \alpha k q + h_m \left[\frac{q^2 (1 - \alpha) (\lambda^n - 1) (\lambda^{n-1} - 1) \lambda}{D(\lambda + 1) (\lambda - 1)^2} + \frac{q^2 (\lambda^n - 1) (2\lambda - \lambda^n - 1)}{2R(\lambda - 1)^2} \right] \\
 &+ T_v k q + T_v \alpha k q
 \end{aligned}$$

Manufacturer's total revenue from the sale of perfect items, design-flawed items,

repairable items, and finally scrap items = $w(1 - \alpha)kq + c_1\theta_1\alpha kq + c_2\theta_2\alpha kq + c_3\theta_3\alpha kq$.

Total profit of the manufacturer (TP_m) = Total revenue – Total cost

$$\begin{aligned} &= w(1 - \alpha)kq + c_1\theta_1\alpha kq + c_2\theta_2\alpha kq + c_3\theta_3\alpha kq - A_m - c_mkq - c_w\alpha kq - s_2\alpha kq \\ &\quad - R_w\theta_1\alpha kq - R_s\theta_2\alpha kq - R_d\theta_3\alpha kq - c_vkq - c_v\alpha kq - c_e\theta_3\alpha kq - T_vkq - T_v\alpha kq \\ &\quad - h_m \left[\frac{q^2(1 - \alpha)(\lambda^n - 1)(\lambda^{n-1} - 1)\lambda}{D(\lambda + 1)(\lambda - 1)^2} + \frac{q^2(\lambda^n - 1)(2\lambda - \lambda^n - 1)}{2R(\lambda - 1)^2} \right] \end{aligned}$$

$$\text{Cycle length}(T) = \frac{kq(1 - \alpha)}{D}.$$

Therefore, average profit of the manufacturer is

$$\begin{aligned} &AP_m(n) \\ &= (c_1\theta_1 + c_2\theta_2 + c_3\theta_3 - s_2 - c_w - c_v - T_v - c_e\theta_3 - R_w\theta_1 - R_s\theta_2 - R_d\theta_3) \frac{\alpha D}{1 - \alpha} \\ &+ wD - \frac{A_mD}{kq(1 - \alpha)} - \frac{(c_m + c_v + T_v)D}{1 - \alpha} - h_m \left[\frac{q\lambda(\lambda^{n-1} - 1)}{(\lambda^2 - 1)} + \frac{qD(2\lambda - \lambda^n - 1)}{2R(\lambda - 1)(1 - \alpha)} \right] \end{aligned} \quad (3.1)$$

B. Retailer's profit

After receiving each delivery, the retailer posts an inspection process to identify the defective ones which are returned to the manufacturer in the same vehicle at the time of next shipment and the perfect items are sold in the market. But, there is a possibility of false acceptance and false rejection due to erroneous inspection. Because of defective items in each batch, shortages occur at the retailer's end and are completely backlogged. The behavior of the inventory level of the retailer is illustrated in Figure 3.3.

Total number of items that are classified as defective and are returned to the manufacturer = $[(1 - \gamma)m_1 + \gamma(1 - m_2)]kq = \alpha kq$, where $\alpha = [(1 - \gamma)m_1 + \gamma(1 - m_2)]$.

Total number of items that are classified as non-defective and are sold in the market = $(1 - \alpha)kq$.

Ordering cost of the retailer = A_r .

Screening cost of the retailer = s_1kq .

The retailer only pays for the perfect items. So, purchase cost of the retailer

$$= w(1 - \alpha)kq.$$

Misclassification cost incurred due to false rejection and false acceptance

$$= c_r(1 - \gamma)m_1kq + c_a\gamma m_2kq.$$

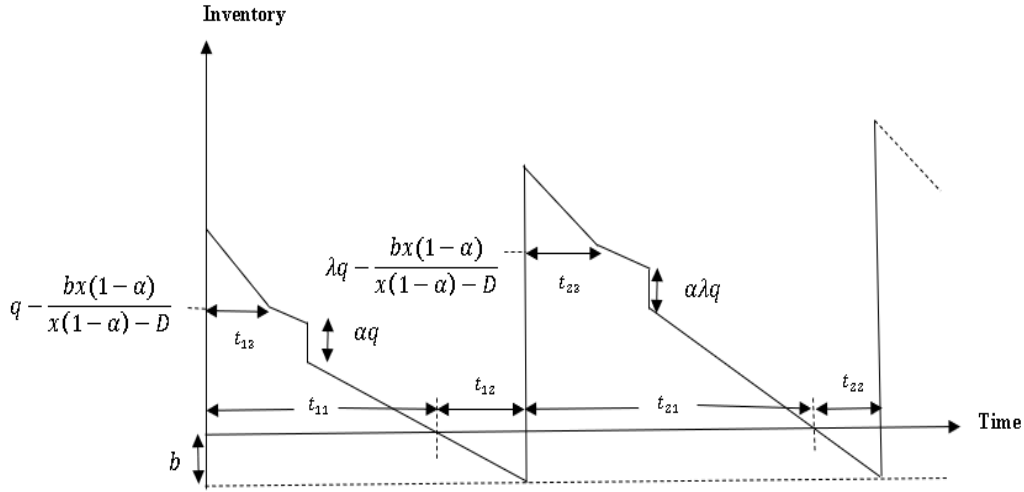


Figure 3.3: Inventory of the retailer

Sales effort cost per cycle = gf^2 .

Similar to Gautam et al., 2019, inventory holding cost of the retailer

$$\begin{aligned}
 &= h_{r1} \sum_{i=0}^{n-1} \left[\frac{\lambda^i q(1-\alpha)}{2} \left(\frac{\lambda^i q(1-\alpha) - b}{D} + \frac{b}{x(1-\alpha) - D} \right) - \frac{bx(1-\alpha)[\lambda^i q(1-\alpha) - b]}{2D[x(1-\alpha) - D]} \right] \\
 &\quad + h_{r2} \sum_{i=0}^{n-1} \left[\lambda^i \alpha q \cdot \frac{\lambda^i q(1-\alpha)}{D} \right] \\
 &= \frac{h_{r1}}{2} \left[\frac{q^2(1-\alpha)^2 l}{D} - \frac{qb(1-\alpha)k}{D} + \frac{kqb(1-\alpha)}{x(1-\alpha) - D} - \frac{bx(1-\alpha)}{x(1-\alpha) - D} \left(\frac{kq(1-\alpha)}{D} - \frac{nb}{D} \right) \right] \\
 &\quad + \frac{h_{r2}\alpha(1-\alpha)lq^2}{D}, \quad \text{where } l = \frac{\lambda^{2n} - 1}{\lambda^2 - 1}
 \end{aligned}$$

$$\text{and backordering cost} = \frac{nc_b b^2}{2} \left[\frac{1}{D} + \frac{1}{x\left(1-\alpha-\frac{D}{x}\right)} \right].$$

$$\text{Revenue of the retailer} = p(1-\alpha)kq.$$

Therefore, the total profit of the retailer (TP_r) = Total revenue – Total cost

$$\begin{aligned}
 &= p(1-\alpha)kq - A_r - s_1 kq - w(1-\alpha)kq - c_r(1-\gamma)m_1 kq - c_a \gamma m_2 kq \\
 &\quad - \frac{h_{r1}}{2} \left[\frac{q^2(1-\alpha)^2 l}{D} - \frac{qb(1-\alpha)k}{D} + \frac{kqb(1-\alpha)}{x(1-\alpha) - D} - \frac{bx(1-\alpha)}{x(1-\alpha) - D} \left(\frac{kq(1-\alpha)}{D} - \frac{nb}{D} \right) \right] \\
 &\quad - \frac{h_{r2}\alpha(1-\alpha)lq^2}{D} - \frac{nc_b b^2}{2} \left[\frac{1}{D} + \frac{1}{x\left(1-\alpha-\frac{D}{x}\right)} \right] - gf^2 T
 \end{aligned}$$

Hence the average profit of the retailer is $AP_r(q, p, b, f) = \frac{TP_r}{T}$

$$\begin{aligned}
&= pD - wD - \frac{A_r D}{kq(1-\alpha)} - \frac{(s_1 + c_r(1-\gamma)m_1 + c_a\gamma m_2)D}{1-\alpha} - \frac{h_{r2}\alpha lq}{k} \\
&\quad - \frac{h_{r1}}{2} \left[\frac{ql(1-\alpha)}{k} - b + \frac{bD}{x(1-\alpha) - D} - \frac{bx(1-\alpha)}{x(1-\alpha) - D} + \frac{nx b^2}{kq[x(1-\alpha) - D]} \right] \\
&\quad - \frac{nc_b b^2 D}{2kq(1-\alpha)} \left[\frac{1}{D} + \frac{1}{x(1-\alpha) - D} \right] - g f^2 \tag{3.2}
\end{aligned}$$

3.4.1 Decentralized Model

In the decentralized decision-making structure, both the manufacturer and the retailer intend to optimize their own individual profits without worrying about others. A Stackelberg gaming approach is adopted to find the optimal decisions considering the retailer as the leader and the manufacturer as the follower. At first, the manufacturer provides his best response n^* optimizing his own profit. Then, taking this reaction into consideration, the retailer determines his optimal retail price, batch size, backorder quantity and sales effort level optimizing his profit function. Now, taking n as real, not just an integer, we differentiate (3.1) with respect to n and get

$$\frac{\partial AP_m}{\partial n} = \frac{\lambda^n \log \lambda}{2q(1-\alpha)(\lambda-1)} \left[\frac{2A_m D(\lambda-1)^2}{(\lambda^n-1)^2} + \frac{h_m q^2 [D(\lambda+1) - 2R(1-\alpha)]}{R(\lambda+1)} \right] \tag{3.3}$$

$$\frac{\partial^2 AP_m}{\partial n^2} = \frac{\lambda^n (\log \lambda)^2}{2q(1-\alpha)(\lambda-1)} \left[\frac{2A_m D(\lambda-1)^2(\lambda^n-3)}{(\lambda^n-1)^3} - \frac{h_m q^2 [2R(1-\alpha) - D(\lambda+1)]}{R(\lambda+1)} \right] \tag{3.4}$$

It can be seen from above that $\frac{\partial^2 AP_m}{\partial n^2} < 0$ if

$$2RA_m D(\lambda+1)(\lambda-1)^2(\lambda^n-3) < h_m q^2 (\lambda^n-1)^3 [2R(1-\alpha) - D(\lambda+1)] \tag{3.5}$$

Therefore, if this condition holds, then the average profit function of the manufacturer is concave in real n and the optimal value of n can be obtained by solving $\frac{\partial AP_m}{\partial n} = 0$ as

$$n^* = \frac{\log \left[1 + \sqrt{\frac{2RA_m D(\lambda-1)(\lambda^2-1)}{h_m q^2 [2R(1-\alpha) - D(\lambda+1)]}} \right]}{\log \lambda} = \frac{\log \left[1 + \frac{X_1}{q} \right]}{\log \lambda}$$

$$\text{where } X_1 = \sqrt{\frac{2RA_mD(\lambda-1)(\lambda^2-1)}{h_m[2R(1-\alpha)-D(\lambda+1)]}} \quad (3.6)$$

After taking this reaction of the manufacturer, the retailer optimizes its batch size, retail price, backorder quantity and sales effort level. Substituting the value of n^* in equation (3.2), the average profit of the retailer can be obtained as

$$\begin{aligned} AP_r(q, p, b, f) = & pD - wD - \frac{A_rD(\lambda-1)}{(1-\alpha)X_1} - \frac{\rho_1D}{(1-\alpha)} - \frac{[2\alpha h_{r2} + h_{r1}(1-\alpha)][2q + X_1]}{2(\lambda+1)} \\ & + h_{r1}b - \frac{xb^2(h_{r1} + c_b)(\lambda-1) \log \left[1 + \frac{X_1}{q}\right]}{2X_1 \log \lambda [x(1-\alpha) - D]} - gf^2 \end{aligned} \quad (3.7)$$

where $\rho_1 = (s_1 + c_r(1-\gamma)m_1 + c_a\gamma m_2)$

To simplify the mathematical derivations, some new notations compacted by other notations are used in this section and are given in Appendix A.2 in 3.7.

The Hessian matrix associated with the average profit function $AP_r(q, p, b, f)$ is given by

$$H = \begin{pmatrix} \frac{\partial^2 AP_r}{\partial b^2} & \frac{\partial^2 AP_r}{\partial b \partial q} & \frac{\partial^2 AP_r}{\partial b \partial p} & \frac{\partial^2 AP_r}{\partial b \partial f} \\ \frac{\partial^2 AP_r}{\partial q \partial b} & \frac{\partial^2 AP_r}{\partial q^2} & \frac{\partial^2 AP_r}{\partial q \partial p} & \frac{\partial^2 AP_r}{\partial q \partial f} \\ \frac{\partial^2 AP_r}{\partial p \partial b} & \frac{\partial^2 AP_r}{\partial p \partial q} & \frac{\partial^2 AP_r}{\partial p^2} & \frac{\partial^2 AP_r}{\partial p \partial f} \\ \frac{\partial^2 AP_r}{\partial f \partial b} & \frac{\partial^2 AP_r}{\partial f \partial q} & \frac{\partial^2 AP_r}{\partial f \partial p} & \frac{\partial^2 AP_r}{\partial f^2} \end{pmatrix} = \begin{pmatrix} V_3 & U_7 & -V_1X_4 & -V_1Y_4 \\ U_7 & V_4 & \beta V_2 & -\mu V_2 \\ -V_1X_4 & \beta V_2 & -2\beta + Z_8 & \mu - U_5 \\ -V_1Y_4 & -\mu V_2 & \mu - U_5 & U_6 - 2g \end{pmatrix} \quad (3.8)$$

The first order principal minor = $-\frac{x(\lambda-1)(c_b+h_{r1})Z_6}{X_1Z_1 \log \lambda} < 0$, since $X_1, Z_1, Z_6 > 0$.

The second order principal minor

$$\begin{aligned} &= V_3V_4 - U_7^2 \\ &= X_1^2 \left[\left(2 + \frac{X_1}{q}\right) \log \left(1 + \frac{X_1}{q}\right) - 2 \right] \\ &> 0 \text{ if } \log \left(1 + \frac{X_1}{q}\right) > 1 \text{ as } q, X_1 > 0 \end{aligned} \quad (3.9)$$

The third order principal minor = $\begin{vmatrix} V_3 & U_7 & -V_1X_4 \\ U_7 & V_4 & \beta V_2 \\ -V_1X_4 & \beta V_2 & -2\beta + Z_8 \end{vmatrix}$

$$= (2\beta - Z_8)(U_7^2 - V_3V_4) - \beta^2V_2^2V_3 - 2\beta V_1V_2X_4U_7 - V_1^2V_4X_4^2$$

which is negative if

$$(2\beta - Z_8)(U_7^2 - V_3V_4) < \beta^2V_2^2V_3 + 2\beta V_1V_2X_4U_7 + V_1^2V_4X_4^2 \quad (3.10)$$

The fourth order principal minor

$$\begin{aligned} &= \begin{vmatrix} V_3 & U_7 & -V_1X_4 & -V_1Y_4 \\ U_7 & V_4 & \beta V_2 & -\mu V_2 \\ -V_1X_4 & \beta V_2 & -2\beta + Z_8 & \mu - U_5 \\ -V_1Y_4 & -\mu V_2 & \mu - U_5 & U_6 - 2g \end{vmatrix} \\ &= U_5^2(U_7^2 - V_3V_4) + V_1^2Y_4^2(\beta^2V_2^2 + 2\beta V_4 - V_4Z_8) - (U_6 - 2g)V_5 - \mu^2V_7 + 2\mu V_1Y_4V_6 \\ &\quad + 2U_5V_8 \end{aligned}$$

which is positive if

$$U_5^2(U_7^2 - V_3V_4) + V_1^2Y_4^2(\beta^2V_2^2 + 2\beta V_4 - V_4Z_8) + 2(\mu V_1Y_4V_6 + U_5V_8) > (U_6 - 2g)V_5 + \mu^2V_7 \quad (3.11)$$

Due to mathematical complexity, it is very difficult to prove analytically that the Hessian matrix is negative definite. We will check the concavity behaviour of the profit function numerically. However, if the conditions in (3.9), (3.10) and (3.11) are satisfied, then AP_r is jointly concave with respect to q , p , b and f . So then there exist unique optimal values of the decision variables q , p , b and f giving global maximum of AP_r .

Proposition 3.1 At the equilibrium, the optimal batch size (q) and optimal backorder quantity (b) of the retailer are given by

$$q^* = \frac{-X_1 + \sqrt{X_1^2 + \frac{2b(\lambda^2-1)X_1V_1}{Z_1Z_2\log\lambda}}}{2} \quad (3.12)$$

$$b^* = \frac{h_{r1}X_1 \log \lambda [x(1-\alpha) - D]}{x(\lambda-1)(c_b + h_{r1}) \log \left(1 + \frac{X_1}{q}\right)} \quad (3.13)$$

Proof. Please see Appendix A.3 in 3.7 for the proof and the notations used in the above proposition.

Proposition 3.2 At the equilibrium, the optimal retail price (p) and optimal sales effort level (f) of the retailer are the positive roots of the equations

$$\begin{aligned} & \left[(1 - \alpha) \{ bV_1 X_4 - 2(D + \beta w - \beta p) \} - 2\rho_1 \beta \right] (\lambda + 1) X_2 Z_4^{\frac{3}{2}} \\ = & \beta \left[2(\lambda + 1) A_r X_2^2 U_4 Z_4 + RDZ_2 (1 - \alpha)^2 (\lambda - 1) \right] \end{aligned} \quad (3.14)$$

$$\begin{aligned} \text{and} & \left[(1 - \alpha) \{ 2\mu(p - w) - 4gf - bV_1 Y_4 \} - 2\rho_1 \mu \right] (\lambda + 1) X_2 Z_4^{\frac{3}{2}} \\ = & \mu \left[2(\lambda + 1) A_r X_2^2 U_4 Z_4 + RDZ_2 (1 - \alpha)^2 (\lambda - 1) \right] \end{aligned} \quad (3.15)$$

respectively.

Proof. Please see Appendix A.4 in 3.7 for the proof and the notations used in the above proposition.

Due to the presence of higher order non-linear terms in equations (3.14) and (3.15), optimal values of p and f are not possible to derive explicitly. Consequently, when the corresponding Hessian matrix is negative definite, the average profit functions of the manufacturer and the retailer in the decentralized model can be optimized and optimal values of the decision variables can be achieved using the following solution algorithm:

- Step 1. Substitute the value of q from equation (3.12) in (3.13). Then, b becomes a function of p and f .
- Step 2. Put this value of b again in equation (3.12). Then q becomes a function of p and f .
- Step 3. Substitution of these values of q and b in equations (3.14) and (3.15) provides two non-linear equations in p and f . Solve these equations to obtain p^* and f^* , the optimal values of p and f , respectively.
- Step 4. Put these values of p^* and f^* in (3.12) and (3.13) to obtain q^* and b^* , the optimal values of q and b , respectively.
- Step 5. Substitute these optimal values of q^* , p^* , b^* and f^* in equation (3.7) and (3.6) to obtain the optimal average profit of the retailer, and calculate the optimal number(real) of batch shipments (n^0).

$$\text{Step 6. Set } n^* = \begin{cases} \lfloor n^0 \rfloor, & \text{if } AP_m(\lfloor n^0 \rfloor) > AP_m(\lceil n^0 \rceil) \\ \lceil n^0 \rceil, & \text{if } AP_m(\lfloor n^0 \rfloor) < AP_m(\lceil n^0 \rceil) \end{cases}$$

Step 7. Using the value of n^* , calculate the optimal average profit of the manufacturer from equation (3.1).

3.4.2 Centralized Model

In the centralized scenario, the manufacturer and the retailer jointly act as a single decision maker in order to achieve collaborative advantages. They unitedly decide the optimal number of batch shipments, smallest batch size, retail price, backorder quantity and sales effort level which maximize the profit of the whole supply chain. The average profit of the whole supply chain is given by

$$\begin{aligned}
& AP_s(n, q, p, b, f) \\
&= (c_1\theta_1 + c_2\theta_2 + c_3\theta_3 - s_2 - c_w - c_v - T_v - c_e\theta_3 - R_w\theta_1 - R_s\theta_2 - R_d\theta_3) \frac{\alpha D}{1 - \alpha} \\
&+ pD - \frac{A_m D}{kq(1 - \alpha)} - \frac{(c_m + c_v + T_v)D}{1 - \alpha} - h_m \left[\frac{q\lambda(\lambda^{n-1} - 1)}{(\lambda^2 - 1)} + \frac{qD(2\lambda - \lambda^n - 1)}{2R(\lambda - 1)(1 - \alpha)} \right] \\
&- \frac{A_r D}{kq(1 - \alpha)} - \frac{(s_1 + c_r(1 - \gamma)m_1 + c_a\gamma m_2)D}{1 - \alpha} - \frac{nc_b b^2 D}{2kq(1 - \alpha)} \left[\frac{1}{D} + \frac{1}{x(1 - \alpha) - D} \right] \\
&- \frac{h_{r1}}{2} \left[\frac{ql(1 - \alpha)}{k} - b + \frac{bD}{x(1 - \alpha) - D} - \frac{bx(1 - \alpha)}{x(1 - \alpha) - D} + \frac{nx b^2}{kq[x(1 - \alpha) - D]} \right] \\
&- \frac{h_{r2}\alpha l q}{k} - g f^2 \tag{3.16}
\end{aligned}$$

Proposition 3.3 The average profit of the whole supply chain is jointly concave in n , q , p , b and f if

$$(i) \quad \mu^2 < 4\beta g \tag{3.17}$$

$$(ii) \quad nx\beta\sigma_2 R b^2 (1 - \alpha)(\lambda - 1)(c_b + h_{r1}) > [x(1 - \alpha) - D]^4 \sigma_4 \tag{3.18}$$

$$(iii) \quad \chi_{11}D_1 + \chi_{13}D_3 + \chi_{15}D_5 < \chi_{12}D_2 + \chi_{14}D_4 \tag{3.19}$$

Proof. Please see Appendix A.5 in 3.7 for the proof and different notations used in the above proposition.

Proposition 3.4 At the equilibrium, the optimal batch size and backorder quantity are given by

$$q^* = \sqrt{\frac{2DZ_1(A_r + A_m) + nbV_1(1 - \alpha)}{2Z_1(1 - \alpha)[kh_m\rho_7 + lZ_2]}} \tag{3.20}$$

$$b^* = \frac{kqh_{r1}[x(1-\alpha) - D]}{nx(c_b + h_{r1})} \quad (3.21)$$

Proof. Please see Appendix A.6 in 3.7 for the proof and different notations used in the above proposition.

Proposition 3.5 At the equilibrium, the retail price and sales effort level of the supply chain are the positive roots of the equations

$$2\beta^3 p^3 + [2\beta\rho_5^2 - 2\beta\rho_5(a + \mu f)] = [(a + \mu f)\beta^2 - 4\beta^2\rho_5]p^2 + [(a + \mu f)\rho_5^2 + \beta\rho_4] \quad (3.22)$$

$$2g\mu^2 f^3 + [2\mu^2 p\rho_6 + 2g\rho_6^2] = [\mu^3 p + 4g\mu\rho_6]f^2 + [\mu p\rho_6^2 - \rho_4\mu] \quad (3.23)$$

Proof. Please see Appendix A.7 in 3.7 for the proof and different notations used in the above proposition.

Whenever the associated Hessian matrix is negative definite, the average profit of the entire supply chain in the centralized model can be optimized using the following solution algorithm:

Step 1. Set $n = 2$.

Step 2. Substitute the value of q from the equation (3.20) in (3.21). Then solving (3.21), one can obtain b as a function of p and f .

Step 3. Put this value of b again in equation (3.20) and then solving this equation, q can be obtained as a function of p and f .

Step 4. Substitute these values of b and q in equations (3.22) and (3.23) yielding two equations with two unknown variables p and f . Solve these equations to obtain the values of p and f .

Step 5. Using these values of p and f , derive the values of q and b from the equations (3.20) and (3.21), respectively.

Step 6. Substitute these values of q , p , b and f in (3.16) and calculate the average supply chain profit $AP_s(n, q, p, b, f)$.

Step 7. Set $n = n + 1$.

Step 8. Repeat Step 2 to Step 6.

Step 9. If $AP_s(n+1, q, p, b, f) > AP_s(n, q, p, b, f)$, repeat Steps 7 and 8. Otherwise, set $n^* = n, q^* = q, p^* = p, b^* = b, f^* = f$ and, from (3.16), calculate the optimal supply chain profit $AP_s(n^*, q^*, p^*, b^*, f^*)$.

3.4.3 Coordinated Model

In order to achieve highest profitability, supply chain players very often adopt various coordination schemes. Here, we address two coordination mechanisms, namely, trade credit mechanism and price discount contract.

3.4.3.1 Trade credit mechanism

In a price-sensitive market, if the retailer reduces the selling price and raises his sales effort level, it can stimulate the customer demand significantly. But, for delivering the product at cheaper price, increasing sales effort and order quantity may cause a loss to him. So, to motivate the retailer to set the pricing and inventory decision according to the centralized scenario, the manufacturer offers a trade credit mechanism to the retailer. In this scheme, the manufacturer asks the retailer to make decision in conformity with the centralized structure and in return he allows the retailer a delay in payment and sets the number of shipments according to the centralized manner. The manufacturer offers a trade credit period ζ to the retailer to delay the payment. The retailer accepts this offer as he can earn interests out of this. Under this trade credit policy, the average profit function of the retailer is

$$\begin{aligned}
& AP_r^{cp}(q^c, p^c, b^c, f^c, \zeta) \\
&= p^c D^c - w D^c - \frac{A_r D^c}{k^c q^c (1 - \alpha)} - \frac{(s_1 + c_r(1 - \gamma)m_1 + c_a \gamma m_2) D^c}{1 - \alpha} - \frac{h_{r2} \alpha l^c q^c}{k^c} \\
&- \frac{h_{r1}}{2} \left[\frac{q^c l^c (1 - \alpha)}{k^c} - b^c + \frac{b^c D^c}{x(1 - \alpha) - D^c} - \frac{b^c x (1 - \alpha)}{x(1 - \alpha) - D^c} + \frac{n^c x b^c c^2}{k^c q^c [x(1 - \alpha) - D^c]} \right] \\
&- \frac{n^c c_b b^c c^2 D^c}{2 k^c q^c (1 - \alpha)} \left[\frac{1}{D^c} + \frac{1}{x(1 - \alpha) - D^c} \right] - g f^{c2} + I_r \zeta w D^c \tag{3.24}
\end{aligned}$$

where $D^c = a - \beta p^c + \mu f^c$, $k^c = \frac{\lambda^{n^c} - 1}{\lambda - 1}$ and $l^c = \frac{\lambda^{2n^c} - 1}{\lambda^2 - 1}$

The last term represents the interest earned by the retailer at an interest rate I_r during the trade credit time ζ .

The average profit of the manufacturer is

$$\begin{aligned}
 & AP_m^{cp}(n^c, \xi) \\
 &= (c_1\theta_1 + c_2\theta_2 + c_3\theta_3 - s_2 - c_w - c_v - T_v - c_e\theta_3 - R_w\theta_1 - R_s\theta_2 - R_d\theta_3) \frac{\alpha D^c}{1-\alpha} \\
 & - \frac{A_m D^c}{k^c q^c (1-\alpha)} - \frac{(c_m + c_v + T_v) D^c}{1-\alpha} - h_m \left[\frac{q^c \lambda (\lambda^{n^c-1} - 1)}{(\lambda^2 - 1)} + \frac{q^c D^c (2\lambda - \lambda^{n^c} - 1)}{2R(\lambda - 1)(1-\alpha)} \right] \\
 & + w D^c - I_m \xi w D^c \tag{3.25}
 \end{aligned}$$

The last term represents the amount of interest the manufacturer losses during the credit period ξ .

Proposition 3.6 The minimum trade credit period required to convince the retailer to accept the coordination scheme is

$$\xi_{min} = \frac{\Delta^d - \Delta^c}{I_r w D^c} \quad \text{where} \tag{3.26}$$

$$\begin{aligned}
 \Delta = & pD - wD - \frac{A_r D}{kq(1-\alpha)} - \frac{(s_1 + c_r(1-\gamma)m_1 + c_a\gamma m_2)D}{1-\alpha} - \frac{h_{r2}\alpha lq}{k} \\
 & - \frac{h_{r1}}{2} \left[\frac{ql(1-\alpha)}{k} - b + \frac{bD}{x(1-\alpha) - D} - \frac{bx(1-\alpha)}{x(1-\alpha) - D} + \frac{nx b^2}{kq[x(1-\alpha) - D]} \right] \\
 & - \frac{nc_b b^2 D}{2kq(1-\alpha)} \left[\frac{1}{D} + \frac{1}{x(1-\alpha) - D} \right] - gf^2 \tag{3.27}
 \end{aligned}$$

Proof. The retailer will accept the trade credit payment offer only if it does not worsen his profit than the decentralized mode. So, the required condition for the retailer to take part in this trade credit mechanism is that his profit must be greater or equal to the decentralized profit, i.e.,

$$AP_r^{cp}(q^c, p^c, b^c, f^c, \xi) \geq AP_r^d(q^d, p^d, b^d, f^d) \tag{3.28}$$

Solving the above inequality using equations (3.2) and (3.24), we get

$$\xi \geq \frac{\Delta^d - \Delta^c}{I_r w D^c} = \xi_{min} \tag{3.29}$$

Proposition 3.7 The maximum trade credit period the manufacturer can offer to motivate the retailer to accept the coordination scheme is

$$\begin{aligned}\zeta_{max} &= \frac{\tau^c - \tau^d}{I_m w D^c} \quad \text{where} \\ \tau &= wD + (c_1\theta_1 + c_2\theta_2 + c_3\theta_3 - s_2 - c_w - c_v - T_v - c_e\theta_3 - R_w\theta_1 - R_s\theta_2 \\ &\quad - R_d\theta_3) \frac{\alpha D}{1 - \alpha} - \frac{A_m D}{kq(1 - \alpha)} - \frac{(c_m + c_v + T_v)D}{1 - \alpha} - h_m \left[\frac{q\lambda(\lambda^{n-1} - 1)}{(\lambda^2 - 1)} \right. \\ &\quad \left. + \frac{qD(2\lambda - \lambda^n - 1)}{2R(\lambda - 1)(1 - \alpha)} \right]\end{aligned}\quad (3.30)$$

Proof. The manufacturer will adopt the centralized shipment policy and allow the retailer to pay later only after being assured of his profit. So, the manufacturer offers the trade credit mechanism when his profit under this coordination mechanism is higher or at least equal to the profit obtained in the decentralized scenario, i.e.,

$$AP_m^{cp}(n^c, \zeta) \geq AP_m^d(n^d) \quad (3.31)$$

Using equations (3.1) and (3.25), the above inequality gives

$$\zeta \leq \frac{\tau^c - \tau^d}{I_m w D^c} = \zeta_{max} \quad (3.32)$$

Now, the coordination is possible if $\zeta_{max} \geq \zeta_{min} > 0$. Then, for any value of ζ lying in the interval $[\zeta_{min}, \zeta_{max}]$, the supply chain members agree to participate in the trade credit mechanism and it becomes profitable for both. If the retailer accepts the minimum delayed payment period, then his profit will be equal to the decentralized profit and the extra surplus profit achieved under the trade credit mechanism is assigned to the manufacturer. But if the retailer claims for longer credit period and wants to achieve more profit, then the manufacturer can provide at most ζ_{max} credit period and in this case, the manufacturer gains the same profit as the decentralized scenario and the retailer obtains the whole surplus profit. This trade credit period and share amount of the additional profit gained in the centralized structure depends on the bargaining power of the supply chain members over each other. If ψ be the bargaining power of the retailer against the manufacturer, then the bargaining power of the manufacturer will be $(1 - \psi)$ and $0 \leq \psi \leq 1$. Then the optimal credit period can be obtained by the relation

$$\zeta_{sharing} = \psi \zeta_{max} + (1 - \psi) \zeta_{min} = \psi \left[\frac{\tau^c - \tau^d}{I_m w D^c} \right] + (1 - \psi) \left[\frac{\Delta^d - \Delta^c}{I_r w D^c} \right] \quad (3.33)$$

Under this mechanism, the average profit of the whole supply chain becomes

$$\begin{aligned}
 AP_s^{cp}(n^c, q^c, p^c, b^c, f^c, \xi) &= AP_m^{cp}(n^c) + AP_r^{cp}(q^c, p^c, b^c, f^c) \\
 &= p^c D^c + (c_1 \theta_1 + c_2 \theta_2 + c_3 \theta_3 - s_2 - c_w - c_v - T_v - c_e \theta_3 - R_w \theta_1 \\
 &\quad - R_s \theta_2 - R_d \theta_3) \frac{\alpha D^c}{1 - \alpha} - \frac{A_m D^c}{k^c q^c (1 - \alpha)} - \frac{(c_m + c_v + T_v) D}{1 - \alpha} \\
 &\quad - h_m \left[\frac{q^c \lambda (\lambda^{n^c - 1} - 1)}{(\lambda^2 - 1)} + \frac{q^c D^c (2\lambda - \lambda^{n^c} - 1)}{2R(\lambda - 1)(1 - \alpha)} \right] - \frac{A_r D^c}{k^c q^c (1 - \alpha)} - \frac{h_{r2} \alpha l^c q^c}{k^c} \\
 &\quad - \frac{(s_1 + c_r(1 - \gamma)m_1 + c_a \gamma m_2) D^c}{1 - \alpha} - \frac{n^c c_b b^{c2} D^c}{2k^c q^c (1 - \alpha)} \left[\frac{1}{D^c} + \frac{1}{x(1 - \alpha) - D^c} \right] \\
 &\quad - \frac{h_{r1}}{2} \left[\frac{q^c l^c (1 - \alpha)}{k^c} - b^c + \frac{b^c D^c}{x(1 - \alpha) - D^c} - \frac{b^c x(1 - \alpha)}{x(1 - \alpha) - D^c} + \frac{n^c x b^{c2}}{k^c q^c [x(1 - \alpha) - D^c]} \right] \\
 &\quad - g f^{c2} + (I_r - I_m) \xi w D^c \tag{3.34}
 \end{aligned}$$

From (3.16) and (3.34), it is observed that

$$AP_s^{cp}(n^c, q^c, p^c, b^c, f^c, \xi) - AP_s^c(n^c, q^c, p^c, b^c, f^c) = (I_r - I_m) \xi w D^c \tag{3.35}$$

Therefore, the whole supply chain's profit under the trade credit mechanism may be greater, equal or less than the supply chain profit of the centralized model according as $I_r > I_m$, $I_r = I_m$ or $I_r < I_m$, respectively. So, depending on the interest rate of the manufacturer and the retailer three cases may arise.

Case (i): $I_r > I_m$

In such circumstances, from equation (3.35), it is observed that

$AP_s^{cp}(n^c, q^c, p^c, b^c, f^c, \xi) > AP_s^c(n^c, q^c, p^c, b^c, f^c)$, i.e., the trade credit mechanism effectively enhances the system profit and it results more supply chain profit than the centralized scenario.

Case (ii): $I_r = I_m$

If the interest rates of both the retailer and manufacturer are the same, then

$AP_s^{cp}(n^c, q^c, p^c, b^c, f^c, \xi) = AP_s^c(n^c, q^c, p^c, b^c, f^c)$. So, in such a situation, coordination is achieved and ideal profit can be obtained. The retailer and the manufacturer can share the surplus profit gained in the centralized mode through bargaining.

Case (iii): $I_r < I_m$

If the interest rate earned by the retailer is smaller than the manufacturer's interest rate, then $AP_s^{cp}(n^c, q^c, p^c, b^c, f^c, \xi) < AP_s^c(n^c, q^c, p^c, b^c, f^c)$. In this case, the trade

credit mechanism fails to coordinate the supply chain and achieves the centralized supply chain profit. So, the supply chain members should look for any other suitable coordination scheme. In the following, a price discount coordination scheme is proposed as an appropriate alternative.

3.4.3.2 Price discount contract

Although integrated decision making leads higher profit level for the entire supply chain but sometimes, the supply chain members may refuse to accept the joint decision making policy. To encourage the retailer for adopting the centralized decision making policy, the manufacturer often offers a price discount contract. In this mechanism, the manufacturer requests the retailer to set his pricing and inventory decisions according to the centralized structure and in return, the manufacturer provides a price discount on its wholesale price. The price discount scheme offered by the manufacturer is

$$w = \begin{cases} w, & \text{if } q < q^{c*} \\ w(1 - \phi), & \text{if } q \geq q^{c*} \end{cases} \quad (3.36)$$

Under this price discount mechanism, the average profit of the retailer is

$$\begin{aligned} & AP_r^{cp}(q^c, p^c, b^c, f^c, \phi) \\ = & p^c D^c - w(1 - \phi) D^c - \frac{A_r D^c}{k^c q^c (1 - \alpha)} - \frac{(s_1 + c_r(1 - \gamma)m_1 + c_a \gamma m_2) D^c}{1 - \alpha} - \frac{h_{r2} \alpha l^c q^c}{k^c} \\ & - \frac{h_{r1}}{2} \left[\frac{q^c l^c (1 - \alpha)}{k^c} - b^c + \frac{b^c D^c}{x(1 - \alpha) - D^c} - \frac{b^c x(1 - \alpha)}{x(1 - \alpha) - D^c} + \frac{n^c x b^c c^2}{k^c q^c [x(1 - \alpha) - D^c]} \right] \\ & - \frac{n^c c_b b^c c^2 D^c}{2 k^c q^c (1 - \alpha)} \left[\frac{1}{D^c} + \frac{1}{x(1 - \alpha) - D^c} \right] - g f^{c2} \end{aligned} \quad (3.37)$$

$$\text{where } D^c = a - \beta p^c + \mu f^c, \quad k^c = \frac{\lambda^{n^c} - 1}{\lambda - 1} \quad \text{and} \quad l^c = \frac{\lambda^{2n^c} - 1}{\lambda^2 - 1}$$

and the average profit of the manufacturer is

$$\begin{aligned} AP_m^{cp}(n^c, \phi) = & w(1 - \phi) D^c + \left(c_1 \theta_1 + c_2 \theta_2 + c_3 \theta_3 - s_2 - c_w - c_v - T_v - c_e \theta_3 - R_w \theta_1 \right. \\ & \left. - R_s \theta_2 - R_d \theta_3 \right) \frac{\alpha D^c}{1 - \alpha} - \frac{A_m D^c}{k^c q^c (1 - \alpha)} - \frac{(c_m + c_v + T_v) D^c}{1 - \alpha} \\ & - h_m \left[\frac{q^c \lambda (\lambda^{n^c} - 1)}{(\lambda^2 - 1)} + \frac{q^c D^c (2\lambda - \lambda^{n^c} - 1)}{2R(\lambda - 1)(1 - \alpha)} \right] \end{aligned} \quad (3.38)$$

Proposition 3.8 The minimum price discount ratio for which the retailer accepts the price discount scheme is given by

$$\phi_{min} = \frac{(\chi^d - \chi^c) - w(D^d - D^c)}{wD^c}$$

where $\chi = pD - \frac{A_r D}{kq(1-\alpha)} - \frac{(s_1 + c_r(1-\gamma)m_1 + c_a\gamma m_2)D}{1-\alpha} - \frac{h_{r2}\alpha lq}{k}$
 $- \frac{h_{r1}}{2} \left[\frac{ql(1-\alpha)}{k} - b + \frac{bD}{x(1-\alpha) - D} - \frac{bx(1-\alpha)}{x(1-\alpha) - D} + \frac{nxb^2}{kq[x(1-\alpha) - D]} \right]$
 $- \frac{nc_b b^2 D}{2kq(1-\alpha)} \left[\frac{1}{D} + \frac{1}{x(1-\alpha) - D} \right] - gf^2$

Proof. The price discount offer is acceptable to the retailer when he achieves higher profit under this scheme than the decentralized profit. So, the necessary condition for the retailer to accept the price discount mechanism is

$$AP_r^{cp}(q^c, p^c, b^c, f^c, \phi) \geq AP_r^d(q^d, p^d, b^d, f^d) \quad (3.39)$$

Using equations (3.2) and (3.37), the above condition results

$$\phi \geq \frac{(\chi^d - \chi^c) - w(D^d - D^c)}{wD^c}.$$

Proposition 3.9 The maximum price discount ratio provided by the manufacturer to convince the retailer is

$$\phi_{max} = \frac{(\rho^c - \rho^d) - w(D^d - D^c)}{wD^c} \quad (3.40)$$

where $\rho = (c_1\theta_1 + c_2\theta_2 + c_3\theta_3 - s_2 - c_w - c_v - T_v - c_e\theta_3 - R_w\theta_1 - R_s\theta_2$
 $- R_d\theta_3) \frac{\alpha D}{1-\alpha} - \frac{A_m D}{kq(1-\alpha)} - \frac{(c_m + c_v + T_v)D}{1-\alpha} - h_m \left[\frac{q\lambda(\lambda^{n-1} - 1)}{(\lambda^2 - 1)} \right.$
 $\left. + \frac{qD(2\lambda - \lambda^n - 1)}{2R(\lambda - 1)(1-\alpha)} \right]$ (3.41)

Proof. Like the retailer, the manufacturer can also participate into the price discount mechanism if his profit increases under this coordination scheme compared to the decentralized structure. Therefore, the manufacturer's condition to offer the price discount scheme is

$$AP_m^{cp}(n^c, \phi) \geq AP_m^d(n^d) \quad (3.42)$$

Simplifying the above inequality using (3.1) and (3.38), we get

$$\phi \leq \frac{(\rho^c - \rho^d) - w(D^d - D^c)}{wD^c}.$$

Therefore, for the price discount ratio lying within the feasible region $[\phi_{min}, \phi_{max}]$, supply chain coordination can be achieved. For any price discount ratio within this acceptable range, desirability of each of the supply chain member to accept the coordination scheme is guaranteed as they could enjoy a higher profitability. Also, the extra profit gained through this coordination is $\Delta AP = AP_s^{cp}(n^c, q^c, p^c, b^c, f^c, \phi) - AP_s^d(n^d, q^d, p^d, b^d, f^d)$. This extra profit is shared between the supply chain members. To share the extra profit fairly between the manufacturer and the retailer, a profit sharing strategy is considered based on the bargaining power of the supply chain members. If ψ be the bargaining power of the retailer against the manufacturer, then $(1-\psi)$ will be the bargaining power of the manufacturer. Therefore, under this price discount contract, ψ fraction of the extra profit ΔAP is acquired by the retailer. Then the average profit of the retailer will be

$$AP_r^{cp}(q^c, p^c, b^c, f^c, \phi) = AP_r^d(q^d, p^d, b^d, f^d) + \psi \Delta AP \quad (3.43)$$

Substituting equations (3.2) and (3.37) into equation (3.43), the exact price discount ratio (ϕ) allowable by both the manufacturer and the retailer can be determined. Needless to say here that the manufacturer receives remaining $(1 - \psi)$ proportion of the extra profit.

3.5 Numerical Analysis

In this section, we consider four different sets of problem to explore the performance of the proposed model under decentralized, centralized and coordinated supply chain scenarios. Data sets used in these four problems are given below and the optimal results are presented in Table 3.1.

Problem 1. $R = 16000; a = 5000; A_r = 300; T_v = 25; c_v = 5; \theta_1 = 0.5; \theta_2 = 0.3; \theta_3 = 0.2; A_m = 150; c_e = 7; c_b = 5; x = 175200; c_1 = 75; c_2 = w = 100; c_3 = 25; R_d = 2; s_1 = 0.4; s_2 = 0.5; h_{r1} = 4.5; h_{r2} = 2; h_m = 1.5; c_m = 5; c_w = 22; R_s = 2; R_w =$

$8; \lambda = 1.7; \beta = 10; \mu = 7; g = 10; c_a = 200; c_r = 100; I_m = 0.2; I_r = 0.23; \psi = 0.6; \gamma = 5\%; m_1 = 1\%; m_2 = 4\%$. (with appropriate units)

Problem 2. $R = 10000; a = 3000; A_r = 250; T_v = 22; c_v = 5; \theta_1 = 0.5; \theta_2 = 0.3; \theta_3 = 0.2; A_m = 150; c_e = 7; c_b = 4; x = 175200; c_1 = 75; c_2 = w = 100; c_3 = 25; R_d = 2; s_1 = 0.3; s_2 = 0.4; h_{r1} = 3; h_{r2} = 2; h_m = 1.5; c_m = 5; c_w = 20; R_s = 2; R_w = 8; \lambda = 1.9; \beta = 10; \mu = 5; g = 8; c_a = 100; c_r = 50; I_m = 0.35; I_r = 0.35; \psi = 0.4; \gamma = 5\%; m_1 = 1\%; m_2 = 4\%$. (with appropriate units)

Problem 3. $R = 14000; a = 4000; A_r = 500; T_v = 25; c_v = 5; \theta_1 = 0.5; \theta_2 = 0.3; \theta_3 = 0.2; A_m = 150; c_e = 7; c_b = 6; x = 175200; c_1 = 75; c_2 = w = 200; c_3 = 25; R_d = 2; s_1 = 0.5; s_2 = 0.6; h_{r1} = 4; h_{r2} = 2.5; h_m = 1.5; c_m = 8; c_w = 25; R_s = 3; R_w = 10; \lambda = 1.8; \beta = 8; \mu = 3; g = 15; c_a = 150; c_r = 90; \psi = 0.5; \gamma = 5\%; m_1 = 1\%; m_2 = 4\%$. (with appropriate units)

Problem 4. All parameter-values are considered same as problem 2 except $\psi = 0.5$.

We observe from the optimal results given in Table 3.1 that centralization generates higher surplus to the profitability of the supply chain compared to the decentralized structure in each problem. In the centralized scenario, the retailer can provide the products to the end-customer at a cheaper price than the decentralized scenario. The retailer can also raise his sales effort to a higher level. Since the market demand is sensitive to the retail price and sales effort level of the product, therefore, in the centralized structure, lower priced product with higher sales effort leads more consumer demand. Then the retailer increases his order quantity from the manufacturer, and allowable shortage level also increases in the integrated mode. Consequently, joint decision making increases the system profitability. However, it could reduce the profitability of an individual member. So, integrated decision making may not be acceptable always for all the members. In test problem 1, $I_r > I_m$ i.e., the interest rate earned by the retailer is more than that of the manufacturer. So, both the manufacturer and the retailer prefer to accept the trade credit mechanism. From Table 3.1, it is observed that, trade credit mechanism not only increases the supply chain members' individual profits but also enhances the profitability of the whole supply chain significantly. Even the trade credit mechanism generates higher supply chain profit than the centralized structure. The minimum credit period for the

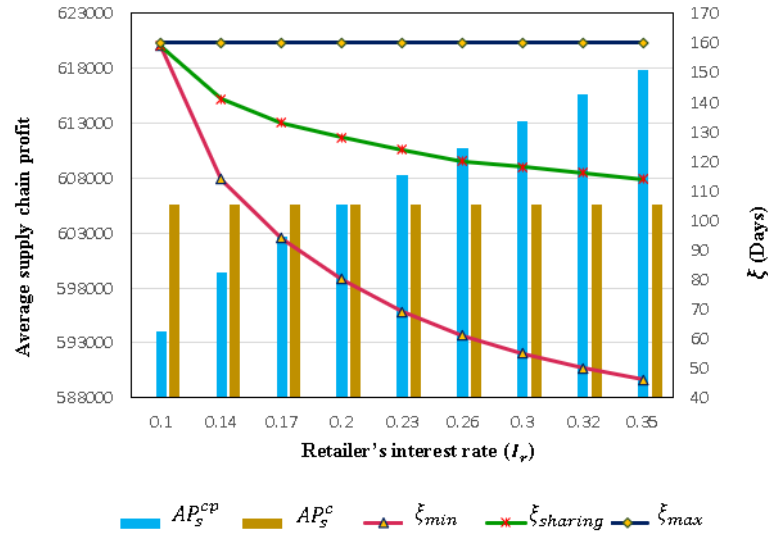


Figure 3.4: Supply chain profit versus retailer's interest rate (I_r) and credit period (ζ)

retailer is 69 days and the maximum allowable credit period from the manufacturer is 160 days. A win-win situation occurs within the interval [69 days, 160 days]. Figure 3.4 reflects the supply chain profitability under the centralized scenario and the trade credit mechanism for different values of I_r and ζ . It can be seen that, supply chain profitability increases for increasing value of I_r . If the retailer's interest rate goes up, he agrees to a short-term trade credit period. So, ζ_{min} gradually decreases but ζ_{max} remains same as I_r has no impact on it. As a result, the optimal credit period $\zeta_{sharing}$ gradually decreases. When I_r is less than 0.2 and credit period is more than 128 days, the supply chain profit under trade credit mechanism is less than the centralized supply chain profit. Then, from equation (3.35), it can be said that the coordination through the trade credit mechanism will not be effective. For $I_r > 0.2$ with $\zeta_{sharing} < 128$ days, both the manufacturer and the retailer participate in the trade credit mechanism which provides more profit to them than the decentralized scenario, and the whole supply chain profit is higher than the centralized structure. A reverse scenario can be observed when the manufacturer's rate of interest (I_m) increases. Figure 3.5 reflects that, rising rate of interest of the manufacturer reduces the supply chain profit progressively. When I_m increases, the manufacturer lowers down the maximum allowable trade credit period (ζ_{max}) as the longer credit period leads more loss to him and ζ_{min} remains unchanged. Hence, the optimal credit period $\zeta_{sharing}$ also decreases. Whenever $I_m > 0.23$ and $\zeta_{sharing} < 111$ days, the trade

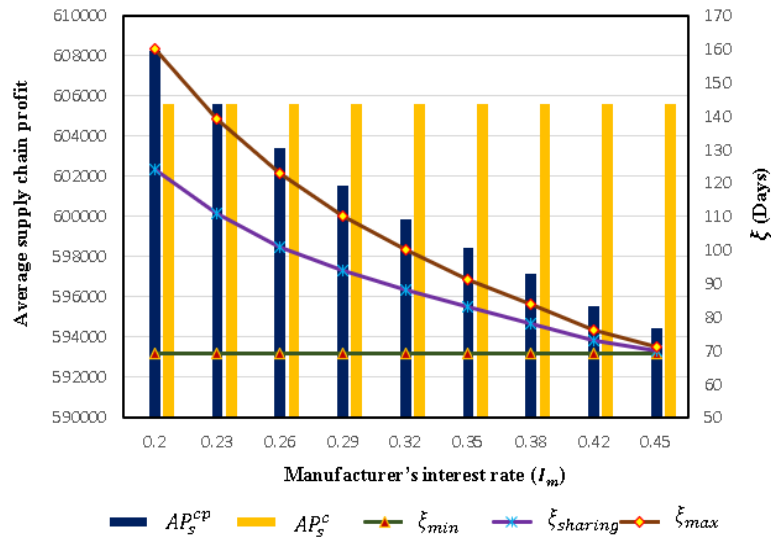


Figure 3.5: Supply chain profit versus manufacturer's interest rate (I_m) and credit period (ξ)

credit mechanism fails to achieve the profit level of the centralized system. Consequently, it can be concluded from both the Figures 3.4 and 3.5 that, the trade credit mechanism generates higher supply chain profit compared to the centralized scenario when $I_m < 0.23$, $I_r > 0.2$ and $111 \text{ days} < \xi_{sharing} < 128 \text{ days}$.

In test problem 2, $I_r = I_m$ (case (ii)). In such situation, the trade credit mechanism leads the same supply chain profit as to the centralized manner and the supply chain members share the additional profit depending on their bargaining power. Table 3.1 shows that, as the bargaining power of the retailer is greater than the manufacturer, the retailer's profit growth is higher compared to the manufacturer.

In test problem 3, $I_r < I_m$ (case (iii)). In such circumstance, the supply chain members would go for the wholesale price contract scheme as the trade credit mechanism fails to reach the profit level of the centralized supply chain. Table 3.1 shows that the price discount mechanism results the same system profit as in the centralized scenario, and increases considerably the profits of the manufacturer and the retailer than those of the decentralized structure. The minimum discount ratio for the retailer to accept the price discount offer is 15%. The manufacturer can provide a maximum 29% price discount to the retailer without making any loss. Therefore, for ϕ lying in $[15\%, 29\%]$, coordination is achievable, and the price discount scheme is capable of making incentives to share the surplus profit obtained in the integrated scenario between the supply chain members.

Table 3.1: Optimal results for different models

	Problem 1	Problem 2	Problem 3	Problem 4
Decentralized Model				
n^{d*}	6	6	6	6
q^{d*}	26.83	19.01	14.87	19.01
p^{d*}	328.82	209.09	353.97	209.09
b^{d*}	65.90	47.35	39.61	47.35
f^{d*}	79.35	33.67	15.17	33.67
AP_m^{d*}	143618	71856	199266	71856
AP_r^{d*}	450399	106654	179982	106654
AP_s^{d*}	594017	178510	379248	178510
Centralized Model				
n^{c*}	5	4	5	4
q^{c*}	63.39	66.06	53.44	66.06
p^{c*}	301.47	178.39	273.22	178.39
b^{c*}	105.02	88.41	89.11	88.41
f^{c*}	92.04	45.04	23.56	45.04
AP_m^{c*}	-	-	-	-
AP_r^{c*}	-	-	-	-
AP_s^{c*}	605574	190717	434585	190717
Coordinated Model				
n^{cp*}	5	4	5	4
q^{cp*}	63.39	66.06	53.44	66.06
p^{cp*}	301.47	178.39	273.22	178.39
b^{cp*}	105.02	88.41	89.11	88.41
f^{cp*}	92.04	45.04	23.56	45.04
$\xi_{min}(day)$	69	88	-	88
$\xi_{max}(day)$	160	176	-	176
$\xi_{share}(day)$	124	123	-	132
$\phi_{min}(\%)$	-	-	15	8
$\phi_{max}(\%)$	-	-	29	17
$\phi_{opt}(\%)$	-	-	22	13
AP_m^{cp*}	148839	79180	226935	77959 [†]
AP_r^{cp*}	459406	111537	207650	112758 [†]
AP_s^{cp*}	608245	190717	434585	190717 [†]

† represents both the trade credit and price discount mechanisms

From figure 3.6, it can be seen that, at $\phi = 15\%$, the retailer earns the same profit as in the decentralized mode and as ϕ increases, the retailer's profit increases gradually. On the contrary, the manufacturer's profitability starts to decrease and the

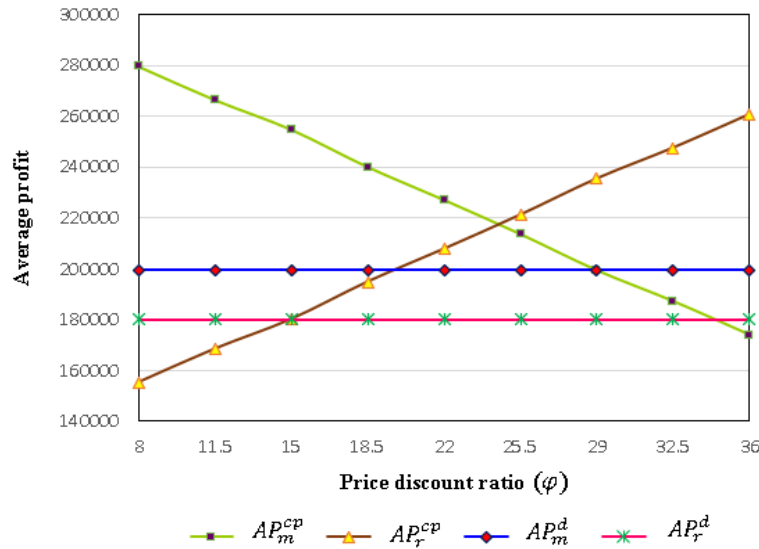


Figure 3.6: Average profit of the manufacturer and the retailer versus ϕ

manufacturer gains the profit same as in the decentralized structure when $\phi = 29\%$. Beyond this, the price discount scheme is no longer effective as it results a loss to the manufacturer. They can share the extra profit by fixing the value of ϕ through bargaining. If the retailer and the manufacturer have equal bargaining power then we take the value of ϕ as the mean of $[15\%, 29\%]$ i.e., 22% .

From test problem 4, it is observed that, when the manufacturer and the retailer have equal interest rates, both the trade credit mechanism and the price discount mechanism act similarly. In both the schemes, as the bargaining power of each member is equal, the increments of the profit levels of the member are the same. In this case, the higher the bargaining power, the greater the profit.

Special Case:

In most traditional business scenarios, defect management is over-looked and the defective items are either sold at a compromised price in a secondary market or disposed which causes carbon emission and landfills. So, in order to find out the efficiency of a skilful defect management, we compare our proposed model with the situation where the manufacturer sells all the returned items at a cheaper price in a

secondary market without any sorting. Problem 1 is considered here and the optimal results presented in Table 3.2 show that, performing an efficient defect management to sort the defective items into repairable, design-flawed and non-repairable categories not only enhances the supply chain profitability but also reduces the carbon emission cost immensely. In Table 3.2, *CEC* represents carbon-emission cost. The manufacturer can generate more revenue and improve his environmental performance by adopting such efficient defect management policy.

Table 3.2: Optimal results for different models

Models	AP_r^d	AP_m^d	AP_s^d	AP_s^c	AP_s^{cp}	CEC^d	CEC^c	CEC^{cp}
Without defect management	450399	139800	590199	601154	604662	5136.42	6786.57	6786.57
With defect management	450399	143618	594017	605574	608245	4845.75	6415.74	6415.74

3.6 Sensitivity Analysis

In this segment, a sensitivity analysis is performed with respect to some crucial key parameters in order to examine both economical and ecological significance of the proposed trade credit mechanism and defect management policy. Data set of the Problem 1 is considered for the analysis and the following observations are made:

(i) From Table 3.3, a drastic change in the profitability of both the retailer and the manufacturer is observed for small increment in β . A higher price elasticity of customer demand indicates that customers are more sensitive to the retail price of the product. In such a high price-sensitive market, the retailer is forced to lower his retail price to retain the market demand and also decreases the sales effort level. Increasing price sensitivity negatively affects the consumer demand and it results lower profit levels for both the chain members. Moreover, in the coordinated scenario, the retailer looks for longer credit period to maintain his profitability. The optimal credit period increases with the increment of β .

Table 3.3: Optimal results for different models

Models	β	n^*	q^*	p^*	b^*	f^*	$\xi_{sharing}$	AP_r^*	AP_m^*	AP_s^*
Decentralized	8	6	28.25	410.82	69.43	108.05	-	645058	156498	801556
	9	6	27.50	364.55	67.56	91.86	-	534875	149647	684522
	10	6	26.83	328.83	65.90	79.35	-	450399	143618	594017
	11	6	26.23	300.41	64.38	69.40	-	383741	138158	521899
	12	6	25.66	277.27	62.97	61.30	-	329940	133109	463049
Centralized	8	5	65.29	384.77	108.07	121.20	-	-	-	811143
	9	5	64.25	337.76	106.41	104.74	-	-	-	695091
	10	5	63.39	301.47	105.02	92.03	-	-	-	605574
	11	5	62.64	272.59	103.82	81.93	-	-	-	534449
	12	5	61.98	249.09	102.75	73.70	-	-	-	476595
Coordinated	8	5	65.29	384.77	108.07	121.20	97	652529	160829	813358
	9	5	64.25	337.76	106.41	104.74	110	543111	154422	697533
	10	5	63.39	301.47	105.02	92.03	124	459406	148839	608245
	11	5	62.64	272.59	103.82	81.93	137	393522	143828	537351
	12	5	61.98	249.09	102.75	73.70	151	340498	139230	479728

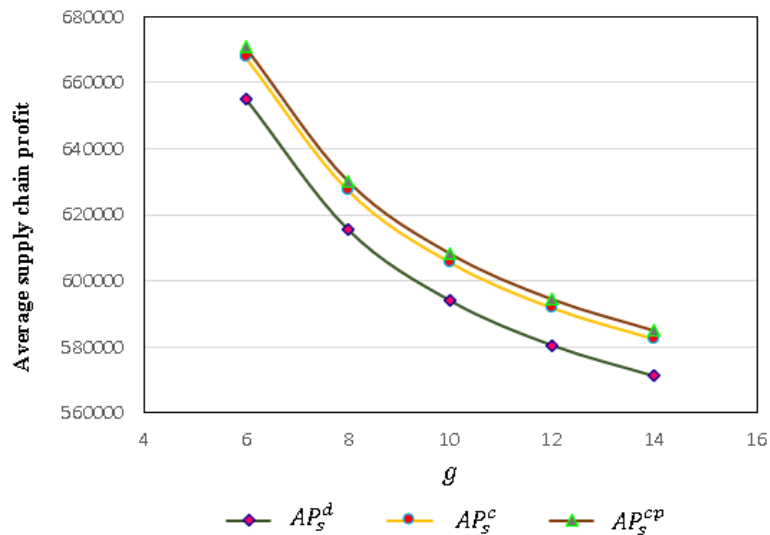


Figure 3.7: Average supply chain profit versus sales effort cost coefficient g

(ii) Figure 3.7 exhibits a huge depletion of the supply chain profitability for the increment of the sales effort cost coefficient g. When the sales effort cost coefficient increases, the retailer's sales effort cost increases. The retailer then reduces its sales

effort level to maintain his profitability. As soon as the sales effort level decreases, customer demand also decreases gradually. So then the retailer is compelled to reduce the selling price of the product to retain the same level of sales. As a consequence, profits of all the members as well as the entire supply chain decrease. In this context, it can be concluded that when the sales effort cost is too high, supply chain players should focus on their pricing strategy rather than sales effort.

Table 3.4: Optimal results for different models

Models	μ	n^*	q^*	p^*	b^*	f^*	$\zeta_{sharing}$	AP_r^*	AP_m^*	AP_s^*
Decentralized	5	6	25.80	314.32	63.33	53.05	-	421533	134392	555925
	6	6	26.26	320.73	64.47	65.59	-	434290	138469	572759
	7	6	26.83	328.83	65.90	79.35	-	450399	143618	594017
	8	6	27.55	338.95	66.67	94.74	-	470536	150054	620590
	9	6	28.42	351.56	69.86	112.26	-	495648	158081	653729
Centralized	5	5	61.07	284.65	101.29	61.53	-	-	-	566743
	6	5	62.10	292.08	102.95	76.07	-	-	-	583094
	7	5	63.39	301.47	105.02	92.03	-	-	-	605574
	8	5	64.98	313.20	107.57	109.88	-	-	-	632662
	9	5	66.93	327.84	110.70	130.21	-	-	-	666444
Coordinated	5	5	61.07	284.65	101.29	61.53	124	429963	139279	569243
	6	5	62.10	292.08	102.95	76.07	124	442976	143504	586480
	7	5	63.39	301.47	105.02	92.03	124	459406	148839	608245
	8	5	64.98	313.20	107.57	109.88	124	479945	155509	635454
	9	5	66.93	327.84	110.70	130.21	124	505558	163826	669385

(iii) Table 3.4 shows the impacts of the customer's sales effort sensitivity on the optimal decisions. When μ increases, the retailer invests more in its sales effort according to customer's satisfaction and it influences customer's purchasing decision. As a consequence, higher sales effort stimulates the consumer demand considerably and the retailer also gets the opportunity to gradually increase the retail price of the product. It results in more order quantity and backorder quantity. Thus, increasing sales effort sensitivity tremendously enhances the profitability of both supply chain members in each model. Impact of μ on the credit period is found to be negligible.

(iv) From Figure 3.8, it can be seen that the hike in rework cost lowers the supply

chain profit moderately. So, if rework cost is excessively high then the manufacturer should dispose the defective items or sell those items in a secondary market without repairing at a lower price. The manufacturer should send the items for repairing or disposal in the same way when the design cost is much higher.

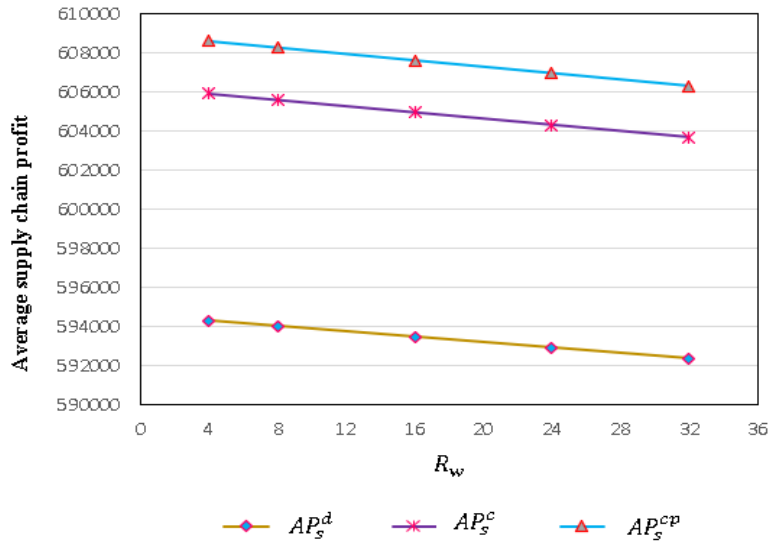


Figure 3.8: Average supply chain profit versus rework cost R_w

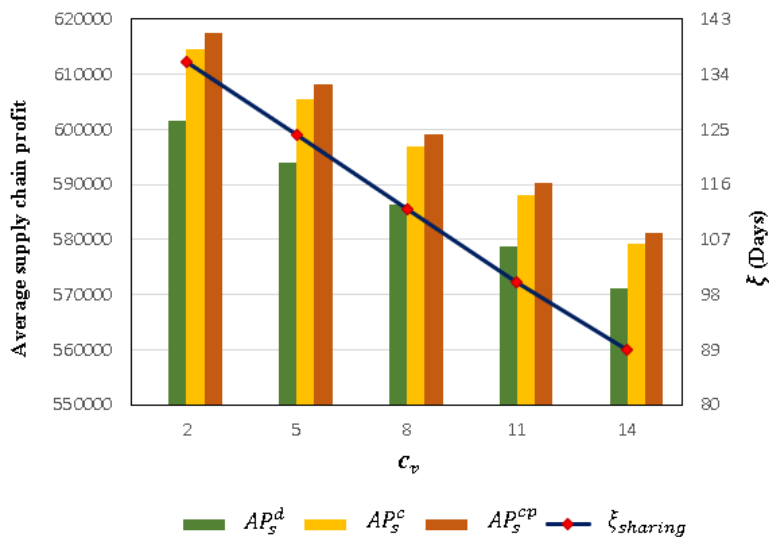


Figure 3.9: Average supply chain profit versus carbon emission cost c_v

(v) It is observed from Figure 3.9 that, whenever carbon emission cost (c_v) due to transportation rises, the manufacturer delivers smaller quantities in shipments to

the retailer and profit level of each supply chain member tends to decrease. Since an increase in carbon emissions cost increases the cost burden of the manufacturer, so the manufacturer intends to shorten the maximum allowable credit period. As a result, the optimal credit period decreases gradually. Moreover, if the carbon emission cost (c_e) due to disposal of scrap items rises, the manufacturer should conduct an effective waste management to reduce carbon emission during disposal.

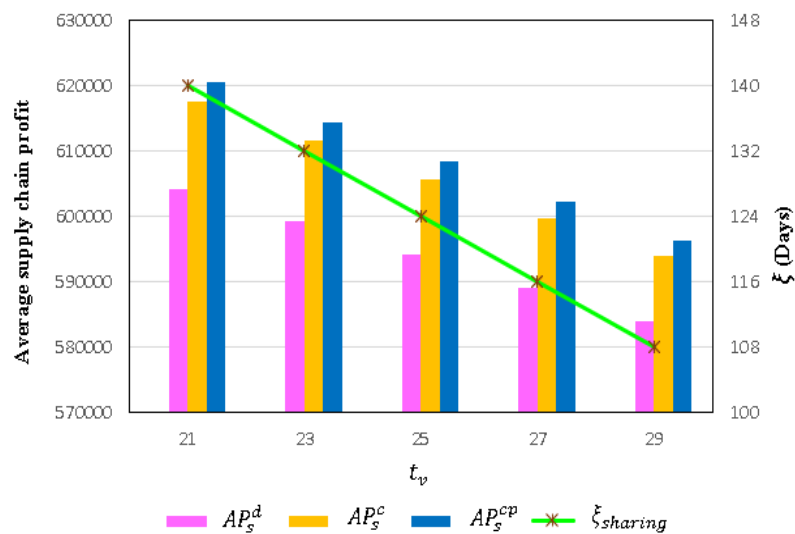


Figure 3.10: Average supply chain profit versus transportation cost t_v

(vi) Figure 3.10 reflects the fact that when the transportation cost increases, the supply chain profit decreases in each model. To avoid high transportation expense, the manufacturer delivers larger quantity in each shipment and shortens the trade credit period for the retailer. As a result, the duration of the optimal credit period as well as profitability of each member decrease.

3.7 Concluding Remarks

In this chapter, a single-manufacturer single-retailer supply chain model is investigated where the production process at the manufacturer is considered to be imperfect and order quantity of the retailer is shipped in some unequal-sized batches. The retailer makes an inspection before delivering the product to end-customers, but the inspection process is erroneous and misclassification causes economic loss

and hampers goodwill. The defective items are reverted to the manufacturer and instead of disposing these items, the manufacturer performs a proficient screening process to categorize the returned items into repairable, design-flawed and non-repairable. Design-flawed items are sold at the price of brand new product after design rectification, and repaired items are sold at a cheaper price in a secondary market. Scrap items are eventually disposed of. Retail price of the product and sales effort influence the market demand. First, the proposed model is investigated in the decentralized scenario using a Stackelberg gaming approach considering the retailer as the leader and the manufacturer as the follower. Afterwards, the performance of the whole supply chain is analyzed under the integrated structure. Two coordination schemes viz. trade credit mechanism and wholesale price contract have been proposed to make incentives to the supply chain members to adopt globally optimal pricing and inventory strategies. Numerical experiment and sensitivity analysis are conducted to exhibit the effectiveness and viability of the proposed coordination schemes and defect management policy. Managerial insights obtained from the numerical examples and sensitivity analysis are summarized as follows:

(i) Companies can improve their economic and environmental performance unprecedentedly by adopting an efficient defect management policy. They can undertake environmental-friendly business practices by reducing waste, carbon emission at the time of disposal and transportation. Salvaging repairable items from waste materials not only benefits financially but also helps to maintain social responsibility. New companies can enhance their brand value and retain their reputation through such eco-friendly practices.

(ii) Companies should reduce their retail prices in a high price sensitive market and they can promote their products by increasing sales efforts to enhance their profitability.

(iii) In order to survive in today's competitive business scenario, traders need to adopt appropriate coordination schemes. Our proposed trade credit mechanism and wholesale price contract are highly efficient to enrich the profitability of the entire supply chain as well as individual profit of each member. Moreover, by accepting these coordination mechanisms, supply chain members can provide their products at a lower price and reduce carbon emissions significantly. Thus, these contracts are found to be very useful to intensify the market demand by making

the products more appealing to the customers. Since the increment in each member's profitability is inevitable under these mechanisms, the proposed coordination schemes effectively encourage the business managers to make decisions in a collaborative manner and adopt such green initiatives. Also, these contracts are effective in all types of market demand, high or low.

(iv) Sensitivity analysis also suggests that, whenever disposal cost becomes higher, firms should look for alternatives such as recycling. In the same way, if the design cost or repair cost becomes higher, firms should dispose the defective items.

The proposed model can be extended in several directions to open up different scopes of future study. To get closer to the real business situations, the current model can be extended considering multiple manufacturers and/or multiple retailers. In the proposed model, defective rate is assumed to be constant, but it would be more realistic to consider the random defective rate in the manufacturer's production process. To further emphasize the aspect of environmental awareness, it would be appropriate to consider green-sensitive customer demand. In the present study, trade credit mechanism and wholesale price discount contract have been considered. Introducing any other suitable coordination schemes such as sales effort cost sharing contract between the manufacturer and the retailer would be another interesting extension.

Appendix A

A.1

From Figure 3.2, inventory holding area of the manufacturer = Bold area - shaded area

$$\begin{aligned}
&= \frac{q(\lambda^n - 1)}{(\lambda - 1)} \left[\frac{q(1 - \alpha)(\lambda^{n-1} - 1)}{D(\lambda - 1)} + \frac{q}{R} \right] - \frac{1}{2} \frac{q(\lambda^n - 1)}{(\lambda - 1)} \frac{q(\lambda^n - 1)}{R(\lambda - 1)} - \left[q \frac{q(1 - \alpha)}{D} + (q + \lambda q) \frac{\lambda q(1 - \alpha)}{D} \right. \\
&\quad \left. + (q + \lambda q + \lambda^2 q) \frac{\lambda^2 q(1 - \alpha)}{D} + \dots + (q + \lambda q + \dots + \lambda^{n-2} q) \frac{\lambda^{n-2} q(1 - \alpha)}{D} \right] \\
&= \frac{q^2(1 - \alpha)(\lambda^{n-1} - 1)(\lambda^n - 1)}{D(\lambda - 1)^2} + \frac{q^2(\lambda^n - 1)}{R(\lambda - 1)} - \frac{1}{2} \frac{q(\lambda^n - 1)}{(\lambda - 1)} \frac{q(\lambda^n - 1)}{R(\lambda - 1)} - \frac{q^2(1 - \alpha)(\lambda^{n-1} - 1)(\lambda^n - 1)}{D(\lambda + 1)(\lambda - 1)^2} \\
&= \frac{\lambda q^2(1 - \alpha)(\lambda^{n-1} - 1)(\lambda^n - 1)}{D(\lambda + 1)(\lambda - 1)^2} + \frac{q^2(\lambda^n - 1)(2\lambda - \lambda^n - 1)}{2R(\lambda - 1)^2}
\end{aligned}$$

A.2

To simplify the mathematical derivation, the following notations compacted by other notations are used:

$$\begin{aligned}
 X_2 &= \sqrt{\frac{h_m}{2RA_m(\lambda+1)}}, X_3 = -\frac{\beta RD(1-\alpha)(\lambda-1)\left[X_1 - (X_1+q)\log\left(1+\frac{X_1}{q}\right)\right]}{X_1^2(X_1+q)X_2[2RD(1-\alpha)-D^2(\lambda+1)]^{\frac{3}{2}}} \\
 Y_3 &= \frac{\gamma RD(1-\alpha)(\lambda-1)\left[X_1 - (X_1+q)\log\left(1+\frac{X_1}{q}\right)\right]}{X_1^2(X_1+q)X_2[2RD(1-\alpha)-D^2(\lambda+1)]^{\frac{3}{2}}}, \\
 X_4 &= \frac{(\lambda-1)\left[X_1X_3\{x(1-\alpha)-D\} - \beta\log\left(1+\frac{X_1}{q}\right)\right]}{X_1[x(1-\alpha)-D]^2\log\lambda}, \\
 Y_4 &= \frac{(\lambda-1)\left[X_1Y_3\{x(1-\alpha)-D\} + \gamma\log\left(1+\frac{X_1}{q}\right)\right]}{X_1[x(1-\alpha)-D]^2\log\lambda}, \\
 X_5 &= Y_5 = \frac{1}{X_1} - \frac{1}{X_1+q} - \frac{\log\left(1+\frac{X_1}{q}\right)}{X_1^2}, X_6 = Y_6 = -\frac{1}{X_1^2} + \frac{1}{(X_1+q)^2} + \frac{\log\left(1+\frac{X_1}{q}\right)}{X_1^3} \\
 X_7 &= \frac{\beta R(1-\alpha)(\lambda-1)}{qX_2} \left[\frac{\beta(\lambda-1)(1-\alpha)RD^2X_6}{X_2[2RD(1-\alpha)-D^2(\lambda+1)]^3} + \frac{DX_3}{X_1[2RD(1-\alpha)-D^2(\lambda+1)]^{\frac{3}{2}}} \right. \\
 &\quad \left. + \frac{\beta X_5[2D^2(\lambda+1)-RD(1-\alpha)]}{[2RD(1-\alpha)-D^2(\lambda+1)]^{\frac{5}{2}}} \right] \\
 Y_7 &= -\frac{\gamma R(1-\alpha)(\lambda-1)}{qX_2} \left[-\frac{\gamma(\lambda-1)(1-\alpha)RD^2Y_6}{X_2[2RD(1-\alpha)-D^2(\lambda+1)]^3} + \frac{DY_3}{X_1[2RD(1-\alpha)-D^2(\lambda+1)]^{\frac{3}{2}}} \right. \\
 &\quad \left. - \frac{\gamma Y_5[2D^2(\lambda+1)-RD(1-\alpha)]}{[2RD(1-\alpha)-D^2(\lambda+1)]^{\frac{5}{2}}} \right] \\
 X_8 &= \frac{[x(1-\alpha)-D]X_7 - \beta X_3}{[x(1-\alpha)-D]^2}, Y_8 = \frac{[x(1-\alpha)-D]Y_7 + \gamma Y_3}{[x(1-\alpha)-D]^2} \\
 X_9 &= \frac{(\lambda-1)}{\log\lambda} \left[\frac{X_1X_8[x(1-\alpha)-D]^3 - \beta X_1X_3[x(1-\alpha)-D] + 2\beta^2\log\left(1+\frac{X_1}{q}\right)}{X_1[x(1-\alpha)-D]^3} \right] \\
 Y_9 &= \frac{(\lambda-1)}{\log\lambda} \left[\frac{X_1Y_8[x(1-\alpha)-D]^3 + \gamma X_1Y_3[x(1-\alpha)-D] + 2\gamma^2\log\left(1+\frac{X_1}{q}\right)}{X_1[x(1-\alpha)-D]^3} \right] \\
 Z_1 &= x(1-\alpha)-D, Z_2 = [2\alpha h_{r2} + (1-\alpha)h_{r1}], Z_3 = 2D^2(\lambda+1) - RD(1-\alpha) \\
 Z_4 &= 2RD(1-\alpha) - D^2(\lambda+1), Z_5 = RD(1-\alpha)(\lambda-1)Z_1 - X_2(q+X_1)Z_4^{\frac{3}{2}} \\
 Z_6 &= \log\left[1+\frac{X_1}{q}\right], Z_7 = q^2 - (q+X_1)^2 \\
 Z_8 &= \frac{\beta^2 R(1-\alpha)[2RX_2^2 A_r Z_4(\lambda+1) - (\lambda-1)Z_2 Z_3]}{2X_2(\lambda+1)Z_4^{\frac{5}{2}}} - \frac{xb^2(c_b+h_{r1})X_9}{2} \\
 Z_9 &= \frac{Z_1 Z_4 + 2DZ_4 - 3D[R(1-\alpha) - D(\lambda+1)]}{Z_1 Z_4} \\
 Z_{10} &= \frac{1}{X_1 Z_1^2 Z_4^{\frac{3}{2}}} - \frac{RD^2(1-\alpha)(\lambda-1)}{X_1^2 X_2 Z_1^2 Z_4^3} + \frac{2D}{X_1 Z_1^3 Z_4^{\frac{3}{2}}} - \frac{3D[R(1-\alpha) - D(\lambda+1)]}{X_1 Z_1^2 Z_4^{\frac{5}{2}}} \\
 U_1 &= X_1^2 [RD^2(1-\alpha)(\lambda-1)(X_1+2q) - q(q+X_1)X_2 Z_9 Z_4^{\frac{3}{2}}], V_1 = xb(c_b+h_{r1}) \\
 U_2 &= \frac{RD(1-\alpha)(\lambda-1)[X_1 - (X_1+q)Z_6]}{X_1^2 X_2 Z_1^2 Z_4^{\frac{3}{2}}(X_1+q)} + \frac{2Z_6}{X_1 Z_1^3}, V_2 = \frac{bU_7 Z_5}{2X_2 Z_4^{\frac{3}{2}}}
 \end{aligned}$$

$$\begin{aligned}
U_3 &= \frac{\beta\mu R(1-\alpha)(\lambda-1)^2[U_1+q(X_1+q)^2X_1^2X_2Z_{10}Z_6Z_1^2Z_4^3]}{qX_1^3X_2^2Z_1^3Z_4^3(X_1+q)^2\log\lambda} - \frac{\beta\mu(\lambda-1)U_2}{\log\lambda}, \\
V_3 &= -\frac{x(\lambda-1)(c_b+h_{r1})Z_6}{X_1Z_1\log\lambda}, \quad U_4 = R(1-\alpha) - D(\lambda+1), \quad V_4 = \frac{bU_7Z_7}{2qX_1(q+X_1)} \\
U_5 &= \frac{\beta\mu A_r X_2[U_4^2 - (\lambda+1)Z_4]}{(1-\alpha)Z_4^{\frac{3}{2}}} - \frac{\beta\mu(1-\alpha)(\lambda-1)RZ_2(Z_4 - 3DU_4)}{2(\lambda+1)X_2Z_4^{\frac{5}{2}}} + \frac{xb^2(c_b+h_{r1})U_3}{2} \\
V_5 &= V_1^2V_4X_4^2 + U_7^2(Z_8 - 2\beta) + 2\beta U_7V_1V_2X_4 + V_3(\beta^2V_2^2 + 2\beta V_4 - V_4Z_8) \\
U_6 &= \frac{2\mu^2Z_8 + xb^2(c_b+h_{r1})(\mu^2X_9 - \beta^2Y_9)}{2\beta^2}, \quad V_6 = U_7V_2(Z_8 - \beta) + V_1X_4(V_4 + \beta V_2^2) \\
U_7 &= \frac{xb(\lambda-1)(c_b+h_{r1})}{q(q+X_1)Z_1\log\lambda}, \quad V_7 = V_3V_4 - (U_7 - V_1V_2X_4)^2 + V_2^2V_3Z_8 \\
V_8 &= U_7V_1V_2(\mu X_4 - \beta Y_4) - V_1^2V_4X_4Y_4 + \mu(-U_7^2 + V_3V_4 + \beta V_2^2V_3)
\end{aligned}$$

The first and second order partial derivatives of AP_r with respect to b , q , p and f are

$$\begin{aligned}
\frac{\partial AP_r}{\partial q} &= \frac{(\lambda-1)(h_{r1}+c_b)xb^2}{2\log\lambda q(q+X_1)[x(1-\alpha)-D]} - \frac{[2\alpha h_{r2} + (1-\alpha)h_{r1}]}{(\lambda+1)} \\
\frac{\partial^2 AP_r}{\partial q^2} &= -\frac{(\lambda-1)(h_{r1}+c_b)xb^2[(q+X_1)^2 - q^2]}{2X_1\log\lambda[x(1-\alpha)-D]q^2(X_1+q)^2} \\
\frac{\partial AP_r}{\partial b} &= h_{r1} - \frac{bx(\lambda-1)(h_{r1}+c_b)\log(1+\frac{X_1}{q})}{X_1\log\lambda[x(1-\alpha)-D]} \\
\frac{\partial^2 AP_r}{\partial b^2} &= -\frac{x(\lambda-1)(h_{r1}+c_b)\log(1+\frac{X_1}{q})}{X_1\log\lambda[x(1-\alpha)-D]} \\
\frac{\partial^2 AP_r}{\partial b\partial q} &= \frac{\partial^2 AP_r}{\partial q\partial b} = \frac{xb(\lambda-1)(c_b+h_{r1})}{q(q+X_1)Z_1\log\lambda} \\
\frac{\partial AP_r}{\partial p} &= D - \beta p + \beta w + \frac{\beta A_r X_2 [R(1-\alpha) - D(\lambda+1)]}{(1-\alpha)\sqrt{2RD(1-\alpha) - D^2(\lambda+1)}} + \frac{\rho_1\beta}{(1-\alpha)} \\
&\quad + \frac{\beta RD(1-\alpha)(\lambda-1)[2\alpha h_{r2} + (1-\alpha)h_{r1}]}{2X_2(\lambda+1)[2RD(1-\alpha) - D^2(\lambda+1)]^{\frac{3}{2}}} - \frac{xb^2X_4(h_{r1}+c_b)}{2} \\
\frac{\partial^2 AP_r}{\partial q\partial p} &= \frac{\partial^2 AP_r}{\partial p\partial q} = \beta V_2
\end{aligned}$$

$$\begin{aligned} \frac{\partial^2 AP_r}{\partial b \partial p} &= \frac{\partial^2 AP_r}{\partial p \partial b} = -xb(c_b + h_{r1})X_4 \\ \frac{\partial^2 AP_r}{\partial p^2} &= -2\beta - \frac{\beta^2 R(1-\alpha)(\lambda-1)[2\alpha h_{r2} + (1-\alpha)h_{r1}][2D^2(\lambda+1) - RD(1-\alpha)]}{2(\lambda+1)X_2[2RD(1-\alpha) - D^2(\lambda+1)]^{\frac{5}{2}}} \\ &\quad + \frac{(1-\alpha)\beta^2 R^2 A_r X_2}{[2RD(1-\alpha) - D^2(\lambda+1)]^{\frac{3}{2}}} - \frac{xb^2 X_9 (h_{r1} + c_b)}{2} \\ \frac{\partial AP_r}{\partial f} &= \mu p - \mu w - \frac{\mu A_r X_2 [R(1-\alpha) - D(\lambda+1)]}{(1-\alpha)\sqrt{2RD(1-\alpha) - D^2(\lambda+1)}} - \frac{\rho_1 \mu}{(1-\alpha)} \\ &\quad - \frac{\mu RD(1-\alpha)(\lambda-1)[2\alpha h_{r2} + (1-\alpha)h_{r1}]}{2X_2(\lambda+1)[2RD(1-\alpha) - D^2(\lambda+1)]^{\frac{3}{2}}} - \frac{xb^2 Y_4 (h_{r1} + c_b)}{2} - 2gf \\ \frac{\partial^2 AP_r}{\partial f^2} &= -2g - \frac{\mu^2 R(1-\alpha)(\lambda-1)[2\alpha h_{r2} + (1-\alpha)h_{r1}][2D^2(\lambda+1) - RD(1-\alpha)]}{2(\lambda+1)X_2[2RD(1-\alpha) - D^2(\lambda+1)]^{\frac{5}{2}}} \\ &\quad + \frac{(1-\alpha)\mu^2 R^2 A_r X_2}{[2RD(1-\alpha) - D^2(\lambda+1)]^{\frac{3}{2}}} - \frac{xb^2 Y_9 (h_{r1} + c_b)}{2} \\ \frac{\partial^2 AP_r}{\partial b \partial f} &= \frac{\partial^2 AP_r}{\partial f \partial b} = -xb(c_b + h_{r1})Y_4 \\ \frac{\partial^2 AP_r}{\partial q \partial f} &= \frac{\partial^2 AP_r}{\partial f \partial q} = -\mu V_2 \\ \frac{\partial^2 AP_r}{\partial p \partial f} &= \frac{\partial^2 AP_r}{\partial f \partial p} = \mu - U_5 \end{aligned}$$

A.3

If the Hessian matrix is negative definite, then there exist unique optimal batch size (q^*) and backorder quantity (b^*) of the retailer. The optimal values of q and b can be derived by solving $\frac{\partial AP_r}{\partial q} = 0$ and $\frac{\partial AP_r}{\partial b} = 0$. The first equation gives $q = \frac{-X_1 \pm \sqrt{X_1^2 + \frac{2b(\lambda^2-1)X_1 V_1}{Z_1 Z_2 \log \lambda}}}{2}$. Since, $q > 0$ and $X_1 > 0$, only positive root of the equation will be considered. After solving these two equations, we get the optimal batch size and backorder quantity as

$$\begin{aligned} q^* &= \frac{-X_1 + \sqrt{X_1^2 + \frac{2b(\lambda^2-1)X_1 V_1}{Z_1 Z_2 \log \lambda}}}{2} \\ b^* &= \frac{h_{r1} X_1 \log \lambda [x(1-\alpha) - D]}{x(\lambda-1)(c_b + h_{r1}) \log \left(1 + \frac{X_1}{q}\right)} \end{aligned}$$

A.4

If the sufficient conditions for concavity of AP_r hold, then the unique optimal retail

price and optimal sales effort level of the retailer can be obtained from $\frac{\partial AP_r}{\partial p} = 0$ and $\frac{\partial AP_r}{\partial f} = 0$. After simplification, these two equations become

$$\begin{aligned} & \left[(1 - \alpha) \{ bV_1 X_4 - 2(D + \beta w - \beta p) \} - 2\rho_1 \beta \right] (\lambda + 1) X_2 Z_4^{\frac{3}{2}} \\ &= \beta \left[2(\lambda + 1) A_r X_2^2 U_4 Z_4 + RDZ_2 (1 - \alpha)^2 (\lambda - 1) \right] \\ & \left[(1 - \alpha) \{ 2\mu(p - w) - 4gf - bV_1 Y_4 \} - 2\rho_1 \mu \right] (\lambda + 1) X_2 Z_4^{\frac{3}{2}} \\ &= \mu \left[2(\lambda + 1) A_r X_2^2 U_4 Z_4 + RDZ_2 (1 - \alpha)^2 (\lambda - 1) \right] \end{aligned}$$

A.5

To simplify the mathematical derivations, the following notations are used:

$$\begin{aligned} \rho_2 &= (c_1 \theta_1 + c_2 \theta_2 + c_3 \theta_3 - s_2 - c_w - c_v - T_v - c_e \theta_3 - R_w \theta_1 - R_s \theta_2 - R_d \theta_3) \alpha \\ \rho_3 &= c_m + c_v + T_v + s_1 + c_r (1 - \gamma) m_1 + c_a \gamma m_2 - \rho_2 \\ \rho_4 &= \frac{1}{2kq} \left[\frac{\rho_3}{1 - \alpha} + \frac{(A_m + A_r)}{kq(1 - \alpha)} + \frac{h_m q (2\lambda - \lambda^n - 1)}{2R(1 - \alpha)(\lambda - 1)} + nxb^2(c_b + h_{r1}) \right] \\ \rho_5 &= x(1 - \alpha) - (a + \mu f) \\ \rho_6 &= x(1 - \alpha) - (a - \beta p) \\ \rho_7 &= \left[\frac{\lambda(\lambda^{n-1} - 1)}{(\lambda^2 - 1)} + \frac{D(2\lambda - \lambda^n - 1)}{2R(\lambda - 1)(1 - \alpha)} \right] \end{aligned}$$

The first and second order partial derivatives of AP_s with respect to the decision variables n (real), q , p , b and f are

$$\begin{aligned} \frac{\partial AP_s}{\partial b} &= h_{r1} - \frac{nbx(c_b + h_{r1})}{kq[x(1 - \alpha) - D]} \\ \frac{\partial^2 AP_s}{\partial b^2} &= -\frac{nx(c_b + h_{r1})}{kq[x(1 - \alpha) - D]} = \chi_{11} \\ \frac{\partial AP_s}{\partial p} &= D - \beta p + \frac{\rho_3 \beta}{(1 - \alpha)} + \frac{(A_m + A_r) \beta}{kq(1 - \alpha)} + \frac{h_m q \beta (2\lambda - \lambda^n - 1)}{2R(1 - \alpha)(\lambda - 1)} + \frac{nx\beta b^2(c_b + h_{r1})}{2kq[x(1 - \alpha) - D]^2} \\ \frac{\partial^2 AP_s}{\partial p \partial b} &= \frac{\partial^2 AP_s}{\partial b \partial p} = \frac{nx\beta b(c_b + h_{r1})}{kq[x(1 - \alpha) - D]^2} = \chi_{12} = \chi_{21} \\ \frac{\partial^2 AP_s}{\partial p^2} &= -\beta \left[2 + \frac{nx\beta b^2(c_b + h_{r1})}{kq[x(1 - \alpha) - D]^3} \right] = \chi_{22} \\ \frac{\partial^2 AP_s}{\partial b \partial f} &= \frac{\partial^2 AP_s}{\partial f \partial b} = -\frac{n\mu x b(c_b + h_{r1})}{kq[x(1 - \alpha) - D]^2} = \chi_{13} = \chi_{31} \end{aligned}$$

$$\begin{aligned} \frac{\partial AP_s}{\partial f} &= -2fg + \mu p - \frac{\rho_3 \mu}{(1-\alpha)} - \frac{(A_m + A_r)\mu}{kq(1-\alpha)} - \frac{h_m q \mu (2\lambda - \lambda^n - 1)}{2R(1-\alpha)(\lambda - 1)} - \frac{nx\mu b^2(c_b + h_{r1})}{2kq[x(1-\alpha) - D]^2} \\ \frac{\partial^2 AP_s}{\partial p \partial f} &= \frac{\partial^2 AP_s}{\partial f \partial p} = \mu + \frac{nx\beta\mu b^2(c_b + h_{r1})}{kq[x(1-\alpha) - D]^3} = \chi_{23} = \chi_{32} \\ \frac{\partial^2 AP_s}{\partial f^2} &= -2g - \frac{nx\mu^2 b^2(c_b + h_{r1})}{kq[x(1-\alpha) - D]^3} = \chi_{33} \\ \frac{\partial AP_s}{\partial q} &= \frac{(A_m + A_r)D}{kq^2(1-\alpha)} - h_m \left[\frac{\lambda(\lambda^{n-1} - 1)}{(\lambda^2 - 1)} + \frac{D(2\lambda - \lambda^n - 1)}{2R(\lambda - 1)(1-\alpha)} \right] - \frac{[\alpha h_{r2} + (1-\alpha)h_{r1}]l}{k} \\ &\quad + \frac{nx b^2(c_b + h_{r1})}{2kq^2[x(1-\alpha) - D]} \\ \frac{\partial^2 AP_s}{\partial q^2} &= -\frac{nx b^2(c_b + h_{r1})}{kq^3[x(1-\alpha) - D]} - \frac{2D(A_r + A_m)}{(1-\alpha)kq^3[x(1-\alpha) - D]} = \chi_{44} \\ \frac{\partial AP_s}{\partial n} &= \frac{(A_r + A_m)D\lambda^n \log \lambda}{k^2 q(1-\alpha)(\lambda - 1)} - \frac{h_m q \lambda^n \log \lambda [2R(1-\alpha) - (\lambda + 1)D]}{2R(1-\alpha)(\lambda^2 - 1)} \\ &\quad - \frac{xb^2(c_b + h_{r1})(\lambda - 1)[(\lambda^n - 1) - n\lambda^n \log \lambda]}{2q(\lambda^n - 1)^2[x(1-\alpha) - D]} - \frac{q\lambda^n \log \lambda [h_{r1}(1-\alpha) + 2\alpha h_{r2}]}{2(\lambda + 1)} \\ \frac{\partial^2 AP_s}{\partial n^2} &= -\frac{(A_r + A_m)D(\lambda - 1)\lambda^n (\log \lambda)^2 (\lambda^n + 1)}{q(1-\alpha)(\lambda^n - 1)^3} - \frac{h_m q \lambda^n (\log \lambda)^2 [2R(1-\alpha) - (\lambda + 1)D]}{2R(1-\alpha)(\lambda^2 - 1)} \\ &\quad - \frac{q\lambda^n (\log \lambda)^2 [h_{r1}(1-\alpha) + 2\alpha h_{r2}]}{2(\lambda + 1)} + \frac{(c_b + h_{r1})(\lambda - 1)xb^2\lambda^n \log \lambda}{2q(\lambda^n - 1)^2[x(1-\alpha) - D]} \\ &\quad + \frac{xb^2\lambda^n (\lambda - 1) \log \lambda (c_b + h_{r1}) \{(\lambda^n - 1)(n \log \lambda + 1) - 2n\lambda^n \log \lambda\}}{2q(\lambda^n - 1)^3[x(1-\alpha) - D]} = \chi_{55} \\ \frac{\partial^2 AP_s}{\partial n \partial q} &= \frac{\partial^2 AP_s}{\partial q \partial n} = -\frac{(A_m + A_r)D\lambda^n \log \lambda}{k^2 q^2(1-\alpha)(\lambda - 1)} - \frac{h_m \lambda^n \log \lambda [2R - (\lambda + 1)D]}{2R(\lambda^2 - 1)} \\ &\quad - \frac{\lambda^n \log \lambda [h_{r1}(1-\alpha) + 2\alpha h_{r2}]}{2(\lambda + 1)} + \frac{xb^2(c_b + h_{r1})(\lambda - 1)[(\lambda^n - 1) - n\lambda^n \log \lambda]}{2q^2(\lambda^n - 1)^2[x(1-\alpha) - D]} = \chi_{45} = \chi_{54} \\ \frac{\partial^2 AP_s}{\partial n \partial b} &= \frac{\partial^2 AP_s}{\partial b \partial n} = -\frac{xb(c_b + h_{r1})(\lambda - 1)[(\lambda^n - 1) - n\lambda^n \log \lambda]}{q[x(1-\alpha) - D](\lambda^n - 1)^2} = \chi_{15} = \chi_{51} \\ \frac{\partial^2 AP_s}{\partial n \partial f} &= \frac{\partial^2 AP_s}{\partial f \partial n} = \frac{(A_m + A_r)\mu \lambda^n \log \lambda}{k^2 q(1-\alpha)(\lambda - 1)} + \frac{h_m \lambda^n q \mu \log \lambda}{2R(\lambda - 1)(1-\alpha)} \\ &\quad - \frac{(\lambda - 1)(c_b + h_{r1})\mu x b^2 [(\lambda^n - 1) - n\lambda^n \log \lambda]}{2q(\lambda - 1)^2[x(1-\alpha) - D]^2} = \chi_{35} = \chi_{53} \\ \frac{\partial^2 AP_s}{\partial n \partial p} &= \frac{\partial^2 AP_s}{\partial p \partial n} = -\frac{(A_r + A_m)\beta \lambda^n \log \lambda}{qk^2(1-\alpha)(\lambda - 1)} + \frac{\beta x b^2 (\lambda - 1)(c_b + h_{r1}) [(\lambda^n - 1) - n\lambda^n \log \lambda]}{2q(\lambda^n - 1)^2[x(1-\alpha) - D]^2} \\ &\quad - \frac{h_m \beta q \lambda^n \log \lambda}{2R(\lambda - 1)(1-\alpha)} = \chi_{25} = \chi_{52} \end{aligned}$$

$$\frac{\partial^2 AP_s}{\partial q \partial p} = \frac{\partial^2 AP_s}{\partial p \partial q} = -\frac{(A_r + A_m)\beta}{k(1-\alpha)q^2} - \frac{n\beta x b^2(c_b + h_{r1})}{2kq^2[x(1-\alpha) - D]^2} + \frac{\beta h_m(2\lambda - \lambda^n - 1)}{2R(\lambda - 1)(1-\alpha)} = \chi_{24} = \chi_{42}$$

$$\frac{\partial^2 AP_s}{\partial q \partial b} = \frac{\partial^2 AP_s}{\partial b \partial q} = \frac{nx b(c_b + h_{r1})}{kq^2[x(1-\alpha) - D]} = \chi_{14} = \chi_{41}$$

$$\frac{\partial^2 AP_s}{\partial q \partial f} = \frac{\partial^2 AP_s}{\partial f \partial q} = \frac{(A_r + A_m)\mu}{k(1-\alpha)q^2} + \frac{n\mu x b^2(c_b + h_{r1})}{2kq^2[x(1-\alpha) - D]^2} - \frac{\mu h_m(2\lambda - \lambda^n - 1)}{2R(\lambda - 1)(1-\alpha)} = \chi_{34} = \chi_{43}$$

Therefore, the Hessian matrix associated with $AP_s(b, p, f, q, n)$ is given by

$$H = \begin{pmatrix} \frac{\partial^2 AP_s}{\partial b^2} & \frac{\partial^2 AP_s}{\partial b \partial p} & \frac{\partial^2 AP_s}{\partial b \partial f} & \frac{\partial^2 AP_s}{\partial b \partial q} & \frac{\partial^2 AP_s}{\partial b \partial n} \\ \frac{\partial^2 AP_s}{\partial p \partial b} & \frac{\partial^2 AP_s}{\partial p^2} & \frac{\partial^2 AP_s}{\partial p \partial f} & \frac{\partial^2 AP_s}{\partial p \partial q} & \frac{\partial^2 AP_s}{\partial p \partial n} \\ \frac{\partial^2 AP_s}{\partial f \partial b} & \frac{\partial^2 AP_s}{\partial f \partial p} & \frac{\partial^2 AP_s}{\partial f^2} & \frac{\partial^2 AP_s}{\partial f \partial q} & \frac{\partial^2 AP_s}{\partial f \partial n} \\ \frac{\partial^2 AP_s}{\partial q \partial b} & \frac{\partial^2 AP_s}{\partial q \partial p} & \frac{\partial^2 AP_s}{\partial q \partial f} & \frac{\partial^2 AP_s}{\partial q^2} & \frac{\partial^2 AP_s}{\partial q \partial n} \\ \frac{\partial^2 AP_s}{\partial n \partial b} & \frac{\partial^2 AP_s}{\partial n \partial p} & \frac{\partial^2 AP_s}{\partial n \partial f} & \frac{\partial^2 AP_s}{\partial n \partial q} & \frac{\partial^2 AP_s}{\partial n^2} \end{pmatrix} = \begin{pmatrix} \chi_{11} & \chi_{12} & \chi_{13} & \chi_{14} & \chi_{15} \\ \chi_{21} & \chi_{22} & \chi_{23} & \chi_{24} & \chi_{25} \\ \chi_{31} & \chi_{32} & \chi_{33} & \chi_{34} & \chi_{35} \\ \chi_{41} & \chi_{42} & \chi_{43} & \chi_{44} & \chi_{45} \\ \chi_{51} & \chi_{52} & \chi_{53} & \chi_{54} & \chi_{55} \end{pmatrix}$$

The first order principal minor is

$$|H_{11}| = |\chi_{11}| = -\frac{nx(c_b + h_{r1})}{kq[x(1-\alpha) - D]} < 0 \text{ as } x(1-\alpha) > D.$$

The second order principal minor is

$$|H_{22}| = \begin{vmatrix} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{vmatrix} = \frac{2\beta nx(c_b + h_{r1})}{kq[x(1-\alpha) - D]} > 0 \text{ as } x(1-\alpha) > D.$$

The third order principal minor is

$$|H_{33}| = \begin{vmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{21} & \chi_{22} & \chi_{23} \\ \chi_{31} & \chi_{32} & \chi_{33} \end{vmatrix} = \frac{nx(c_b + h_{r1})(\mu^2 - 4\beta g)}{kq[x(1-\alpha) - D]}$$

Therefore, the third order principal minor will be negative if $\mu^2 < 4\beta g$.

The fourth order principal minor is

$$|H_{44}| = \begin{vmatrix} \chi_{11} & \chi_{12} & \chi_{13} & \chi_{14} \\ \chi_{21} & \chi_{22} & \chi_{23} & \chi_{24} \\ \chi_{31} & \chi_{32} & \chi_{33} & \chi_{34} \\ \chi_{41} & \chi_{42} & \chi_{43} & \chi_{44} \end{vmatrix}$$

$$\begin{aligned}
&= \frac{2n^2x^2\beta^2b^2g(c_b+h_{r1})^2}{k^3q^5(1-\alpha)[x(1-\alpha)-D]^3} + \frac{h_m n^2 x^2 \beta b^2 (c_b+h_{r1})^2 (2\lambda-\lambda^n-1)(\mu^2-2\beta g)}{2Rk^2q^3(1-\alpha)(\lambda-1)[x(1-\alpha)-D]^3} \\
&\quad - \frac{2nx\beta^2g(c_b+h_{r1})(A_m+A_r)^2}{k^3q^5(1-\alpha)^2[x(1-\alpha)-D]} + \frac{2h_m nx\beta^2g(c_b+h_{r1})(A_m+A_r)(2\lambda-\lambda^n-1)}{Rk^2q^3(1-\alpha)^2(\lambda-1)[x(1-\alpha)-D]} \\
&\quad + \frac{n^3x^3\beta^2b^3g(c_b+h_{r1})^3}{2k^3q^5[x(1-\alpha)-D]^5} - \frac{h_m nx\beta^2g(c_b+h_{r1})(2\lambda-\lambda^n-1)^2}{2R^2kq(1-\alpha)^2(\lambda-1)^2[x(1-\alpha)-D]} \\
&\quad - \frac{2Dnx(c_b+h_{r1})(A_m+A_r)(\mu^2-4\beta g)}{k^2q^4(1-\alpha)[x(1-\alpha)-D]}
\end{aligned}$$

Simplifying the inequality $|H_{44}| > 0$, we have $nx\beta\sigma_2Rb^2(1-\alpha)(\lambda-1)(c_b+h_{r1}) > [x(1-\alpha)-D]^4\sigma_4$, where

$$\begin{aligned}
\sigma_1 &= 4\beta g(\lambda-1)R + h_mkq^2(2\lambda-\lambda^n-1)(\mu^2-2\beta g) \\
\sigma_2 &= \sigma_1[x(1-\alpha)-D]^2 + nxR\beta b g(1-\alpha)(\lambda-1)(c_b+h_{r1}) \\
\sigma_3 &= R\beta^2g(\lambda-1)(A_m+A_r) + DRkq(1-\alpha)(\lambda-1)(\mu^2-4\beta g) - h_mk\beta^2q^2g(2\lambda-\lambda^n-1) \\
\sigma_4 &= 4R(\lambda-1)(A_m+A_r)\sigma_3 + h_m^2\beta^2k^2q^4g(2\lambda-\lambda^n-1)^2
\end{aligned}$$

The fifth order principal minor

$$|H_{55}| = |H| = \chi_{11}D_1 - \chi_{12}D_2 + \chi_{13}D_3 - \chi_{14}D_4 + \chi_{15}D_5$$

where D_i is the i -th fourth order principal minor in the expansion of $|H|$ with respect to the first row. Therefore, $|H_{55}| < 0$ gives $\chi_{11}D_1 + \chi_{13}D_3 + \chi_{15}D_5 < \chi_{12}D_2 + \chi_{14}D_4$.

Because of higher complexity of the model, it is very difficult to prove the negative definiteness of the Hessian matrix analytically. So, we examine it numerically. Hence, the average supply chain profit is globally optimal with respect to its decision variables if the Hessian matrix H is negative definite.

A.6

If the sufficient conditions for concavity of the supply chain profit function (AP_s) are satisfied, then there exist unique optimal values of the decision variables. Therefore, by solving the equations $\frac{\partial AP_s}{\partial q} = 0$ and $\frac{\partial AP_s}{\partial b} = 0$, we obtain the optimal values of q and b as

$$\begin{aligned}
q^* &= \sqrt{\frac{2DZ_1(A_r+A_m) + nbV_1(1-\alpha)}{2Z_1(1-\alpha)[kh_m\rho_7 + lZ_2]}} \\
b^* &= \frac{kqh_{r1}[x(1-\alpha)-D]}{nx(c_b+h_{r1})}
\end{aligned}$$

A.7

If the sufficient conditions for concavity of the function AP_s hold, then the unique optimal retail price and optimal sales effort level of the supply chain can be obtained by solving $\frac{\partial AP_s}{\partial p} = 0$ and $\frac{\partial AP_s}{\partial f} = 0$. After simplification, these two equations become

$$\begin{aligned}
2\beta^3p^3 + [2\beta\rho_5^2 - 2\beta\rho_5(\bar{a} + \mu f)] &= [(a + \mu f)\beta^2 - 4\beta^2\rho_5]p^2 + [(a + \mu f)\rho_5^2 + \beta\rho_4] \\
2g\mu^2f^3 + [2\mu^2p\rho_6 + 2g\rho_6^2] &= [\mu^3p + 4g\mu\rho_6]f^2 + [\mu p\rho_6^2 - \rho_4\mu]
\end{aligned}$$

Two-echelon supply chain model with price and green sensitive demand under equal and unequal batch shipment policy

4.1 Introduction

In recent years, various industries and firms are paying much attention in producing environment friendly products to save our nature from being over-polluted. Demand of environment conscious customer is also influenced by the environmental performance of the products. It is found from a survey of European Commission (2013) that there are about 40.7 percent such customers who are willing to pay more for more greener products (Zanoni et al., 2014). Companies are taking initiatives to improve the environmental performance or greenness of the goods in many different ways such as reducing carbon emissions caused by the production system (Hewlett-Packard, Disney), using some renewable energies rather than fossil fuels for transportation and production (Ford, Nike), reducing waste with less packaging (Johnson and Johnson), using recycled materials as raw materials to reduce greenhouse gas and harmful gas emissions from the wastes (Ford, Nike, ebay). This chapter* focuses on such widespread adoption of green practices. However, to

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manufacture such eco-friendly products, the manufacturers have to invest a huge amount of money. So, to motivate the manufacturer to take such initiatives, the retailers sometimes coordinate with the manufacturer under some contracts such as cost sharing contract, profit sharing contract, revenue sharing contract, etc. Therefore, bearing in mind these facts, this chapter incorporates a cost-sharing contract in a two-level green supply chain model with price and green sensitive demand under combined equal and unequal sized batch shipments and planned backorders. Our aim is to put into effect a cost sharing contract between the supply chain participants and assess how this contract between the manufacturer and the retailer affects their profits as well as the key decisions of the proposed green supply chain. Additionally, it is intended to determine whether this contract can enhance the supply chain's performance on both an economic and environmental level.

4.2 Problem Definition and Assumptions

In this chapter, we consider a two-echelon supply chain model consisting of a single manufacturer and a single retailer. The manufacturer produces the retailer's order quantity in one set up and transfers the lot to the retailer in some equal and unequal sized batches. All unequal sized batches are in a geometric progression with a fixed common ratio. The customer's demand is treated as linearly dependent on the product's retail price and its greening improvement level. A planned shortage is allowed and the customers during the shortage period wait for the next shipment to meet up their unsatisfied demands. In practice, there are inventory systems where such shortages are allowed to occur in order to reduce the average total cost (Hsu & Hsu, 2013). For example, in a jewellery shop, the shopkeeper does not always stock enough jewellery to meet up the demand of all customers. Customer often have to wait for delivery. Assuming that the retailer has higher power than the manufacturer, we develop non-coordinated decentralized model and coordinated model with cost sharing contract, and the integrated centralized model. The assumptions of the proposed model are given below:

1. We consider a two-echelon supply chain model with one manufacturer and one retailer. The retailer has dominant power over the manufacturer.

2. Demand at the retailer is a linear function of the product's retail price (p) and the greening improvement level (θ) i.e., $D(p, \theta) = a - \beta p + \alpha \theta$, where α and β are positive constants and $a + \alpha \theta > \beta p$ i.e., demand is always positive.
3. The manufacturer manufactures the retailer's order quantity in a single lot and delivers to the retailer in a combination of some equal and unequal sized batches.
4. The retailer decides the order quantity and number of deliveries, and pay the transportation cost.
5. The annual investment for greening cost of the product is taken as $I\theta^2$, which is increasing and convex in the greening improvement level θ .
6. Shortages are allowed and completely backlogged.

4.3 Notations

The following notations are used for developing the proposed model:

A_1	: Production set-up cost (\$/set-up)
A_2	: Transportation cost of the retailer (\$/shipment)
A_3	: Retailer's ordering cost (\$/order)
h_1	: Manufacturer's holding cost (\$/unit/unit time)
h_2 ($h_2 > h_1$)	: Retailer's holding cost (\$/unit/unit time)
c_2	: Shortage cost at the retailer (\$/unit/unit time)
P	: Manufacturer's production rate (units/unit time)
D	: Demand rate at the retailer ($P > D$) (units/unit time)
b	: Shortage quantity at the retailer (units)
z	: Smallest batch size (units)
p	: Retailer's unit retail price (\$/unit)
c	: Manufacturer's unit production cost (\$/unit)
I	: Greening investment parameter (\$)
m_1	: Price margin (%) at the retailer, [$p = (1 + m_1)w$]
a	: Total market potential (units/unit time)
α	: Consumer sensitivity coefficient to greening level

β	: Consumer sensitivity coefficient to retail price
k	: A real positive number ($1 < k \leq \frac{P}{D}$)
AP_m	: Average profit of the manufacturer (\$/unit time)
AP_r	: Average profit of the retailer (\$/unit time)
AP_s	: Average profit of the supply chain system (\$/unit time)

Decision

Variables

w	: Manufacturer's unit wholesale price (\$/unit)
θ	: Greening improvement level
n	: Total number of batches (positive integer)
m	: Total number of unequal sized batches (positive integer)
Q	: Retailer's order quantity (units)
ϕ	: Proportion of greening cost which the retailer shares ($0 < \phi < 1$)

4.4 Model Development

We suppose that the manufacturer produces a single product at a constant production rate P and sells it to the retailer. The retailer faces an annual demand D and places an order of size Q . Therefore, the number of cycles per unit time is $\frac{D}{Q}$. The manufacturer manufactures a lot size Q and delivers to the retailer in n shipments with m unequal and $(n - m)$ equal sized batches. We assume that the smallest batch size is z . All unequal sized batches are increased to the next by a known factor k ($1 < k \leq \frac{P}{D}$). The batch sizes are, therefore, $z, kz, k^2z, k^3z, \dots, k^{m-1}z, k^{m-1}z, \dots, k^{m-1}z$. A planned shortage is allowed at the retailer and is completely backlogged. When the shortage level reaches to b , the retailer requests the manufacturer for the next shipment. Figures 4.1 and 4.2 illustrate the inventory level of the manufacturer and the retailer with respect to time. Since the manufacturer meets the ordering quantity Q of the retailer in m unequal sized batches $z, kz, k^2z, k^3z, \dots, k^{m-1}z$ and $(n - m)$ equal sized batches $k^{m-1}z, k^{m-1}z, \dots, k^{m-1}z$, therefore, we have

$$z + kz + k^2z + k^3z + \dots + k^{m-1}z + (n - m)k^{m-1}z = Q$$

$$\text{i.e., } z = \frac{Q}{f(n, m)}, \text{ where } f(n, m) = \sum_{r=0}^{m-1} k^r + (n - m)k^{m-1}$$

The total inventory holding area of the manufacturer-retailer system

$$\begin{aligned}
 &= \text{Area of the trapezoid ABLM (see Figure 4.1).} \\
 &= \frac{1}{2}(AB + LM)Q \\
 &= \frac{Q}{2} \left[\frac{z}{P} + \left(\frac{Q}{D} + \frac{z}{P} - \frac{Q}{P} \right) \right] \\
 &= \frac{Qz}{2P} + \frac{Q^2}{2} \left(\frac{1}{D} - \frac{1}{P} \right)
 \end{aligned}$$

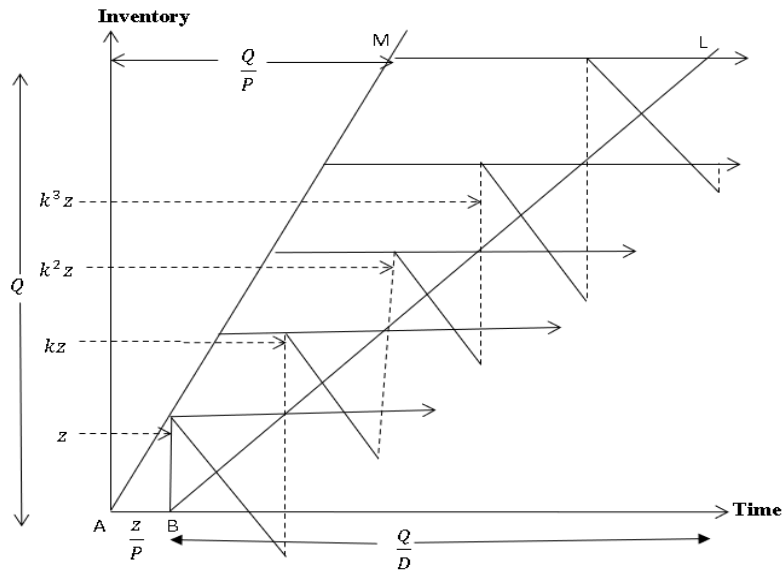


Figure 4.1: Inventory of the manufacturer and retailer system

The total inventory holding area of the retailer

$$\begin{aligned}
 &= \text{Sum of the triangular areas (shaded) in Figure 4.2} \\
 &= \left[\frac{1}{2}(z - b) \frac{(z - b)}{D} \right] + \left[\frac{1}{2}(kz - b) \frac{(kz - b)}{D} \right] + \left[\frac{1}{2}(k^2z - b) \frac{(k^2z - b)}{D} \right] \\
 &\quad + \left[\frac{1}{2}(k^{m-1}z - b) \frac{(k^{m-1}z - b)}{D} \right] + (n - m) \left[\frac{1}{2}(k^{m-1}z - b) \frac{(k^{m-1}z - b)}{D} \right] \\
 &= \frac{g}{2D}, \quad \text{where } g = \sum_0^{m-1} (k^r z - b)^2 + (n - m)(k^{m-1}z - b)^2
 \end{aligned}$$

Inventory holding area of the manufacturer

$$\begin{aligned}
 &= (\text{Inventory holding area of the manufacturer-retailer system}) - (\text{inventory} \\
 &\quad \text{holding area of the retailer}) = \frac{Qz}{P} + \frac{Q^2}{2} \left(\frac{1}{D} - \frac{1}{P} \right) - \frac{g}{2D}
 \end{aligned}$$

$$\text{Shortage quantity at the retailer} = \frac{nb^2}{2D}$$

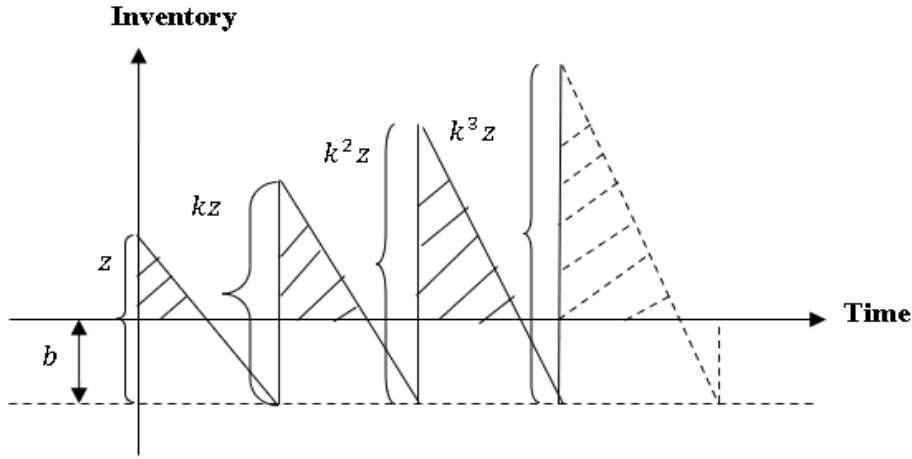


Figure 4.2: Inventory of the retailer

A. Retailer's average profit

The retailer's total cost includes ordering cost, transportation cost, inventory holding cost and shortage cost. The ordering cost = A_3 , transportation cost for the n shipments = nA_2 , holding cost of the retailer = $\frac{h_2g}{2D}$ and shortage cost = $\frac{nc_2b^2}{2D}$. Hence retailer's total profit is given by

$$TP_r = pQ - wQ - A_3 - nA_2 - \frac{h_2g}{2D} - \frac{nc_2b^2}{2D}$$

As there are $\frac{D}{Q}$ cycles per year, the average profit function of the retailer is given by

$$AP_r(n, m, Q, m_1) = \frac{D}{Q} \left[pQ - wQ - A_3 - nA_2 - \frac{h_2g}{2D} - \frac{nc_2b^2}{2D} \right] \quad (4.1)$$

B. Manufacturer's average profit

The manufacturer's total cost includes set up cost, production cost, inventory holding cost and greening cost. Set up cost = A_1 , production cost = cQ , holding cost = $h_1 \left[\frac{Qz}{P} + \frac{Q^2}{2} \left(\frac{1}{D} - \frac{1}{P} \right) - \frac{g}{2D} \right]$ and greening cost = $I\theta^2$.

Hence the average profit of the manufacturer is given by

$$AP_m(w, \theta) = \frac{D}{Q} \left[wQ - cQ - A_1 - h_1 \left[\frac{Qz}{P} + \frac{Q^2}{2} \left(\frac{1}{D} - \frac{1}{P} \right) - \frac{g}{2D} \right] \right] - I\theta^2 \quad (4.2)$$

Average profit of the manufacturer-retailer integrated system is

$$AP_s(n, m, Q, \theta, m_1) = pD - cD - \frac{h_1 D}{Q} \left[\frac{Qz}{P} + \frac{Q^2}{2} \left(\frac{1}{D} - \frac{1}{P} \right) - \frac{g}{2D} \right] - \frac{h_2 g}{2Q} - \frac{A_1 D}{Q} - \frac{n A_2 D}{Q} - \frac{A_3 D}{Q} - \frac{n c_2 b^2}{2Q} - I\theta^2 \quad (4.3)$$

4.4.1 Decentralized Model

In the decentralized case, the manufacturer and the retailer do not co-operate with each other in maximizing their mutual profits and they make their own decisions independently. Here we assume that the retailer acts as the leader because of his higher power and the manufacturer as the follower. So knowing the retailer's number of shipments (n), the order size (Q) and margin (m_1), the manufacturer first decides his wholesale price (w) and product's greening level (θ) by maximizing his profit. Getting these response from the manufacturer, the retailer finally decides his optimal decisions, i.e., the number of shipments (n), the order size (Q) and the price margin (m_1). Taking the first and the second order derivatives of $AP_m(w, \theta)$ with respect to w we get

$$\begin{aligned} \frac{\partial(AP_m)}{\partial w} &= a - \beta w(1 + m_1) + \alpha\theta - \beta(1 + m_1) \left[w - c - \frac{A_1}{Q} - \frac{h_1 z}{P} + \frac{h_1 Q}{2P} \right] \\ \text{and } \frac{\partial^2(AP_m)}{\partial w^2} &= -2\beta(1 + m_1) < 0, \text{ as } \beta > 0 \end{aligned}$$

Thus $AP_m(w, \theta)$ is concave in w . So there exists a unique value of w which maximizes AP_m . The optimal value of w can be obtained by solving $\frac{\partial(AP_m)}{\partial w} = 0$ which gives

$$w = \frac{a + \alpha\theta}{2\beta(1 + m_1)} + \frac{S}{2}, \text{ where } S = c + \frac{A_1}{Q} + \frac{h_1 z}{P} - \frac{h_1 Q}{2P} \quad (4.4)$$

Similarly, taking the first and the second order derivatives of $AP_m(w, \theta)$ with respect to θ , we get

$$\begin{aligned} \frac{\partial(AP_m)}{\partial \theta} &= \alpha(w - S) - 2I\theta \\ \text{and } \frac{\partial^2(AP_m)}{\partial \theta^2} &= -2I < 0 \end{aligned}$$

This shows that AP_m is concave in θ and it attains the maximum value for a unique value of θ obtained by solving $\frac{\partial(AP_m)}{\partial\theta} = 0$ which gives

$$\theta = \frac{\alpha(w - S)}{2I} \quad (4.5)$$

Now solving (4.4) and (4.5), we get

$$w^* = \frac{2aI + 2I\beta S(1 + m_1) - \alpha^2 S}{4I\beta(1 + m_1) - \alpha^2} \quad (4.6)$$

$$\theta^* = \frac{\alpha[a - \beta S(1 + m_1)]}{4I\beta(1 + m_1) - \alpha^2} \quad (4.7)$$

Using these reaction functions of the manufacturer, the retailer sets up his optimal margin, optimal order quantity and optimal number of shipments. Substituting these values of w^* and θ^* in (4.1), the average profit of the retailer can be obtained as

$$\begin{aligned} AP_r(n, m, Q, m_1) &= \left[\frac{m_1(2aI + 2I\beta S(1 + m_1) - \alpha^2 S)}{4I\beta(1 + m_1) - \alpha^2} - \frac{A_3}{Q} - \frac{nA_2}{Q} \right] \\ &\times \left[\frac{2I\beta(1 + m_1)[a - \beta S(1 + m_1)]}{4I\beta(1 + m_1) - \alpha^2} \right] - \frac{h_2 g}{2Q} - \frac{nc_2 b^2}{2Q} \end{aligned}$$

Due to complexity, the formal proof of the concavity of the retailer's profit function (given above) with respect to decision variables n, m, Q and m_1 can not be provided. Instead, any suitable numerical method can be applied to examine the concavity behavior of this profit function.

4.4.2 Coordinated Model with Cost Sharing Contract

Nowadays people are more concerned about major environmental issues. Along with the retail price, environmental impacts of the product are also taking attention of the customers. In our problem, the demand rate (D) is linearly dependent on the greening improvement level (θ) of the product. So, the customer demand increases if the product's greening level improves. But the cost of greening falls upon the manufacturer who is not always able to improve the greening level because of cost

burden. In such situations, to motivate the manufacturer to undertake green initiatives the retailer offers to share the cost burden of the manufacturer. In this section, we consider a cost sharing contract between the manufacturer and the retailer. In order to increase customer's demand, the retailer shares a portion of the greening cost because the profit of the retailer will increase while demand increases. In that case, the manufacturer will accept the offer because this will lead to a cost reduction to him. Since the market demand increases, the manufacturer has the opportunity to earn more profit by increasing the wholesale price also. We assume that the retailer shares a portion ϕ of the greening cost and the manufacturer spends $(1 - \phi)$ portion of the greening cost.

Under this assumption the profit functions of the manufacturer and retailer are given by

$$AP_m(w, \theta) = wD - cD - \frac{A_1D}{Q} - \frac{h_1zD}{P} - \frac{h_1Q}{2} + \frac{h_1QD}{2P} + \frac{h_1g}{2Q} - (1 - \phi)I\theta^2 \quad (4.8)$$

$$AP_r(n, m, Q, m_1, \phi) = pD - wD - \frac{A_3D}{Q} - \frac{nA_2D}{Q} - \frac{h_2g}{2Q} - \frac{nc_2b^2}{2Q} - \phi I\theta^2 \quad (4.9)$$

The manufacturer first announces his optimal wholesale price and greening improvement level. Then, using these reaction functions, the retailer decides his optimal order quantity (Q^*), number of batch shipments (n^*), number of unequal sized batches (m^*), optimal margin (m_1^*) and optimal proportion of cost sharing (ϕ^*). Taking first and second order derivatives of AP_m with respect to w and θ , we get

$$\begin{aligned} \frac{\partial(AP_m)}{\partial w} &= a - \beta w(1 + m_1) + \alpha\theta - \beta(1 + m_1) \left[w - c - \frac{A_1}{Q} - \frac{h_1z}{P} + \frac{h_1Q}{2P} \right] \\ \frac{\partial^2(AP_m)}{\partial w^2} &= -2\beta(1 + m_1) < 0 \text{ as } \beta > 0 \\ \frac{\partial(AP_m)}{\partial \theta} &= \alpha(w - S) - 2I\theta(1 - \phi) \\ \text{and } \frac{\partial^2(AP_m)}{\partial \theta^2} &= -2I(1 - \phi) < 0 \end{aligned}$$

The profit function AP_m is concave with respect to w as well as θ . Solving the first order necessary conditions for optimality, i.e., solving $\frac{\partial(AP_m)}{\partial w} = 0$ and $\frac{\partial(AP_m)}{\partial \theta} = 0$, we get

$$w = \frac{a + \alpha\theta}{2\beta(1 + m_1)} + \frac{S}{2} \quad \text{where } S = c + \frac{A_1}{Q} + \frac{h_1z}{P} - \frac{h_1Q}{2P} \quad (4.10)$$

$$\text{and } \theta = \frac{\alpha(w - S)}{2I(1 - \phi)} \quad (4.11)$$

Now solving (4.10) and (4.11) we get

$$w^* = \frac{2aI(1 - \phi) + 2I\beta S(1 + m_1)(1 - \phi) - \alpha^2 S}{[4I\beta(1 + m_1)(1 - \phi) - \alpha^2]} \quad (4.12)$$

$$\theta^* = \frac{\alpha[a - \beta S(1 + m_1)]}{[4I\beta(1 + m_1)(1 - \phi) - \alpha^2]} \quad (4.13)$$

Using the responses given in (4.12) and (4.13), the retailer finally derives his optimal decisions. Substituting these values of w^* and θ^* in (4.9), the retailer's average profit reduces to

$$\begin{aligned} AP_r(n, m, Q, m_1, \phi) = & \left[\frac{m_1[2aI(1 - \phi) + 2I\beta S(1 + m_1)(1 - \phi) - \alpha^2 S]}{[4I\beta(1 + m_1)(1 - \phi) - \alpha^2]} - \frac{A_3}{Q} - \frac{nA_2}{Q} \right] \\ & \times \left[\frac{2I\beta(1 + m_1)(1 - \phi)[a - \beta S(1 + m_1)]}{4I\beta(1 + m_1)(1 - \phi) - \alpha^2} \right] - \frac{h_2 g}{2Q} - \frac{nc_2 b^2}{2Q} \\ & - \frac{\phi I \alpha^2 [a - \beta S(1 + m_1)]^2}{[4I\beta(1 + m_1)(1 - \phi) - \alpha^2]^2} \end{aligned}$$

Since the profit function AP_r is highly non-linear, it is not possible to give a formal proof of its concavity. Instead, we optimize it numerically with a computational software. Using the optimal results of the retailer in (4.8), (4.12) and (4.13), we obtain the optimal values of the manufacturer's profit, wholesale price, and greening level.

4.4.3 Integrated Model

In this section, we consider the situation where the manufacturer and the retailer collaborate with each other and agree to share information to maximize the average profit of the whole supply chain. The average profit of the integrated system is

$$\begin{aligned} AP_s(n, m, Q, \theta, m_1) = & pD - cD - \frac{h_1 D}{Q} \left[\frac{Qz}{P} + \frac{Q^2}{2} \left(\frac{1}{D} - \frac{1}{P} \right) - \frac{g}{2D} \right] - \frac{h_2 g}{2Q} \\ & - \frac{A_1 D}{Q} - \frac{nA_2 D}{Q} - \frac{A_3 D}{Q} - \frac{nc_2 b^2}{2Q} - I\theta^2 \end{aligned} \quad (4.14)$$

The optimal greening improvement level is obtained from the first order condition

$\frac{\partial(AP_s)}{\partial\theta} = 0$, which gives

$$\theta^* = \frac{\alpha[(1+m_1)w - S_1]}{2I}$$

where $S_1 = c + \frac{A_1 + nA_2 + A_3}{Q} + \frac{h_1z}{P} - \frac{h_1Q}{2P}$

and the optimal margin of the retailer is obtained from the condition $\frac{\partial(AP_s)}{\partial m_1} = 0$, which gives

$$m_1^* = \frac{a + \alpha\theta + \beta S_1 - 2\beta w}{2\beta w}$$

Again, due to complexity, it is difficult to examine the concavity of the profit function AP_s and derive the optimal values of the decision variables analytically. However, we optimize the profit function AP_s numerically to get optimal results.

The following algorithm can be used to obtain numerically the optimal solutions of the models developed in sub-sections 4.4.1, 4.4.2 and 4.4.3.

Solution Algorithm

Step 1: Set $n = 2, m = 1$.

Step 2: Optimize AP_r taking the responses w and θ from the manufacturer.

Step 3: Set $n = n + 1$ and optimize AP_r for all possible values of $m, 1 \leq m < n$.

Step 4: Compare AP_r with the previous value for each m . Suppose that AP_r attains the greatest value at $m = M_n$.

Step 5: If $AP_r(n+1, M_{n+1}) \geq AP_r(n, M_n)$, go to step 3. Otherwise, the optimal solution is (n, M_n) .

Step 6: Evaluate the optimal values of Q, m_1 and ϕ for the Model developed in sub-section 4.4.2.

Step 7: Using optimal results of the retailer, finally calculate the optimal values of w, θ and AP_m .

4.5 Numerical Study

In this section, we consider a numerical example to find the optimal solution of each model proposed in sub-sections 4.4.1, 4.4.2 and 4.4.3. We assume the following data

set:

$P = 2500; A_1 = 400; A_2 = 40; A_3 = 150; h_1 = 4; h_2 = 5; c = 5; c_2 = 7; a = 1500; k = 1.2; I = 20; \alpha = 3; \beta = 6; b = 150; w = 27$ (for integrated model). (with appropriate units)

The optimal results of the decentralized model, the coordinated model under cost sharing contract and the integrated model are summarized in Table 4.1. From Table 4.1 we notice that, in the model under cost sharing contract, the wholesale price(w) of the product increases and profit margin(m_1) at the retailer also increases than the decentralized case. Because the retailer bears a portion of the greening cost and he has to face a higher wholesale price. So, in order to maintain his profitability, the retailer increases his margin(m_1). Although in the cost sharing contract model, the retail price and wholesale price of the product have higher values but the product's more eco-friendly behavior satisfies customer's desire and both the retailer and the manufacturer earn more profits than their profits in decentralized case. This happens because by sharing the greening cost burden, they can raise the greenness of the product to a higher level. As the market demand is sensitive to the product's greening level, a higher greening level leads to a more profitable circumstance for the manufacturer as well as the retailer. The manufacturer gets more profit as his greening cost reduces, and with the increment of demand, he has a scope to increase wholesale price slightly. Though the retailer's cost increases due to cost sharing, the retailer can compensate it from the increased demand and by increasing his margin slightly. It is also observed that the total system profit increases under cost sharing contract when the customers are more green conscious. This approach encourages the supply chain players to make attempt to produce higher greening products to gain more profit as well as save the environment. It can be seen that in the integrated model, greening level of the product is maximum and retailer's margin (m_1) decreases than the decentralized model and cost sharing contract model. Then the customers can get a more greener product at a lower price. So, more and more customers are attracted and as a result customer's demand for the product as well as total system profit increases in the integrated scenario.

Table 4.1: Optimal results for different models

Models	n^*	m^*	Q^*	ϕ^*	m_1^*	θ^*	w^*	AP_r^*	AP_m^*	AP_s^*
Decentralized	4	3	669.01	—	4.04775	1.65670	27.6018	74165.1	13446.4	87611.5
Coordinated	4	3	679.63	0.81497	4.09026	9.02043	27.7469	75228.8	13526.8	88755.6
Integrated	4	3	694.86	—	3.82035	9.33901	—	—	—	89751.9

4.6 Sensitivity Analysis

In order to study how does the changes in various parameter-values of the proposed model influence the profits of the manufacturer and the retailer under cost sharing contract, we perform a sensitivity analysis. We change one parameter-value at a time and keep all other values unchanged. Table 4.2 shows the sensitivity analysis with respect to the parameters $I, \alpha, \beta,$ and k . From Table 4.2, we observe that:

(i) The profits of both the manufacturer and the retailer decrease with the increment of I because when I increases, the greening cost also increases i.e., the manufacturer has to invest more for producing green products. Then the retailer decreases his share in greening cost and the greening improvement level of the item gradually decreases. So the consumer demand also decreases in a green sensitive market and this results in a lesser profit of the manufacturer and the retailer.

Table 4.2: Optimal results for different values of I, α, β and k

Parameter	Value	Q^*	ϕ^*	w^*	θ^*	m_1^*	AP_r^*	AP_m^*	AP_s^*
I	18	681.105	0.815196	27.767	10.0445	4.09615	75410	13537.9	88947.9
	19	680.327	0.815083	27.7564	9.50494	4.09305	75314.6	13532.1	88846.7
	20	679.628	0.814982	27.7469	9.02043	4.09026	75228.8	13526.8	88755.6
	21	678.997	0.81489	27.7383	8.58289	4.08744	75157.4	13522.1	88679.5
	22	678.424	0.814807	27.7305	8.18585	4.08546	75081.1	13517.7	88598.8
α	2.8	677.922	0.814734	27.7237	8.39793	4.08345	75019.6	13514.0	88533.6
	2.9	678.759	0.814856	27.7351	8.70861	4.0868	75122.3	13520.3	88642.6
	3	679.628	0.814982	27.7469	9.02043	4.09026	75228.8	13526.8	88755.6
	3.1	680.528	0.815112	27.7591	9.33344	4.09385	75339.3	13533.6	88872.9
	3.2	681.461	0.815247	27.7718	9.64772	4.09757	75453.6	13540.6	88994.2
β	5.6	683.653	0.82077	28.6737	9.70242	4.26524	81489.3	14211.1	95700.4
	5.8	681.611	0.817845	28.1981	9.34937	4.17543	78245.9	13859.9	92105.8
	6	679.628	0.814982	27.7469	9.02043	4.09026	75228.8	13526.8	88755.6
	6.2	677.698	0.812177	27.3181	8.71318	4.00934	72415.5	13210.4	85625.9
	6.4	675.817	0.809429	26.9099	8.42562	3.93232	69786.3	12909.3	82695.6
k	1.1	679.407	0.81452	27.8184	9.01599	4.07843	75168.8	13561.0	88729.8
	1.2	679.628	0.814982	27.7469	9.02043	4.09026	75228.8	13526.8	88755.6
	1.3	679.817	0.815373	27.6858	9.02403	4.10042	75276.8	13499.7	88776.5
	1.4	679.979	0.815707	27.6333	9.02702	4.10919	75315.2	13478.5	88793.7
	1.5	680.119	0.815993	27.5879	9.02948	4.11979	75346.3	13462.0	88808.3

(ii) With increment of α , the profits of the manufacturer and the retailer increase. Since a higher value of α indicates that the consumers are more sensitive to the greenness of the product. In such cases, in order to increase the customer demand the retailer will increase his share in greening cost. Then the supply chain players

can provide a higher level of greening of the product. So, both the manufacturer and retailer charge a higher price for more greener product. As a result, both the manufacturer and the retailer earn more profits.

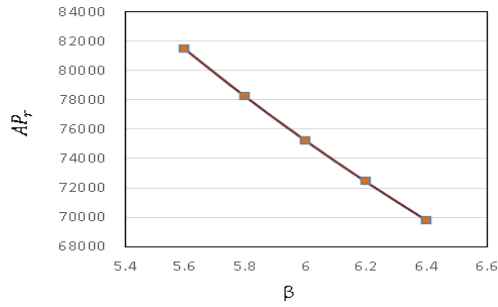


Figure 4.3: retailer’s average profit vs. β

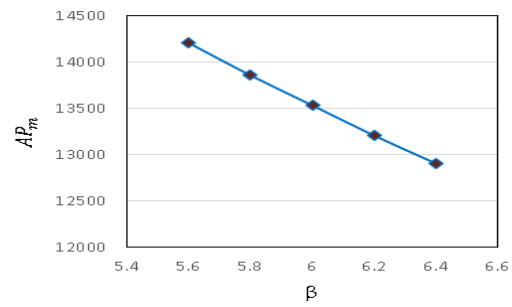


Figure 4.4: manufacturer’s average profit vs. β

(iii) For a small increment of β , the profits of the manufacturer and the retailer decrease by a large amount i.e., when a customer’s demand is more sensitive to the retail price of a product, consumer demand decreases as the selling price increases. As a result, the retailer reduces his greening cost sharing portion. The manufacturer and the retailer are then forced to decrease the wholesale price and margin respectively to hold the market demand. The greening cost sharing option is not effective as it does not cause a more profit for higher values of β . Figures 4.3 and 4.4 show the behavior of the profit functions of the players with changes in β .

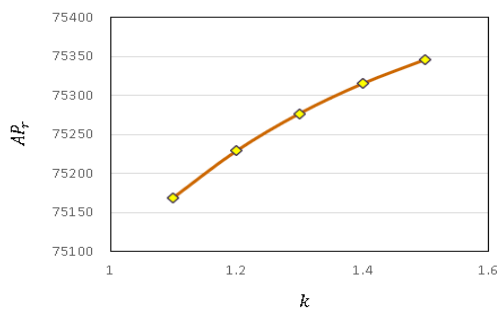


Figure 4.5: retailer’s average profit vs. k

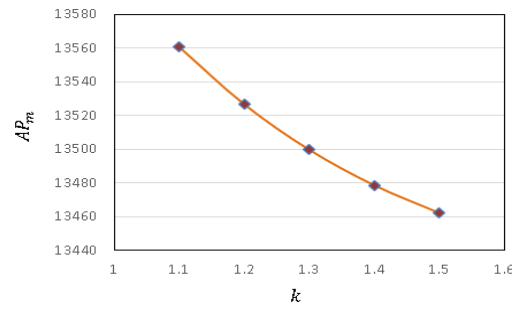


Figure 4.6: manufacturer’s average profit vs. k

(iv) With an increment of k , the retailer’s margin, his share in greening cost and the greenness of the product becomes higher and as a result the retailer’s profit increases but the wholesale price at the manufacturer varies inversely with k . While

k increases, the wholesale price of the product decreases and then manufacturer's profit also starts decreasing. Figures 4.5 and 4.6 show the behavior of the profit functions of the players with the changing values of k .

4.7 Concluding Remarks

This chapter proposes a two-echelon single-manufacturer single-retailer supply chain model with price and green sensitive demand, batch shipment policy, and planned backorder. Assuming that the retailer has higher power than the manufacturer, the decentralized model and coordinated model with cost sharing contract are developed. Optimal values of the ordering quantity, margin, wholesale price, greening improvement level, cost sharing proportion and number of batches have been derived in both the decentralized model and coordinated model with cost sharing contract. Sensitivity analysis is performed to investigate the impact of the key model-parameters on the average profits of the manufacturer and the retailer under cost sharing contract. It is observed from the numerical results that, the model with cost sharing contract leads to better profit level for both players than the non-coordinated model in the situation when the customers are environment conscious and the manufacturer delivers the ordered quantity to the retailer in some equal and unequal sized batches. It is also found that the manufacturer and the retailer can improve the product's greening level to protect the environment by adopting cost sharing contract and such contract improves system profit and player's individual profits significantly. This result can grow interest to the supply chain players to undertake such green initiatives.

Global warming and contamination of the natural resources are rapidly increasing day by day and causing a harmful and irreparable damage to the environment and human society. The contribution of our proposed model in the literature is that, it focuses on such environmental issues and provides optimal ways for the business managers to produce green products and raising awareness to prevent pollution. Secondly, the results confirm that the supply chain players can improve the environmental performance of the product as well as the overall performance of the system by adopting a suitable cost sharing contract rather than optimizing the profit levels independently in the decentralized case. Thirdly, our developed model suggests

that the integrated policy leads to a better profit level of the supply chain as in the integrated scenario the supply chain players can deliver the product at a lower price to the customers, maintaining a higher environmental performance.

The managerial implications of our present research are as follows: Firms and industries can take anti-pollution initiatives through producing eco-friendly products and by extending awareness among the consumers for a sustainable future through their advertisements.

We have considered completely backlogged shortage in our model which may not be implemented in some real world situations. So, further research can be carried out by assuming partially backlogged shortage where some customers are not ready to wait for the next shipment. It can also be considered a price discount offer to hold the market demand during the shortage period. The model can be extended by considering single-manufacturer multi-retailer case. Researchers can also consider other contracts such as revenue sharing contract instead of cost sharing contract. We have solved the model assuming that the retailer dominates the manufacturer. So, for future study, one can consider the situation where the manufacturer dominates the retailer.

CHAPTER 5

Optimal batch shipment policy for an imperfect production system under price, advertisement and green-sensitive demand

5.1 Introduction

In recent times, environmental protection and pollution reduction have drawn global attention. Most developed countries are focusing on reduction of carbon footprint and harmful impacts of various supply chain activities on the environment. In this context, green supply chain management balances environmental issues and economic performance quite efficiently. Also, demand forecasting and demand planning are essential components of supply chain planning and price is a crucial factor affecting it. A lower priced product attracts more customers and increases profit margin of the supply chain. Price sensitive customers expect to obtain products at a lower price even if its environmental performance is low. On the contrary, environment sensitive customers require higher environmental performance of the product. These facts have already been lighted in the previous two chapters. In addition, this chapter* incorporates advertising policy which is another key component influencing customer demand. New companies can promote their products through advertisement by comparing the features, quality, prices and benefits of their products

*This chapter is based on the work published in *Journal of Management Analytics*, 9(1), 86-119.

with other substitutable products in the market. Advertisement attracts new customers as well as helps to sell more products and services to the existing customers. Companies can influence customers by advertising their environmental friendly initiatives and realizing them the benefits and necessity of going green. Therefore, it has become an important managerial concern to determine the best pricing, advertising and greening strategy to improve profitability of the supply chain. On the other hand, efficient quality management of the product is a crucial aspect of successful business strategies and it maintains the reputation of the company and customer satisfaction (Xu, 2011). In the manufacturing sector, despite the developed and sophisticated technologies, various factors such as machine problem, imperfect raw materials, mishandling, etc. may lead production of imperfect items. So, it is unrealistic to assume that all produced items are of good quality. Delivery of defective items may ruin the company's goodwill and causes financial loss. Therefore, to be consistent with today's competitive market and provide better customer services, identification and efficient management of defective items and quality control (Peruzzini & Stjepandić, 2018) are very important.

Motivated by the above mentioned practical issues, in this chapter we integrate environmental thinking and advertising strategy under one umbrella with an imperfect production system in a two-level supply chain model consisting of a single manufacturer and a retailer. The manufacturer delivers ordered quantity of the retailer in some unequal-sized batches. The retailer performs a 100% error-free screening process to rectify the defective items. Customer demand is influenced by retail price, advertisement frequency and greening level of the product. Our main objective is to find answers to the following research questions:

- What are the optimal pricing, advertising and greening strategies for the proposed supply chain?
- What is the impact of a cost sharing contract on the optimal strategies and profitability of the supply chain members ?
- Is such contract capable to enhance environmental and social performance of the supply chain ?
- How does the greening investment affect the profitability of the supply chain members ?

5.2 Problem Definition and Assumptions

We consider an imperfect production-inventory system which consists of a single manufacturer and a single retailer. The manufacturer delivers the order quantity to the retailer in some unequal-sized batches. To separate the defective items, the retailer performs an error-free screening process after receiving each delivery from the manufacturer. Shortage in retailer's inventory is allowed and completely backlogged. The customer demand is influenced by the retail price, advertisement frequency and greening level of the product. The centralized model and the decentralized model based on a Stackelberg gaming approach are developed to determine optimal pricing, advertising and inventory decisions. The cooperation between the manufacturer and the retailer is demonstrated by implementing a cost sharing contract. The basic assumptions of the proposed model are given below:

1. A single manufacturer and a single retailer are involved for trading a single product.
2. The consumer's demand is considered as an iso-elastic function of the selling price (p), advertisement frequency (a), and greenness (e) of the product i.e., $D(p, a, e) = kp^{-\alpha}a^{\beta}e^{\gamma}$, where $k(> 0)$ is a scaling factor and $\alpha > 1, 0 < \beta < 1$ and $0 < \gamma < 1$.
3. The production process of the manufacturer is imperfect due to some unavoidable circumstances. The defective rate (y) of items is uniformly distributed over $[L, J], 0 \leq L \leq J \leq 1$.
4. The manufacturer produces the order quantity of the retailer in a single lot and delivers it to the retailer in some unequal sized batches.
5. After receiving each delivery, the retailer performs a 100% screening process to remove the defective items with a screening rate of x_r . The screening process is assumed to be error-free.
6. To avoid intricacy, we assume that shortages are fulfilled from the next batch before screening, and those items satisfying the backorders are faultless.
7. After the completion of inspection process, the retailer pays the purchasing cost only for the perfect items and the manufacturer pays an warranty cost for

each defective item.

8. Shortages at the retailer are allowed and considered to be completely backlogged.
9. For avoiding shortages during inspection process, it is considered that there are perfect items in each shipment at least equal to the customer demand during screening period i.e., $\lambda^i q(1 - y) \geq \frac{\lambda^i q D}{x_r} \Rightarrow \rho_r \leq (1 - y)$, where $\rho_r = \frac{D}{x_r}$ is known and $i = 0, 1, 2, \dots, n - 1$.
10. Annual greening investment for the product is taken as Ie^2 , which is increasing and convex in the greening improvement level e , and the advertisement cost per unit time is Ga .

5.3 Notations

The following notations are used for developing the proposed model:

k	: Scaling factor of the demand function
c	: Unit production cost of the manufacturer (\$/unit)
h_m	: Holding cost of the manufacturer (\$/unit/unit time)
h_r	: Holding cost of the retailer ($h_r > h_m$) (\$/unit/unit item)
c_2	: Shortage cost at the retailer (\$/unit/unit item)
c_w	: Unit warranty cost for the defective items (\$/unit)
D	: Demand rate at the retailer (units/unit time)
Q	: Order quantity of the retailer (units)
R	: Production rate of the manufacturer (units/unit time)
I_r	: Unit screening cost at the retailer (\$/unit)
S	: Manufacturer's set up cost (\$/ set up)
A	: Retailer's ordering cost (\$/ order)
F	: Transportation cost (\$/ shipment)
G	: Advertisement cost coefficient (\$/ advertisement)
α	: Index of price elasticity in the demand function
β	: Index of advertisement elasticity in the demand function
γ	: Index of green sensitivity in the demand function

m	: Retailer's margin (> 1) so that $p = mw$
Y	: Mean rate of defective
x_r	: Screening rate of the retailer (units/unit time)
ρ	: Ratio of the demand rate to the production rate i.e., $\rho = \frac{D}{R}$
ρ_r	: Ratio of the demand rate to the screening rate i.e., $\rho_r = \frac{D}{x_r}$
λ	: Rate of increasing shipment size ($1 < \lambda \leq \frac{R}{D}$)
I	: Greening investment parameter (\$)
T	: Cycle length (time)
ϕ	: Proportion of greening cost which the retailer shares ($0 < \phi < 1$)
y	: Defective rate, uniformly distributed over $[L, J]$
AEP_m	: Manufacturer's average expected profit (\$/unit time)
AEP_r	: Retailer's average expected profit (\$/unit time)
AEP_s	: Average expected profit of the whole supply chain (\$/unit time)
$(.)^d$: Value in the decentralized policy
$(.)^{cs}$: Value in the cost sharing contract policy
$(.)^c$: Value in the centralized policy

Decision**Variables**

q	: Smallest batch size (units)
p	: Unit retail price of the product (\$/unit)
b	: Retailer's shortage quantity (units)
a	: Frequency of advertisement per unit time
n	: Total number of batches (positive integer)
w	: Unit wholesale price of the manufacturer (\$/unit)
e	: Greening improvement level of the product

5.4 Model Formulation

We assume that the retailer orders a lot of size Q . The manufacturer produces the products at a rate R and delivers to the retailer in n unequal sized batches. The batch sizes are $q, \lambda q, \lambda^2 q, \dots, \lambda^{n-1} q$. Therefore, $Q = \left(\frac{\lambda^n - 1}{\lambda - 1}\right)q$. The production process at the manufacturer is imperfect due to various reasons such as mechanical problem, human error, mismanagement, etc. After receiving each delivery, the retailer carries out an 100% inspection process to find out the defective items. After completion of

screening process, the customer demand is meet up by the perfect items. The manufacturer pays an warranty cost for defective items. Because of imperfect or defective items in each shipment, shortage may occur at the retailer.

A. Average expected profit of the retailer

The expected cost of the retailer includes holding cost, ordering cost, purchasing cost, transportation cost, advertising cost, screening cost and shortage cost. Variation of inventory level of the retailer with respect to time is shown in Figure 5.1.

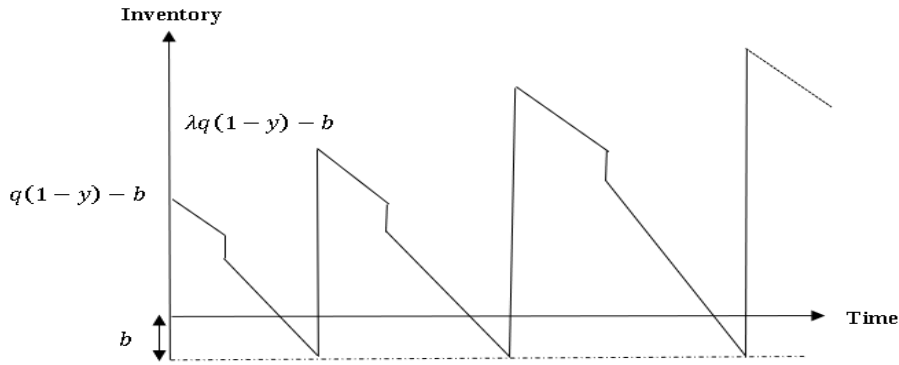


Figure 5.1: Inventory of the retailer

Inventory holding area of the retailer can be obtained as

$$\begin{aligned} & \frac{[q(1-y)-b]^2}{2D} + \frac{[\lambda q(1-y)-b]^2}{2D} + \frac{[\lambda^2 q(1-y)-b]^2}{2D} + \dots + \frac{[\lambda^{n-1} q(1-y)-b]^2}{2D} \\ & + \frac{q^2 y (\lambda^{2n} - 1)}{x_r (\lambda^2 - 1)} \\ = & \frac{q^2 (1-y)^2 (\lambda^{2n} - 1)}{2D (\lambda^2 - 1)} - \frac{q(1-y)(\lambda^n - 1)b}{D(\lambda - 1)} + \frac{nb^2}{2D} + \frac{q^2 y (\lambda^{2n} - 1)}{x_r (\lambda^2 - 1)} \end{aligned}$$

Therefore, the holding cost of the retailer is

$$h_r \left[\frac{q^2 (1-y)^2 (\lambda^{2n} - 1)}{2D (\lambda^2 - 1)} - \frac{q(1-y)(\lambda^n - 1)b}{D(\lambda - 1)} + \frac{nb^2}{2D} + \frac{q^2 y (\lambda^{2n} - 1)}{x_r (\lambda^2 - 1)} \right]$$

Shortage quantity at the retailer is $\frac{nb^2}{2D}$.

Total profit of the retailer (TP_r) = sales revenue - total cost

$$= \frac{pq(1-y)(\lambda^n-1)}{(\lambda-1)} - \frac{wq(1-y)(\lambda^n-1)}{(\lambda-1)} - A - nF - GTa - \frac{I_rq(\lambda^n-1)}{(\lambda-1)} - \frac{nc_2b^2}{2D} \\ - h_r \left[\frac{q^2(1-y)^2(\lambda^{2n}-1)}{2D(\lambda^2-1)} - \frac{q(1-y)(\lambda^n-1)b}{D(\lambda-1)} + \frac{nb^2}{2D} + \frac{q^2y(\lambda^{2n}-1)}{x_r(\lambda^2-1)} \right]$$

The cycle length (T)

$$= \frac{[q(1-y)-b]}{D} + \frac{b}{D} + \frac{[\lambda q(1-y)-b]}{D} + \frac{b}{D} + \frac{[\lambda^2 q(1-y)-b]}{D} + \frac{b}{D} + \dots \\ + \frac{[\lambda^{n-1}q(1-y)-b]}{D} + \frac{b}{D} \\ = \frac{(\lambda^n-1)(1-y)q}{D(\lambda-1)}$$

Average expected profit of the retailer is

$$AEP_r(p, q, a, b) \\ = \frac{E[TP_r]}{E[T]} \\ = (p-w)D - \frac{(A+nF)(\lambda-1)D}{(\lambda^n-1)(1-Y)q} - Ga - \frac{I_rD}{(1-Y)} - \frac{nc_2b^2(\lambda-1)}{2(\lambda^n-1)(1-Y)q} \\ - h_r \left[\frac{qE[(1-y)^2](\lambda^n+1)}{2(\lambda+1)(1-Y)} - b + \frac{qY(\lambda^n+1)\rho_r}{(\lambda+1)(1-Y)} + \frac{nb^2(\lambda-1)}{2q(\lambda^n-1)(1-Y)} \right] \quad (5.1)$$

B. Average expected profit of the manufacturer

The expected cost of the manufacturer consists of production cost, set up cost, warranty cost, holding cost and greening cost. Figure 5.2 represents the joint total inventory level of both the manufacturer and the retailer.

Holding inventory of the manufacturer is given by (Giri & Sharma, 2014)

$$\frac{q^2(1-y)(\lambda^n-1)(\lambda^{n-1}-1)\lambda}{D(\lambda-1)^2(\lambda+1)} + \frac{q^2(\lambda^n-1)(2\lambda-\lambda^n-1)}{2R(\lambda-1)^2}$$

Total profit of the manufacturer (TP_m) = (sales revenue) - (total cost)

$$= \frac{wq(1-y)(\lambda^n-1)}{(\lambda-1)} - \frac{cq(\lambda^n-1)}{(\lambda-1)} - S - \frac{c_wqy(\lambda^n-1)}{(\lambda-1)} - ITe^2 \\ - h_m \left[\frac{q^2(1-y)(\lambda^n-1)(\lambda^{n-1}-1)\lambda}{D(\lambda-1)^2(\lambda+1)} + \frac{q^2(\lambda^n-1)(2\lambda-\lambda^n-1)}{2R(\lambda-1)^2} \right]$$

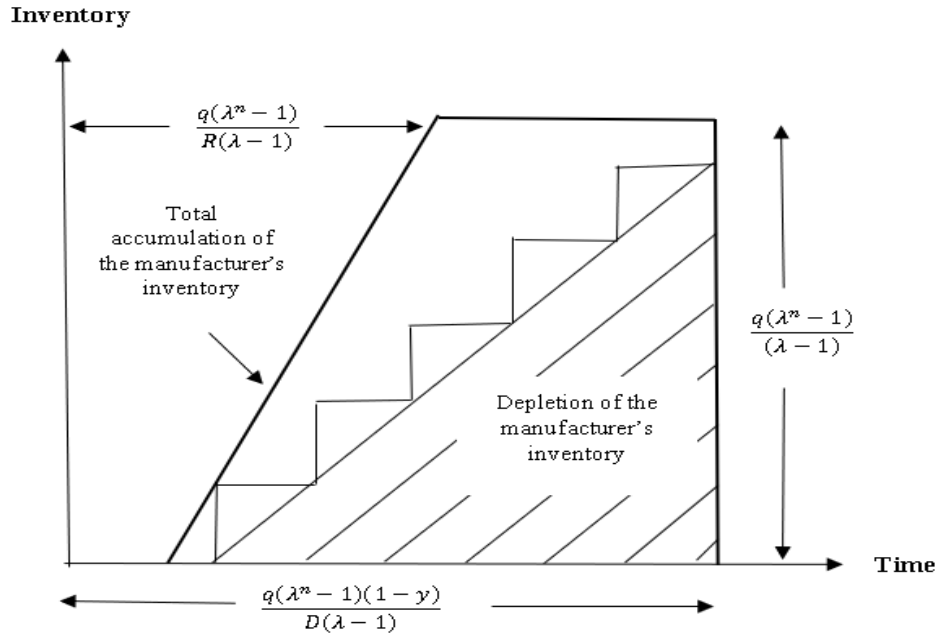


Figure 5.2: The manufacturer's inventory accumulation and depletion

Average expected profit of the manufacturer is

$$\begin{aligned}
 AEP_m(n, w, e) &= \frac{E[TP_m]}{E[T]} \\
 &= wD - \frac{cD}{(1 - \gamma)} - h_m \left[\frac{q\lambda(\lambda^{n-1} - 1)}{(\lambda^2 - 1)} + \frac{q\rho(2\lambda - \lambda^n - 1)}{2(\lambda - 1)(1 - \gamma)} \right] \\
 &\quad - \frac{SD(\lambda - 1)}{(\lambda^n - 1)(1 - \gamma)q} - \frac{c_w \gamma D}{(1 - \gamma)} - Ie^2
 \end{aligned} \tag{5.2}$$

5.4.1 Decentralized policy

In the decentralized scenario, the manufacturer and the retailer independently make their own decisions aiming at optimizing their individual profits. Here we adopt the Stackelberg gaming approach considering the retailer as the leader and the manufacturer as the follower. The manufacturer gives its best reactions on the number of shipments, wholesale price and greening level of the product. Taking these reactions into consideration, the retailer finally sets the retail price, advertisement frequency, backorder quantity and order quantity.

Proposition 5.1 The average expected profit function of the retailer is concave in q for given values of p , a and b and the optimal smallest batch size of the retailer is given by

$$q^* = \sqrt{\frac{(\lambda^2 - 1)[2D(A + nF) + nc_2b^2 + nh_rb^2]}{(\lambda^{2n} - 1)h_r[E[(1 - y)^2] + 2Y\rho_r]}} \quad (5.3)$$

Proof: Differentiating equation (5.1) partially with respect to q , we get

$$\begin{aligned} \frac{\partial(AEP_r)}{\partial q} &= \frac{(A + nF)D(\lambda - 1)}{(\lambda^n - 1)(1 - Y)q^2} + \frac{nc_2b^2(\lambda - 1)}{2(\lambda^n - 1)(1 - Y)q^2} - \frac{h_rE[(1 - y)^2](\lambda^n + 1)}{2(\lambda + 1)(1 - Y)} \\ &\quad + \frac{h_rnb^2(\lambda - 1)}{2(\lambda^n - 1)(1 - Y)q^2} - \frac{h_rY(\lambda^n + 1)\rho_r}{(\lambda + 1)(1 - Y)} \end{aligned} \quad (5.4)$$

$$\frac{\partial^2(AEP_r)}{\partial q^2} = -\frac{2(A + nF)D(\lambda - 1)}{(\lambda^n - 1)(1 - Y)q^3} - \frac{nc_2b^2(\lambda - 1)}{(\lambda^n - 1)(1 - Y)q^3} - \frac{h_rnb^2(\lambda - 1)}{(\lambda^n - 1)(1 - Y)q^3} \quad (5.5)$$

From equation (5.5), it is observed that all the terms in the right hand side are negative which implies $\frac{\partial^2(AEP_r)}{\partial q^2} < 0$. So, the expected average profit function of the retailer is concave in q for given p , a and b .

Now, from the first order optimality condition i.e., by solving $\frac{\partial(AEP_r)}{\partial q} = 0$ for q , we obtain the optimal batch size as $q^* = \sqrt{\frac{(\lambda^2 - 1)[2D(A + nF) + nc_2b^2 + nh_rb^2]}{(\lambda^{2n} - 1)h_r[E[(1 - y)^2] + 2Y\rho_r]}}$.

Proposition 5.2 The average expected profit function of the retailer is concave in b for given values of p , a and q and the optimal backorder quantity of the retailer is given by

$$b^* = \frac{qh_r(\lambda^n - 1)(1 - Y)}{n(\lambda - 1)(h_r + c_2)} \quad (5.6)$$

Proof: Differentiating equation (5.1) partially with respect to b , we obtain

$$\frac{\partial(AEP_r)}{\partial b} = -\frac{nc_2b(\lambda - 1)}{(\lambda^n - 1)(1 - Y)q} + h_r - \frac{nbh_r(\lambda - 1)}{(\lambda^n - 1)(1 - Y)q} \quad (5.7)$$

$$\frac{\partial^2(AEP_r)}{\partial b^2} = -\frac{nc_2(\lambda - 1)}{(\lambda^n - 1)(1 - Y)q} - \frac{nh_r(\lambda - 1)}{(\lambda^n - 1)(1 - Y)q} \quad (5.8)$$

It is clear from equation (5.8) that, $\frac{\partial^2(AEP_r)}{\partial b^2} < 0$. Therefore, the average expected

profit function of the retailer is concave in b and then by solving $\frac{\partial(AEP_r)}{\partial b} = 0$ for b , we get the optimal backorder quantity as $b^* = \frac{qh_r(\lambda^n-1)(1-Y)}{n(\lambda-1)(h_r+c_2)}$.

Proposition 5.3 The average expected profit function of the retailer is concave in a for given values of p , b and q if $w > \frac{H_1}{m-1}$ and the optimal advertisement frequency of the retailer is given by

$$a^* = \left[\frac{k\beta p^{-\alpha} e^{\gamma} [(m-1)w - H_1]}{G} \right]^{\frac{1}{1-\beta}} \quad \text{where } H_1 = \frac{(A+nF)(\lambda-1)}{(\lambda^n-1)(1-Y)q} + \frac{I_r}{(1-Y)}. \quad (5.9)$$

Proof: From equation (5.1), we get

$$\frac{\partial(AEP_r)}{\partial a} = \left[(m-1)w - \frac{(A+nF)(\lambda-1)}{(\lambda^n-1)(1-Y)q} - \frac{I_r}{(1-Y)} \right] k\beta p^{-\alpha} a^{\beta-1} e^{\gamma} - G \quad (5.10)$$

$$\frac{\partial^2(AEP_r)}{\partial a^2} = - \left[(m-1)w - \frac{(A+nF)(\lambda-1)}{(\lambda^n-1)(1-Y)q} - \frac{I_r}{(1-Y)} \right] k\beta(1-\beta)p^{-\alpha} a^{\beta-2} e^{\gamma} \quad (5.11)$$

Clearly, the average expected profit function of the retailer is concave in a for given p , b and q if $\frac{\partial^2(AEP_r)}{\partial a^2} < 0$, i.e., $w > \frac{H_1}{m-1}$. Then, the optimal advertising frequency obtained by solving $\frac{\partial(AEP_r)}{\partial a} = 0$ is given by

$$a^* = \left[\frac{k\beta p^{-\alpha} e^{\gamma} [(m-1)w - H_1]}{G} \right]^{\frac{1}{1-\beta}}.$$

Proposition 5.4 The average expected profit function of the retailer is concave in p for given values of a , b and q if $p < \frac{m(\alpha+1)H_1}{(m-1)(\alpha-1)}$ and the optimal retail price of the retailer is given by

$$p^* = \frac{\alpha m H_1}{(\alpha-1)(m-1)}. \quad (5.12)$$

Proof: Differentiating equation (5.1) partially with respect to p , we get

$$\frac{\partial(AEP_r)}{\partial p} = \left(1 - \frac{1}{m}\right) k p^{-\alpha} a^{\beta} e^{\gamma} - \alpha k p^{-\alpha-1} a^{\beta} e^{\gamma} \left[p - \frac{p}{m} - H_1 \right] \quad (5.13)$$

$$\frac{\partial^2(AEP_r)}{\partial p^2} = -2 \left(1 - \frac{1}{m}\right) \alpha k p^{-\alpha-1} a^{\beta} e^{\gamma} + \alpha(\alpha+1) \left(p - \frac{p}{m} - H_1 \right) k p^{-\alpha-2} a^{\beta} e^{\gamma} \quad (5.14)$$

It is clear that, $\frac{\partial^2(AEP_r)}{\partial p^2} < 0$ if $p < \frac{m(\alpha+1)H_1}{(m-1)(\alpha-1)}$. So, the average expected profit function of the retailer is concave in p for fixed a , b and q , if this condition holds. Then

by solving $\frac{\partial(AEP_r)}{\partial p} = 0$ for p , we obtain the optimal retail price as $p^* = \frac{\alpha m H_1}{(\alpha-1)(m-1)}$.

Proposition 5.5 The average expected profit function of the manufacturer is concave in w for given values of n and e if $w < \frac{(\alpha+1)H_2}{(\alpha-1)}$ and the optimal wholesale price is given by

$$w^* = \frac{\alpha H_2}{(\alpha-1)} \text{ where } H_2 = \frac{c}{(1-Y)} + \frac{S(\lambda-1)}{(\lambda^n-1)(1-Y)q} + \frac{c_w Y}{(1-Y)}. \quad (5.15)$$

Proof: Differentiating equation (5.2) partially with respect to w , we get

$$\frac{\partial(AEP_m)}{\partial w} = km^{-\alpha}w^{-\alpha}a^\beta e^\gamma - \alpha km^{-\alpha}w^{-\alpha-1}a^\beta e^\gamma(w-H_2) \quad (5.16)$$

$$\frac{\partial^2(AEP_m)}{\partial w^2} = \alpha km^{-\alpha}w^{-(\alpha+2)}a^\beta e^\gamma [(\alpha-1)w - (\alpha+1)H_2] \quad (5.17)$$

Clearly, $\frac{\partial^2(AEP_m)}{\partial w^2} < 0$ when $w < \frac{(\alpha+1)H_2}{(\alpha-1)}$. If this condition holds, then the average expected profit function of the manufacturer is concave in w for given n and e . Then, the optimal wholesale price can be obtained by solving $\frac{\partial(AEP_m)}{\partial w} = 0$ for w as $w^* = \frac{\alpha H_2}{(\alpha-1)}$.

Proposition 5.6 The average expected profit function of the manufacturer is concave in e for given values of n and w if $H_2 < \frac{k\gamma(1-\gamma)m^{-\alpha}a^\beta e^{\gamma-2}w + 2Iw^\alpha}{k\gamma(1-\gamma)m^{-\alpha}a^\beta e^{\gamma-2}}$ and the optimal greening level is

$$e^* = \left[\frac{k\gamma m^{-\alpha} w^{-\alpha} a^\beta (w-H_2)}{2I} \right]^{\frac{1}{2-\gamma}} \text{ where } H_2 = \frac{c}{(1-Y)} + \frac{S(\lambda-1)}{(\lambda^n-1)(1-Y)q} + \frac{c_w Y}{(1-Y)} \quad (5.18)$$

Proof: From equation (5.2), we have

$$\frac{\partial(AEP_m)}{\partial e} = k\gamma m^{-\alpha} w^{-\alpha} a^\beta e^{\gamma-1} (w-H_2) - 2Ie \quad (5.19)$$

$$\frac{\partial^2(AEP_m)}{\partial e^2} = -k\gamma(1-\gamma)m^{-\alpha}w^{-\alpha}a^\beta e^{\gamma-2}(w-H_2) - 2I \quad (5.20)$$

Now, $\frac{\partial^2(AEP_m)}{\partial e^2} < 0$ implies that $H_2 < \frac{k\gamma(1-\gamma)m^{-\alpha}a^\beta e^{\gamma-2}w + 2Iw^\alpha}{k\gamma(1-\gamma)m^{-\alpha}a^\beta e^{\gamma-2}}$. Therefore, the average expected profit function of the manufacturer is concave in e for given values of w and n , if the above condition is satisfied. Then, using the first order optimality condition, the optimal greening level of the product is obtained as $e^* = \left[\frac{k\gamma m^{-\alpha} w^{-\alpha} a^\beta (w-H_2)}{2I} \right]^{\frac{1}{2-\gamma}}$.

Proposition 5.7 The average expected profit function of the manufacturer is concave in n for given values of e and w if $\rho < \frac{2[h_m q^2 (\lambda^n - 1)^3 (1 - Y) + SD(\lambda - 1)(\lambda^2 - 1)(\lambda^n + 1)]}{h_m q^2 (\lambda^n - 1)^3 (\lambda + 1)}$ and the optimal number of shipments is

$$n^* = \frac{\log \left[1 + \sqrt{\frac{2SD(\lambda - 1)(\lambda^2 - 1)}{h_m q^2 [2(1 - Y) - \rho(\lambda + 1)]}} \right]}{\log \lambda}. \quad (5.21)$$

Proof: From equation (5.2), we have

$$\frac{\partial(AEP_m)}{\partial n} = -\frac{h_m q \lambda^n \log \lambda [2(1 - Y) - \rho(\lambda + 1)]}{2(\lambda^2 - 1)(1 - Y)} + \frac{SD(\lambda - 1) \lambda^n \log \lambda}{(\lambda^n - 1)^2 (1 - Y) q} \quad (5.22)$$

$$\frac{\partial^2(AEP_m)}{\partial n^2} = -\frac{h_m q \lambda^n (\log \lambda)^2 [2(1 - Y) - \rho(\lambda + 1)]}{2(\lambda^2 - 1)(1 - Y)} - \frac{SD(\lambda - 1) \lambda^n (\log \lambda)^2 (\lambda^n + 1)}{(\lambda^n - 1)^3 (1 - Y) q} \quad (5.23)$$

Since, $\rho < 1$ and $\lambda > 1$, it implies that $\frac{\partial^2(AEP_m)}{\partial n^2} < 0$ if

$$\rho < \frac{2[h_m q^2 (\lambda^n - 1)^3 (1 - Y) + SD(\lambda - 1)(\lambda^2 - 1)(\lambda^n + 1)]}{h_m q^2 (\lambda^n - 1)^3 (\lambda + 1)}.$$

i.e., the average expected profit function of the manufacturer is concave in n for given w and e if the above condition holds. Then solving $\frac{\partial(AEP_m)}{\partial n} = 0$ we get the optimal number of batch shipments as $n^* = \frac{\log \left[1 + \sqrt{\frac{2SD(\lambda - 1)(\lambda^2 - 1)}{h_m q^2 [2(1 - Y) - \rho(\lambda + 1)]}} \right]}{\log \lambda}$.

Solution algorithm

- Step 1. Take the manufacturer's response functions for w , e and n from equations (5.15), (5.18) and (5.21).
- Step 2. Put these values in equations (5.3), (5.6), (5.9) and (5.12). Then solve the resulting equations to find the optimal values of q , b , a and p .
- Step 3. Put the optimal values of q , p , a and b in equation (5.1) to find the optimal average expected profit of the retailer, and using these optimal results, calculate the optimal values of w , e and n from equations (5.15), (5.18) and (5.21).
- Step 4. Finally, using the optimal values of w , e and n , calculate the optimal average expected profit of the manufacturer from equation (5.2).

5.4.2 Centralized policy

In the centralized scenario, the manufacturer and the retailer agree to accept the joint decision making policy for achieving collaborative advantage. They jointly make optimal pricing and inventory decisions to acquire the best profit level for the entire supply chain. Note that the wholesale price has no role in this case.

Now, the average expected profit of the supply chain is given by

$$\begin{aligned}
& AEP_s(n, e, q, p, a, b) \\
= & pD - \frac{cD}{(1-Y)} - h_m \left[\frac{q\lambda(\lambda^{n-1} - 1)}{(\lambda^2 - 1)} + \frac{q\rho(2\lambda - \lambda^n - 1)}{2(\lambda - 1)(1-Y)} \right] - \frac{SD(\lambda - 1)}{(\lambda^n - 1)(1-Y)q} \\
& - \frac{c_w Y D}{(1-Y)} - Ie^2 - \frac{(A + nF)(\lambda - 1)D}{(\lambda^n - 1)(1-Y)q} - Ga - \frac{I_r D}{(1-Y)} - \frac{nc_2 b^2 (\lambda - 1)}{2(\lambda^n - 1)(1-Y)q} \\
& - h_r \left[\frac{qE[(1-y)^2](\lambda^n + 1)}{2(\lambda + 1)(1-Y)} - b + \frac{qY(\lambda^n + 1)\rho_r}{(\lambda + 1)(1-Y)} + \frac{nb^2(\lambda - 1)}{2q(\lambda^n - 1)(1-Y)} \right] \quad (5.24)
\end{aligned}$$

For convenience, we use the following notations:

$$X_1 = \frac{c}{(1-Y)} + \frac{S(\lambda - 1)}{q(\lambda^n - 1)(1-Y)} + \frac{c_w Y}{(1-Y)} + \frac{(A + nF)(\lambda - 1)}{q(\lambda^n - 1)(1-Y)} + \frac{I_r}{(1-Y)} \quad (5.25)$$

$$X_2 = h_m \left[\frac{\lambda(\lambda^{n-1} - 1)}{(\lambda^2 - 1)} + \frac{\rho(2\lambda - \lambda^n - 1)}{2(\lambda - 1)(1-Y)} \right] + h_r \left[\frac{E[(1-y)^2](\lambda^n + 1)}{2(\lambda + 1)(1-Y)} + \frac{Y(\lambda^n + 1)\rho_r}{(\lambda + 1)(1-Y)} \right] \quad (5.26)$$

$$X_3 = \frac{(A + nF)(\lambda - 1)D}{(\lambda^n - 1)(1-Y)} + \frac{nc_2 b^2 (\lambda - 1)}{2(\lambda^n - 1)(1-Y)} + \frac{SD(\lambda - 1)}{(\lambda^n - 1)(1-Y)} + \frac{nh_r b^2 (\lambda - 1)}{2(\lambda^n - 1)(1-Y)} \quad (5.27)$$

Proposition 5.8 The average expected profit function of the supply chain is concave in e for given values of n, q, b, a and p if $X_1 < \frac{k\gamma(1-\gamma)a^\beta e^{\gamma-2}p + 2Ip^\alpha}{k\gamma(1-\gamma)a^\beta e^{\gamma-2}}$ and the optimal greening level is

$$e^* = \left[\frac{k\gamma p^{-\alpha} a^\beta (p - X_1)}{2I} \right]^{\frac{1}{2-\gamma}}. \quad (5.28)$$

Proof: Differentiating equation (5.24) partially with respect to e yields

$$\frac{\partial(AEP_s)}{\partial e} = k\gamma p^{-\alpha} a^\beta e^{\gamma-1} (p - X_1) - 2Ie \quad (5.29)$$

$$\frac{\partial^2(AEP_s)}{\partial e^2} = -k\gamma(1-\gamma)p^{-\alpha} a^\beta e^{\gamma-2} (p - X_1) - 2I \quad (5.30)$$

It is observed that $\frac{\partial^2(AEP_S)}{\partial e^2} < 0$ if $X_1 < \frac{k\gamma(1-\gamma)a^\beta e^{\gamma-2}p+2Ip^\alpha}{k\gamma(1-\gamma)a^\beta e^{\gamma-2}}$. So, if this condition holds, the average expected profit function is concave in e for given values of n , q , b , a and p . Solving $\frac{\partial(AEP_S)}{\partial e} = 0$ for e , we obtain the optimal greening level as $e^* = \left[\frac{k\gamma p^{-\alpha} a^\beta (p-X_1)}{2I} \right]^{\frac{1}{2-\gamma}}$.

Proposition 5.9 The average expected profit function of the supply chain is concave in q for given values of n , e , b , a and p and the optimal batch size is given by

$$q^* = \sqrt{\frac{X_2}{X_3}}. \quad (5.31)$$

Proof: Differentiating equation (5.24) partially with respect to q , we get

$$\frac{\partial(AEP_S)}{\partial q} = -X_2 + \frac{X_3}{q^2} \quad (5.32)$$

$$\frac{\partial^2(AEP_S)}{\partial q^2} = -\frac{2X_3}{q^3} \quad (5.33)$$

Clearly, $\frac{\partial^2(AEP_S)}{\partial q^2} < 0$ as $X_3 > 0$. So, the average expected profit function of the supply chain is concave in q for given values of n , e , b , a and p . Solving $\frac{\partial(AEP_S)}{\partial q} = 0$ for q , the optimal batch size is derived as $q^* = \sqrt{\frac{X_2}{X_3}}$.

Proposition 5.10 For given values of n , e , b , a and q , the average expected profit function of the supply chain is concave in p if $p < \frac{(\alpha+1)X_1}{(\alpha-1)}$ and the optimal retail price is given by

$$p^* = \frac{\alpha X_1}{\alpha - 1}. \quad (5.34)$$

Proof: We have

$$\frac{\partial(AEP_S)}{\partial p} = kp^{-\alpha}a^\beta e^\gamma - \alpha(p-X_1)kp^{-\alpha-1}a^\beta e^\gamma \quad (5.35)$$

$$\frac{\partial^2(AEP_S)}{\partial p^2} = -2\alpha kp^{-\alpha-1}a^\beta e^\gamma + \alpha(\alpha+1)(p-X_1)kp^{-\alpha-2}a^\beta e^\gamma \quad (5.36)$$

It can be shown that $\frac{\partial^2(AEP_S)}{\partial p^2} < 0$ provided that $p < \frac{(\alpha+1)X_1}{(\alpha-1)}$.

Therefore, if the above condition holds, then using the first order optimality condition one can get the optimal retail price as $p^* = \frac{\alpha X_1}{\alpha-1}$.

Proposition 5.11 For given values of n , e , b , p and q , the average expected profit function of the supply chain is concave in a if $p > X_1$ and the optimal advertisement frequency is

$$a^* = \left[\frac{k\beta p^{-\alpha} e^\gamma (p - X_1)}{G} \right]^{\frac{1}{1-\beta}}. \quad (5.37)$$

Proof: Differentiating (5.24) partially with respect to a , we obtain

$$\frac{\partial(AEP_S)}{\partial a} = k\beta p^{-\alpha} a^{\beta-1} e^\gamma (p - X_1) - G \quad (5.38)$$

$$\frac{\partial^2(AEP_S)}{\partial a^2} = -k\beta(1-\beta)p^{-\alpha} a^{\beta-2} e^\gamma (p - X_1) \quad (5.39)$$

Now, $X_1 > 0$ and $\beta < 1$ implies $\frac{\partial^2(AEP_S)}{\partial a^2} < 0$ if $p > X_1$ i.e., the average expected profit function of the supply chain is concave in a . Using the first order optimality condition, the optimal advertisement frequency is obtained as $a^* = \left[\frac{k\beta p^{-\alpha} e^\gamma (p - X_1)}{G} \right]^{\frac{1}{1-\beta}}$.

Proposition 5.12 For given values of n , e , a , p and q , the average expected profit function of the supply chain is concave in b and the optimal shortage quantity is

$$b^* = \frac{h_r q (\lambda^n - 1) (1 - Y)}{n(\lambda - 1)(h_r + c_2)}. \quad (5.40)$$

Proof: Differentiating (5.24) partially with respect to b , we get

$$\frac{\partial(AEP_S)}{\partial b} = -\frac{nc_2 b (\lambda - 1)}{q(\lambda^n - 1)(1 - Y)} + h_r - \frac{nh_r b (\lambda - 1)}{q(\lambda^n - 1)(1 - Y)} \quad (5.41)$$

$$\frac{\partial^2(AEP_S)}{\partial b^2} = -\frac{n(h_r + c_2)(\lambda - 1)}{q(\lambda^n - 1)(1 - Y)} \quad (5.42)$$

Clearly, $\frac{\partial^2(AEP_S)}{\partial b^2} < 0$. Hence, the average expected profit function is concave in b and $\frac{\partial(AEP_S)}{\partial b} = 0$ provides the optimal shortage quantity as $b^* = \frac{h_r q (\lambda^n - 1) (1 - Y)}{n(\lambda - 1)(h_r + c_2)}$.

5.4.3 Cost Sharing Contract

In this section, we propose a cost sharing contract between the manufacturer and the retailer which encourages them to take green initiative. Although, from the environmental perspective, it is always desirable to produce eco-friendly products but

as the manufacturer has to pay the entire cost of greening, it may not be interested to produce such product always. Under a cost sharing contract, the retailer offers to share a certain portion of greening cost to reduce the manufacturer's cost burden, which motivates the manufacturer to adopt green manufacturing. Here, we assume that, the retailer offers to share ϕ ($0 < \phi < 1$) fraction of the greening cost. So the manufacturer has to bear the remaining $(1 - \phi)$ fraction of the greening cost. Under this contract, the manufacturer sets its optimal number of shipments, wholesale price and greening level of the product. The retailer then decides its optimal batch size, retail price, advertisement frequencies and shortage quantity considering the reactions of the manufacturer and the sharing proportion ϕ .

The average expected profits of the retailer and the manufacturer are given by

$$\begin{aligned} & AEP_r^{cs}(p, q, a, b, \phi) \\ = & (p - w)D - h_r \left[\frac{qE[(1 - y)^2](\lambda^n + 1)}{2(\lambda + 1)(1 - Y)} - b + \frac{qY(\lambda^n + 1)\rho_r}{(\lambda + 1)(1 - Y)} + \frac{nb^2(\lambda - 1)}{2q(\lambda^n - 1)(1 - Y)} \right] \\ & - \frac{(A + nF)(\lambda - 1)D}{(\lambda^n - 1)(1 - Y)q} - Ga - \frac{I_r D}{(1 - Y)} - \frac{nc_2 b^2(\lambda - 1)}{2(\lambda^n - 1)(1 - Y)q} - \phi I e^2 \end{aligned} \quad (5.43)$$

$$\begin{aligned} AEP_m^{cs}(n, w, e) = & wD - \frac{cD}{(1 - Y)} - h_m \left[\frac{q\lambda(\lambda^{n-1} - 1)}{(\lambda^2 - 1)} + \frac{q\rho(2\lambda - \lambda^n - 1)}{2(\lambda - 1)(1 - Y)} \right] \\ & - \frac{SD(\lambda - 1)}{(\lambda^n - 1)(1 - Y)q} - \frac{c_w Y D}{(1 - Y)} - (1 - \phi)I e^2 \end{aligned} \quad (5.44)$$

Proposition 5.13 The optimal values of the greening level, wholesale price, retail price and advertisement frequency under the cost sharing contract follow the order : $e^{cs*} > e^{d*}$, $w^{cs*} > w^{d*}$, $p^{cs*} > p^{d*}$ and $a^{cs*} > a^{d*}$, where $(.)^{cs}$ and $(.)^d$ denote the results of the model under cost sharing contract and decentralized model, respectively.

Proof. If the retailer shares a portion ϕ of the greening cost, from (5.44) we have

$$\frac{\partial(AEP_m^{cs})}{\partial e} = k\gamma m^{-\alpha} w^{-\alpha} a^\beta e^{\gamma-1} (w - H_2) - 2I(1 - \phi)e \quad (5.45)$$

$$\frac{\partial^2(AEP_m^{cs})}{\partial e^2} = -k\gamma(1 - \gamma)m^{-\alpha} w^{-\alpha} a^\beta e^{\gamma-2} (w - H_2) - 2I(1 - \phi) \quad (5.46)$$

Since $\frac{\partial^2(AEP_m^{cs})}{\partial e^2} < 0$, therefore, the average expected profit function of the manufacturer is concave in e for given values of w and n . So, the average expected profit function attains its maximum value for an optimal greening level. Solving

$\frac{\partial(AEP_m^{CS})}{\partial e} = 0$, we get the optimal greening level of the product as

$$e^{CS*} = \left[\frac{k\gamma m^{-\alpha} w^{-\alpha} a^\beta (w - H_2)}{2I(1 - \phi)} \right]^{\frac{1}{2-\gamma}} \quad (5.47)$$

Now, comparing the results given in (5.47) and (5.18), it is clear that $e^{CS*} > e^{d*}$ as $0 < \phi < 1$.

Again, it is observed from (5.47) that, $e^{CS*} \propto \frac{w - H_2}{w^\alpha} = f(w)$ (say). Therefore,

$$\hat{f}(w) = \frac{w - \alpha(w - H_2)}{w^{\alpha+1}}$$

Since, $\alpha > 1$ and $H_2 > 0$, it implies that $\hat{f}(w) > 0$ i.e., $f(w)$ is an increasing function of w as well as e . Also, the greening level of the product under cost sharing contract is more than that of the decentralized case. Hence, it can be concluded that $w^{CS*} > w^{d*}$.

Also, we have, $p = mw$ where $m > 1$. It implies that the retail price of the product increases with increment of wholesale price. From the above two results, it is concluded that $p^{CS*} > p^{d*}$.

Equation (5.47) reflects that the advertisement frequency varies proportionally with the retail price, wholesale price and greening level of the product under the cost sharing contract. The advertisement frequency also increases under the cost sharing contract i.e., $a^{CS*} > a^{d*}$.

From the above proposition we observe that, under a cost sharing contract, the wholesale price, the retail price, the advertisement frequency and the greening improvement level of the product are more than those in decentralized scenario. Cost sharing contract offered by the retailer reduces the cost burden of the manufacturer and encourages the manufacturer to improve the greening level of the product. Consequently, higher greening level of the product and more advertisement frequency enhance customer demand significantly in an environment conscious market. A higher wholesale price then generates a surplus to the profitability of the manufacturer. Although, cost sharing contract leads to an additional cost burden and higher purchasing price to the retailer, the retailer meets this deficit by increasing market demand and charging higher selling price from the end customers. Therefore, both the manufacturer and the retailer get benefit from this cost sharing contract. This

leads to the following result:

Proposition 5.14 Under cost sharing contract, the average expected profits of the manufacturer and the retailer satisfy the following order: $AEP_m^{cs*} > AEP_m^{d*}$ and $AEP_r^{cs*} > AEP_r^{d*}$, where $(.)^{cs}$ and $(.)^d$ denote the results of the model under cost sharing contract and decentralized model, respectively.

5.5 Numerical Analysis

In this section, we develop a numerical example with some parameter-values from Rad et al., 2018 to illustrate and analyze our proposed models in different scenarios. We consider the following parameter-values:

$k = 35000; A = 100; F = 100; S = 1200; I = 3000; I_r = 0.1; G = 1000; c_w = 10;$
 $\gamma = 0.85; c = 2.5; h_m = 0.86; \rho = 0.8; \alpha = 1.25; \lambda = 1.1; c_2 = 2.5; h_r = 1.2; \beta = 0.45;$
 $\rho_r = 0.3.$ (with appropriate units)

We suppose that the percentage of defective items(y) is uniformly distributed continuous random variable in $(0, z), 0 < z < 1$ and

$$f(y) = \begin{cases} 25 & 0 \leq y \leq 0.04 \\ 0 & \text{elsewhere} \end{cases}$$

Optimal results of different models are represented in Table 5.1 which indicates that collaborative decision-making significantly increases the profitability of the system. Optimal order quantities of the retailer are $Q^{d*} = 141856$ units, $Q^{cs*} = 211644$ units and $Q^c = 521114$ units, respectively. The supply chain members achieve the highest system profit while acting as a single decision maker. By cooperating with each other, they can deliver the product to the end customer at a lower price than the non-coordinated decision making scenario. Also, the manufacturer takes a proactive stance for improving the environmental performance of the product and rises the greening level in production. The retailer makes effort on promoting the product through advertising more often to attract the attention of customers. Furthermore,

optimal allowable backorder quantity is also increased enough in the centralized decision making model than the decentralized model. Thus, in a price sensitive market where the customers are environmentally conscious and the advertising of the products influences their purchase decision, a cheaper product with higher greening improvement level and frequent advertising make the customers attractive towards this product and increase the market demand sufficiently. Consequently, the retailer increases its order quantity from the manufacturer. In the centralized model, a higher customer demand improves the profitability of the entire supply chain terribly than the decentralized model.

Table 5.1: Optimal results for different models

Model	n^*	w	e^*	q^*	p^*	a^*	b^*	ϕ	AEP_r^*	AEP_m^*	AEP_s^*
Decentralized	11	13.80	20	7655	25.79	2247	4099	-	618313	1527700	2146013
Cost sharing contract	11	13.82	43	11421	46	5875	6115	0.7	1623360	2232810	3856170
Centralized	12	-	138	24369	14.31	60962	13802	-	-	-	16876000

Table 5.1 shows that, adoption of cost sharing contract between the manufacturer and the retailer becomes beneficial for both the supply chain members. Participating in the cost sharing contract, both the manufacturer and the retailer earn more profit than that of the independent optimization policy. When the retailer offers to share a certain fraction of the greening cost of the product, it reduces the cost burden of the manufacturer, and the manufacturer accepts the offer. Under this cost sharing contract, since the retailer bears a certain portion of the greening cost, it makes an incentive to the manufacturer for producing more and more greener product. Optimal results reflect that the retailer shares 70% cost of greening. It is seen from Table 5.1 that the greening level of the product under cost sharing contract increases significantly than that of the decentralized model. Herewith, in an environment conscious market, a higher greening product makes a positive attribute to the customers. Also, the manufacturer charges a higher wholesale price from the retailer for more greener product. Thus, greening cost reduction, increasing market demand and higher wholesale price together increase the profit level of the manufacturer. On the other hand, though cost sharing contract increases the retailer's expenditure but the retailer compensates it by increasing customer demand. It is also observed that, under the cost sharing contract, advertisement frequency is more than that of the decentralized case. The retailer mandates the corresponding company to increase its

advertisement frequency to promote the product. A higher greening product and more advertisement increase customer demand. Furthermore, the retailer demands higher selling price from consumers to provide more eco-friendly product. These lead to better profit level for the retailer than that of the non-coordinated optimization. Thus, the cost sharing contract provides higher surplus to the profitability of the entire supply chain. The average expected profit increases about 79% under cost sharing contract compared to the profit in decentralized scenario. It improves both the economical and environmental performances of the supply chain members.

Special case:

If shortages are not allowed at the retailer end i.e., $c_2 \rightarrow \infty$ and $b = 0$ then the optimal results are as given in Table 5.2.

Table 5.2: Optimal results for different models (without shortage)

Model	n^*	w^*	e^*	q^*	p^*	a^*	ϕ	AEP_r^*	AEP_m^*	AEP_s^*
Decentralized	11	13.82	19.44	6423	25.81	2243	-	616069	1526430	2142249
Cost sharing contract	11	13.80	42	9589	46	5869	0.7	1620010	2232760	3852770
Centralized	12	-	138	22790	14.31	60947	-	-	-	16868000

It is observed from Table 5.2 that, the order quantity decreases in each case. The average expected profits of the retailer, the manufacturer and the entire supply chain decrease if shortages are not considered. Therefore, the models with shortages provide better performance than the models without shortages. This confirms earlier observations made by Rad et al., 2018.

5.6 Sensitivity Analysis

In this section, sensitivity analysis is carried out to study the impacts of some crucial model-parameters on the optimal solution and profitability of each supply chain member under different scenarios. Here, we change one parameter value at a time keeping remaining parameter-values unchanged. Our findings are summarized in the following:

(i) We analyze the impact of the basic market demand on the optimal solution. Table 5.3 shows that, as the basic market demand increases, the profitability of the supply chain members and the entire supply chain increases at a decreasing rate. This

is because, growing market demand generates more revenue for the supply chain members. So, along with the increment of the customer demand, the manufacturer reduces its wholesale price and raises the greening level of the product.

Table 5.3: Sensitivity with respect to k

Model	k	n^*	w	e^*	q^*	p^*	a^*	b^*	ϕ^*
Decentralized	33000	11	13.83	15.36	6038	25.8207	1399	3233	-
	34000	11	13.82	17.34	6811	25.79	1779	3647	-
	35000	11	13.818	19.46	7655	25.78	2247	4099	-
	36000	11	13.814	21.82	8574	25.76	2818	4591	-
	37000	11	13.810	24.34	9574	25.75	3511	5126	-
Cost sharing contract	33000	11	13.812	33.97	9016	46.13	3664	4827	0.70
	34000	11	13.808	38.29	10166	46.11	4656	5443	0.70
	35000	11	13.804	43.01	11421	46.09	5875	6115	0.70
	36000	11	13.801	48.15	12789	46.08	7365	6848	0.70
	37000	11	13.798	53.74	14276	46.06	9174	7644	0.70
Centralized	33000	12	-	109.43	19245	14.315	38039	10900	-
	34000	12	-	123.34	21694	14.313	48324	12287	-
	35000	12	-	138.54	24369	14.310	60962	13802	-
	36000	12	-	155.07	27283	14.307	76400	15453	-
	37000	12	-	173.07	30450	14.305	95154	17247	-

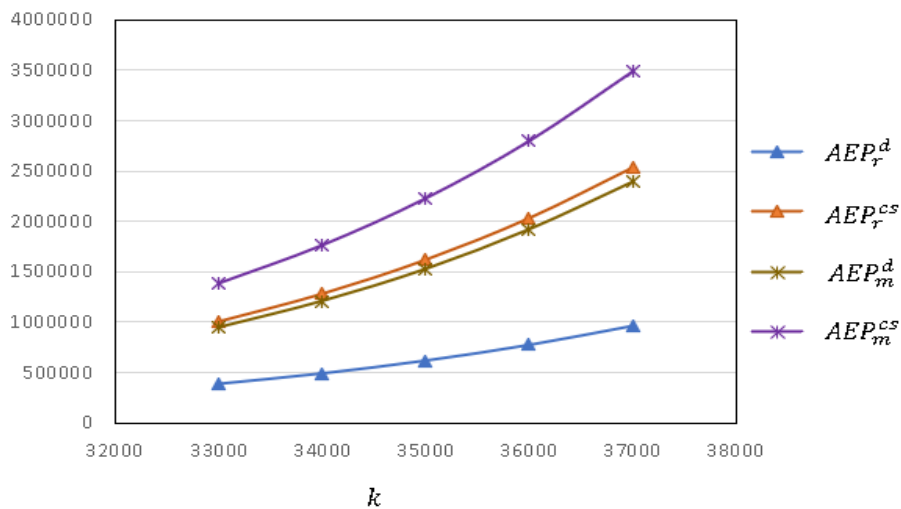


Figure 5.3: Average expected profit of the manufacturer and the retailer vs. k

Furthermore, the retailer increases its order quantity to meet up the growing customer demand. As it earns more revenue, it increases advertisement frequency and reduces the selling price of the product. Also, the optimal backorder quantity increases with the increasing basic market demand. All these factors enhance the average expected profits of the supply chain members significantly. From Figures 5.3 and 5.4, we observe that, when k changes from 33000 to 34000, the expected average profits of the manufacturer, the retailer and the entire supply chain grow up about 27%, but when k changes from 35000 to 36000, the growth rate of their profitability is about 24% in each model.

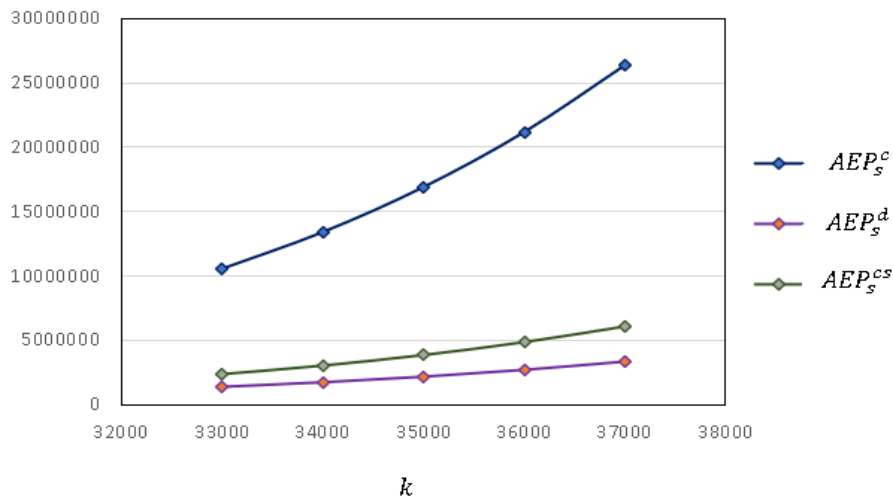


Figure 5.4: Average expected profit of the supply chain vs. k

(ii) Table 5.4 presents the effect of α , the price sensitivity of the demand, on the optimal decisions. Initially, advertising is beneficial for new companies promoting their products but after reaching its maturity stage, pricing strategy becomes key point to survive in the competitive market. A drastic change is observed in the profitability of the manufacturer and the retailer for a small increment in price elasticity. Whenever price sensitivity of customers increases, the retailer reduces its retail price to hold the market demand and also decreases its advertisement frequency as it does not remain profitable in a high price sensitive market. The order quantity is also reduced for high price elasticity. Moreover, in case of cost sharing contract, the retailer lowers its share in the greening cost because customers are more worried about the

price of the product in a higher price sensitive market. Then the manufacturer reduces its wholesale price, and since the sharing proportion of the manufacturer increases, environmental performance of the product also depleted.

Table 5.4: Sensitivity with respect to α

Model	α	n^*	w	e^*	q^*	p^*	a^*	b^*	ϕ^*
Decentralized	1.24	11	14.26	21.73	8393	26.82	2819	4494	-
	1.245	11	14.03	20.93	8015	25.86	2516	4291	-
	1.25	11	13.80	20	7655	25.79	2247	4099	-
	1.255	11	13.58	18.43	7312	25.31	2008	3915	-
	1.26	11	13.38	17.75	6985	24.47	1795	3740	-
Cost sharing contract	1.24	11	14.27	47.14	12667	50.62	7574	6782	0.708
	1.245	11	14.04	46.64	12027	46.72	6667	6439	0.704
	1.25	11	13.82	43	11421	46	5875	6115	0.70
	1.255	11	13.60	40.39	10309	44.76	5182	5809	0.696
	1.26	11	13.39	38.59	10309	42.84	4576	5519	0.692
Centralized	1.24	12	-	154.22	26581	14.78	75543	15055	-
	1.245	12	-	146.141	25449	14.54	67840	14414	-
	1.25	12	-	138	24369	14.31	60962	13802	-
	1.255	12	-	131.36	23337	14.08	54185	13212	-
	1.25	12	-	128	22351	14.01	49319	12659	-

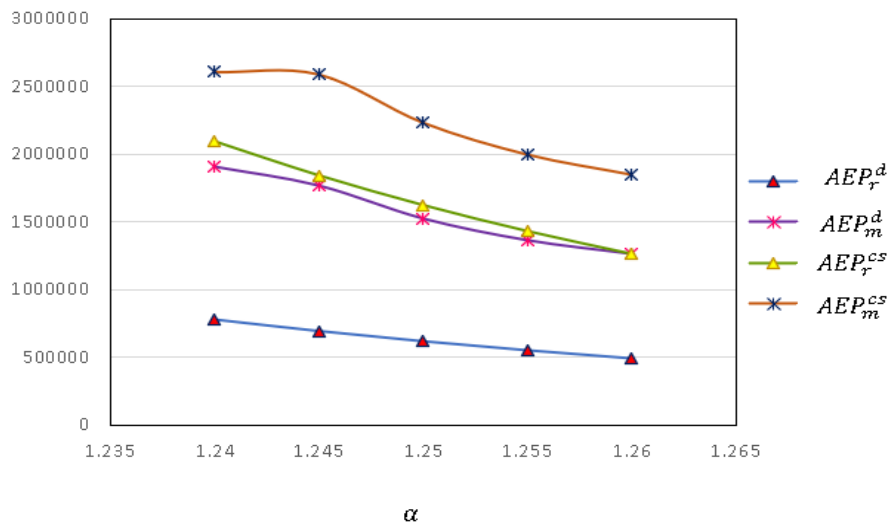


Figure 5.5: Average expected profit of the manufacturer and the retailer vs. α

The optimal backorder quantity also decreases with higher price elasticity. Figure 5.5 reflects a rapid decline in the profit levels of the manufacturer and the retailer in both the decentralized model and the model under cost sharing contract. A harsh reduction in the profitability of the entire supply chain in all cases is observed in Figure 5.6.

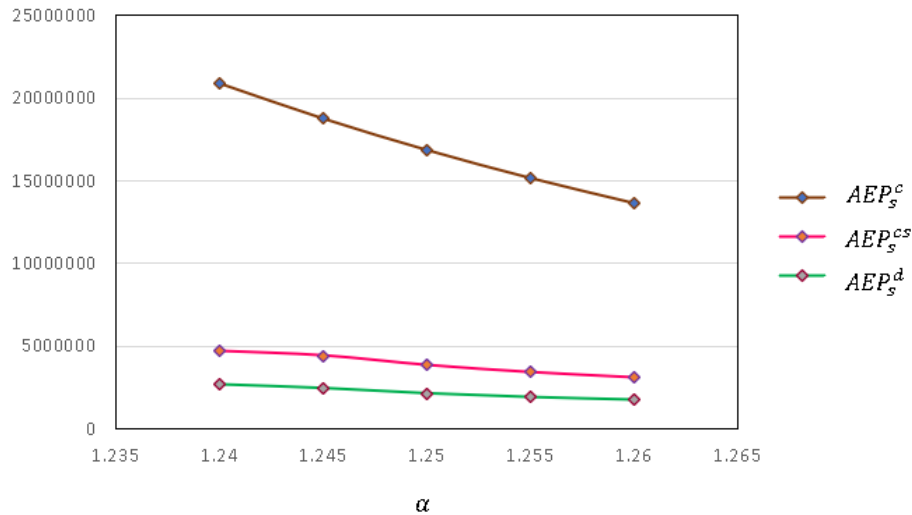


Figure 5.6: Average expected profit of the supply chain vs. α

(iii) The impact of customers' green sensitivity on the optimal decisions is shown in Table 5.5. It is observed that customers green sensitivity exhibits a positive impact on the profitability of the supply chain players. When the value of γ increases, it indicates that the customers are more environment conscious. The environmental performance of the product affects more on the purchasing decision. In green sensitive market, the manufacturer raises greening level of the product as per customer's satisfaction. But along with the increment of product's greening level e , greening cost increases rapidly. So, the retailer reduces its share in the greening cost gradually. A higher greening level of the product results in more consumer demand in an environment conscious market and this generates more revenue for both the manufacturer and the retailer. So, the retailer lowers its retail price and increases its ordering quantity, advertisement frequency and optimal backorder quantity. The manufacturer reduces its wholesale price slightly. Figures 5.7 and 5.8 reflect a significant growth in the profit level of both the members as well as the whole supply chain with growing green sensitiveness. It is observed that the growth rates of profits of both the manufacturer and the retailer are higher in the model under cost

sharing contract than those in the decentralized model. When γ changes from 0.86 to 0.87, the expected average profits of the retailer and the manufacturer increase 37.8% and 43.4%, respectively in the decentralized model. However, in the model under cost sharing contract, these growth rates are 43.2% and 49.2%, respectively.

Table 5.5: Sensitivity with respect to γ

Model	γ	n^*	w	e^*	q^*	p^*	a^*	b^*	ϕ^*
Decentralized	0.83	11	13.83	14.22	5682	26.22	1246	3042	-
	0.84	11	13.82	16.55	6557	25.98	1654	3511	-
	0.85	11	13.80	19.47	7655	25.79	5875	6115	-
	0.86	11	13.812	23.24	9055	25.56	3134	4848	-
	0.87	11	13.80	28.10	10870	25.37	4502	5820	-
Cost sharing contract	0.83	11	13.84	30.64	8218	46.69	3053	4400	0.704
	0.84	11	13.83	36.08	9627	46.37	4182	5154	0.702
	0.85	11	13.818	43	11421	46	5875	6115	0.70
	0.86	11	13.815	52.02	13747	45.80	8496	7360	0.698
	0.87	11	13.812	63.97	16817	48.49	12693	9004	0.696
Centralized	0.83	12	-	91.52	16284	14.32	27246	9223	-
	0.84	12	-	11.725	19765	14.312	40122	11195	-
	0.85	12	-	138	24369	14.31	60962	13802	-
	0.86	12	-	174.82	30577	14.305	95948	17319	-
	0.87	12	-	225.02	39139	14.301	157152	22168	-

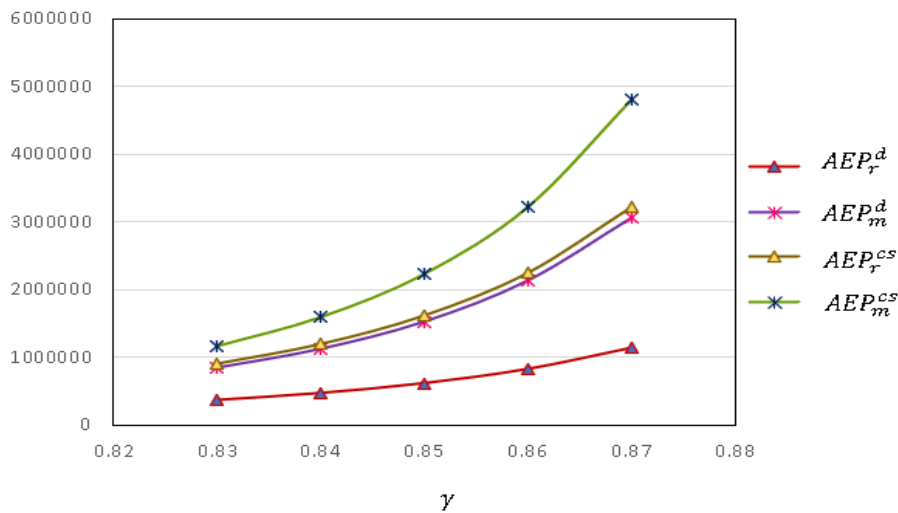


Figure 5.7: Average expected profit of the manufacturer and the retailer vs. γ

Similar changes are observed for all values of γ . This reveals that, in an environment sensitive market, adopting cost sharing contract is more beneficial for supply chain members.

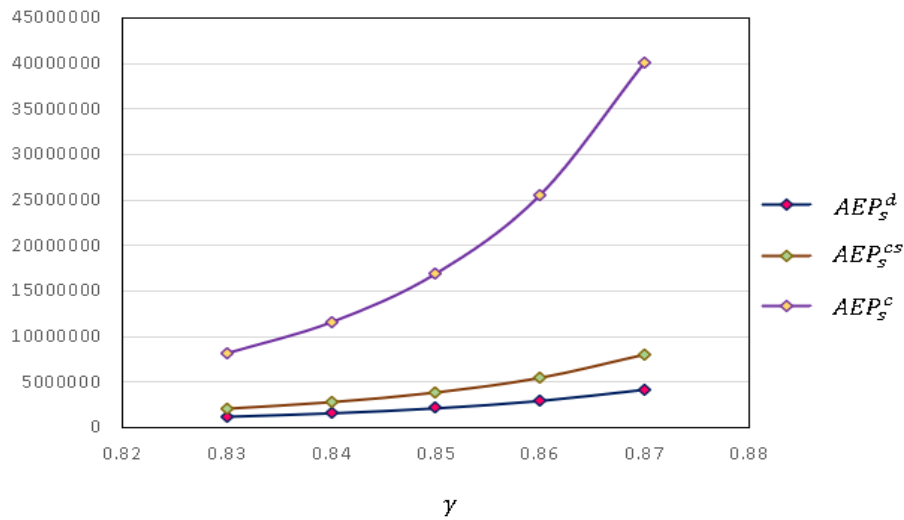


Figure 5.8: Average expected profit of the supply chain vs. γ

Table 5.6: Sensitivity with respect to β

Model	β	n^*	w	e^*	q^*	p^*	a^*	b^*	ϕ^*
Decentralized	0.41	11	13.87	6.75	2646	25.98	246	1417	-
	0.42	11	13.85	8.37	3277	25.87	386	1754	-
	0.43	11	13.83	10.67	4181	25.83	642	2238	-
	0.44	11	13.81	14.10	5531	25.80	1149	2961	-
	0.45	11	13.80	20	7655	25.79	2247	4099	-
Cost sharing contract	0.41	11	13.90	13.32	3526	46.37	513	1888	0.7009
	0.42	11	13.88	16.88	4466	46.22	842	2391	0.7007
	0.43	11	13.85	22.08	5847	46.15	1475	3131	0.7006
	0.44	11	13.84	30.07	7970	46.09	2801	4267	0.7005
	0.45	11	13.82	43	11421	46	5875	6115	0.70
Centralized	0.41	12	-	32.32	5669	14.39	3023	3211	-
	0.42	12	-	43.31	7603	14.36	5560	4306	-
	0.43	12	-	60.46	10623	14.31	11094	6016	-
	0.44	12	-	88.72	15599	14.32	24449	8835	-
	0.45	12	-	138	24369	14.31	60962	13802	-

(iv) Table 5.6 shows changes in the optimal decisions for increasing advertisement sensitivity. When advertisement sensitivity increases, it indicates that advertisement influences more and more consumers perception for the product and encourages them to purchase this product. It can be seen that, the retailer increases its advertisement frequency to stimulate the customer demand and reduces its sharing proportion in greening cost though it is found to be negligible. More advertisement enhances customer demand and results in more ordering and backorder quantities. Both the manufacturer and the retailer achieve more revenue by increasing advertisement frequency. Therefore, the manufacturer and the retailer reduce their wholesale price and selling price, respectively. Also, as the greening level of the product has a positive impact on customer demand, the manufacturer uses increased revenue to raise the greenness of the product. Altogether, a higher advertisement sensitiveness results in higher market demand and it improves the profit level of both the members as well as the whole supply chain.

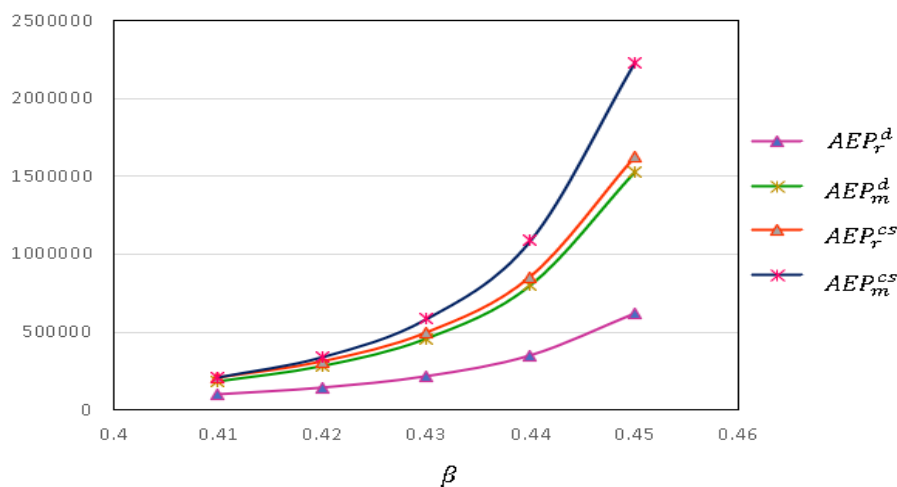


Figure 5.9: Average expected profit of the manufacturer and the retailer vs. β

It is found to be interesting that, cost sharing contract offered by the retailer is not only beneficial for higher green sensitive market but also for an advertisement sensitive market. It is seen from Figures 5.9 and 5.10 that, when β changes from 0.41 to 0.42 in the decentralized model, the average expected profits of the retailer and the manufacturer are increased by 44% and 54%, respectively but those increase 50%

and 61%, respectively in the model under cost sharing contract. Similar kind of observation is made for the parameter β in entire supply chain profitability also.

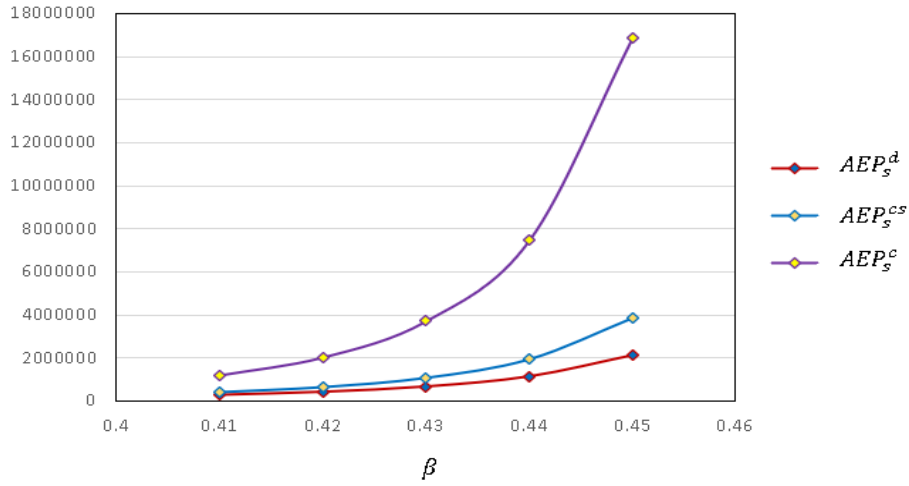


Figure 5.10: Average expected profit of the supply chain vs. β

Table 5.7: Sensitivity with respect to I

Model	I	n^*	w	e^*	q^*	p^*	a^*	b^*	ϕ^*
Decentralized	2600	11	13.79	26.70	9778	25.77	3662	5235	-
	2800	11	13.795	22.66	8614	25.79	2844	4612	-
	3000	11	13.80	20	7655	25.79	2247	4099	-
	3200	11	13.82	16.88	6854	25.80	1802	3670	-
	3400	11	13.83	14.75	6177	25.83	1465	3307	-
Cost sharing contract	2600	11	13.80	58.95	14580	46.07	9569	7806	0.7005
	2800	11	13.81	50.06	12848	46.09	7433	6879	0.7004
	3000	11	13.82	43	11421	46	5875	6115	0.70
	3200	11	13.823	37.30	10230	46.11	4715	5477	0.695
	3400	11	13.829	32.63	9223	46.12	3834	4938	0.69
Centralized	2600	12	-	189.86	31098	14.304	99239	17613	-
	2800	12	-	161.27	27409	14.307	77108	15524	-
	3000	12	-	138	24369	14.31	60962	13802	-
	3200	12	-	120.17	21830	14.313	48931	12364	-
	3400	12	-	105.15	19686	14.316	39800	11150	-

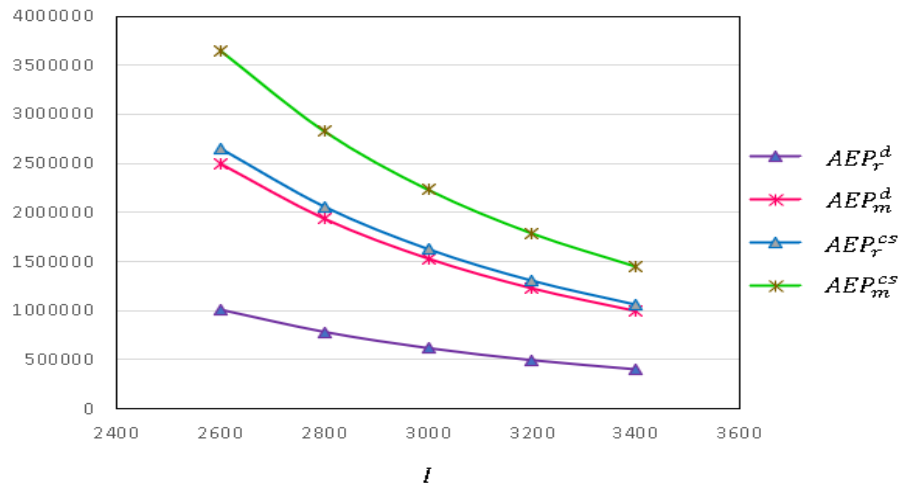


Figure 5.11: Average expected profit of the manufacturer and the retailer vs. I

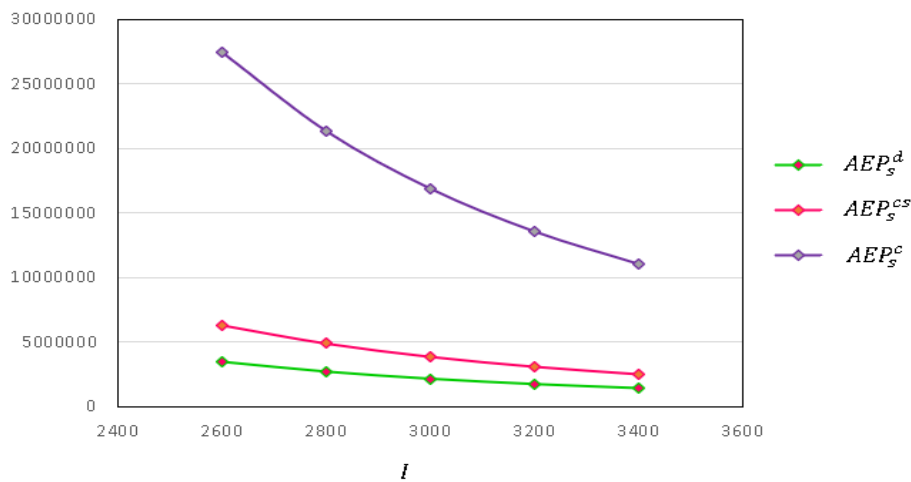


Figure 5.12: Average expected profit of the supply chain vs. I

(v) A decreasing impact of the greening investment parameter I on the decision variables is observed from the results given in Table 5.7. When the greening investment increases, the retailer lowers its sharing proportion in the cost sharing contract. As a result, the greening level of the product falls gradually. Also, it results in higher cost to both the retailer and the manufacturer. Therefore, they compensate this by increasing the wholesale price and the retail price of the product, respectively. A higher priced product with lower environmental performance reduces customer demand significantly. Figures 5.11 and 5.12 reflect the negative impact of greening investment parameter on the profitability of the manufacturer and the retailer in each model.

5.7 Concluding Remarks

In this chapter, we have considered a two-level supply chain model comprised of a single manufacturer and a single retailer trading a single product. The manufacturer transfers the order quantity of the retailer in some unequal sized batch shipments in which successive batch sizes are increased by a constant factor. The production process at the manufacturer is imperfect due to some unavoidable reasons such as machine failure, human errors, etc. On arrival of each batch, the retailer performs an error-free screening process to identify the defective items and returns those to the manufacturer. The manufacturer pays warranty cost for those defective items. The market demand is influenced by three important factors viz. selling price, advertisement frequency and greening level of the product. First, we have investigated the performance of the supply chain members in a decentralized scenario where they independently make decisions to optimize their own profits without cooperating with others. We have used a Stackelberg gaming approach considering the retailer as the leader and the manufacturer as the follower. A solution algorithm is provided to find the optimal decisions. Afterwards, we have investigated the performance of the entire supply chain in a centralized decision making scenario. Although, the joint optimization provides more profit for the whole supply chain and enriches its environmental performance but sometimes it may not be beneficial to each supply chain member individually. In this context, we have proposed a suitable cost sharing contract between the manufacturer and the retailer, which effectively improves the environmental performance, enhances advertisement frequency and results in better profit level for both supply chain members. The following key managerial insights are derived from the numerical and sensitivity analyses of our proposed model:

(1) New companies can build their brand value and enhance their economic efficiency by adopting eco-friendly manufacturing practices and advertising strategies. From the sensitivity analysis, it is found that customers' environmental consciousness and advertisement sensitiveness play an important role in the profitability of the supply chain members. So, business managers should estimate profitable growth before determining advertisement strategy and entering into green channel. With the proposed cost sharing contract, the supply chain members can raise the environmental performance of the product to a remarkable higher level and promote

their product through more advertising to the customers. It helps them to maintain their social responsibility and improve their economical and environmental performances both.

(2) Production process may be imperfect due to various reasons. Therefore, firms should conduct screening procedure to remove the defective items and maintain their goodwill in the market. Also, they should reduce selling price of the product for higher price sensitive customer demand for smooth running of their business in the competitive market.

(3) Companies can create environmental awareness among people by producing green products and highlight the importance of protecting the environment through advertising. It is also beneficial for them to survive in today's cut-throat competitive market.

(4) Through advertising, business firms can grab the market demand and gain popularity by changing customer's attitude and perception towards their products. Thereby, in an advertisement-sensitive market, an adept advertising strategy strengthens their positions.

To enhance the scope of study, the current model can be extended in many different directions. In this chapter, we have considered a single-manufacturer and a single-retailer trading a single product. For more realistic view, it can be extended to a multi-manufacturer multi-retailer model with multiple products. Screening procedure at the retailer end is assumed to be error-free in our model. One can consider errors in the screening process of the retailer. We have proposed a cost sharing contract between the manufacturer and the retailer, any other suitable coordinating scheme can be implemented such as the manufacturer shares a certain proportion of advertising cost of the retailer, etc. Also, bargaining between the manufacturer and the retailer on the sharing proportion of greening cost will be another interesting area.

Coordination of a sustainable supply chain under unequal-sized shipments, sample inspection and set-up cost reduction

6.1 Introduction

The last few decades, especially recent few years have witnessed an unanticipated declination in the global economy. The swift and widespread shock of pandemics, incessant trade deficits, depletion of essential resources, wars, etc., have drowned the global economy into a grievous contraction. As a consequence, surviving in today's competitive market has become a challenging issue for the business firms. Companies are facing considerable adversity in meeting all their customer needs as well as maintaining their desired profit levels. Companies are adopting various innovative marketing strategies and business models to maintain top-quality relationships with customers and keep their brands relevant. One of these most viable strategies is the promotional effort. It is an exemplary way to develop a strong brand reputation and make the product different out from the crowd. The impact of promotional efforts on consumer demand is investigated in this chapter*. Commercial enterprises are maintaining a robust effort in advertising and promotional activities to maintain their importance in the marketplace. Hindustan Unilever Ltd., Maruti

*This chapter is based on the work submitted to *Computers & Industrial Engineering*, 2023.

Suzuki, Ford, Samsung Electronics, Vivo, and Oppo are some leading companies who spend huge amounts on advertising and promotion. Researchers are also doing considerable research focusing on such a crucial factor to find the best pricing and promotional strategies. Another major concern around the world is environmental degradation and it is high time to pave the way for a cleaner environment for the future generation. With this concern, business firms are looking at environmental protection through various green initiatives such as reducing the environmental impacts of production, green packaging, emission reduction, recycling of waste, and so on. Numerous pioneer companies have stepped forward to achieve environmental goals. They are going green and making tremendous efforts to mitigate their environmental impacts. Besides this government is also taking drastic measures to prevent environmental degradation. Various laws and regulations are being enacted to build environmental awareness among the people. Moreover, the government is offering many subsidies and tax reductions to the manufacturing firms adopting eco-friendly business practices. Practitioners and researchers are also emphasizing the integration of environmental thinking into the traditional business scenarios.

In spite of massive efforts and utilization of sophisticated equipment, it would not be reasonable to assume that all the products are flawless. Delivering defective products to the customer not only causes financial damage, but also undermines the goodwill of the company. Customers foster a negative perception about their products. Hence, for successful and smooth running of business, it is desirable to undergo a thorough inspection to ensure the quality of the product before reaching in the customer's hand. However, in case of a large size lot, a full inspection may be extremely expensive and time consuming. In such situations, business managers often conduct a sub-lot inspection on some randomly selected items. If identified defective items satisfy an acceptable criterion, the entire lot is accepted; otherwise, a thorough inspection is conducted. This effectively reduces the inspection costs and work volume. Nevertheless, companies should consider the customers' preferences, product type and its utility while choosing the inspection process. For instance, in the case of aviation industry and medical equipment, accepting of all items on the basis of sample screening may result in fatal circumstances.

Integration in supply chain provides the organizations the utmost profit margins as well as stimulates the product and information flow related activities. It nurtures the joint-decision making policy aimed at achieving optimal supply chain profits,

even though it may not be beneficial to each member of the supply chain system. Under integrated decision-making, supply chain members may suffer losses individually than the decentralized decision-making scenario. In such circumstances, suitable coordination schemes play an important role to motivate the different units of the supply chain to adopt integrated decision making. Strengthening cooperation among supply chain members through effective coordination strategies helps increase productivity and reduce costs. For a smooth-functioning, supply chain executives should monitor its every aspect and assess in a timely manner to ensure speedy delivery of goods and services while meeting the ever-changing consumer needs. An effective coordination scheme affirms smooth-running of these activities and provides efficiencies to adapt to disruptions, ever-evolving markets and emerging industry trends. Researchers are paying close attention to derive suitable coordination policies to achieve improved business strategies.

Keeping all the aforementioned factors in mind, in this chapter, we have incorporated sample inspection policy in a single-manufacturer single-retailer inventory model with price, green and promotional effort sensitive customer demand under unequal batch shipment policy. The manufacturer makes a continuous investment to reduce set up costs. The retailer invests in greening and promotional activities to boost the market demand. The manufacturer bears a variable carbon emission cost due to the production process and transportation. Both decentralized and centralized models are investigated. We also propose a trade credit mechanism and a wholesale price contract for channel coordination, which improve the profit margin of each member as well as enhance the environmental performance of the supply chain. The main purpose of this chapter is to address the following research questions:

- Is it always beneficial to conduct a 100% inspection before selling the product? When to choose a sample inspection policy over full inspection?
- What are the implications of inspection cost and product defect rate in choosing an appropriate inspection policy?
- How do the investments in greening, promotional efforts and set up cost reduction affect the overall supply chain performance? Are such investments capable to stimulate the market demand and enhance profit levels of the manufacturer and the retailer?

- Is the trade credit mechanism and wholesale price contract lucrative for each individual supply chain entity? Are these coordination schemes adept at enriching the economical as well as the environmental performances of the supply chain?

6.2 Problem Definition and Assumptions

This chapter considers a single-manufacturer single-retailer green supply chain model that deals with a single type of product. The manufacturer transfers the order quantity of the retailer with an unequal batch shipment policy. The manufacturer makes a continuous investment to reduce the set up cost. The retailer implements two different marketing efforts to boost the market demand, such as promotional effort and greening effort. The customer demand at the retailer is dependent on the retail price, the level of greening, and promotional efforts. The production system at the manufacturer is imperfect and the lot is shifted to the retailer without any inspection. So, on arrival of each lot, the retailer performs a rigorous inspection on a sample. The defective items are sold at a discount in a secondary market and the retailer bears a penalty cost for each unidentified faulty item. Both the decentralized and centralized models are formulated. Subsequently, a trade credit mechanism and a wholesale price discount contract are implemented to coordinate the supply chain. Figure 6.1 presents a schematic diagram of the inventory flow.

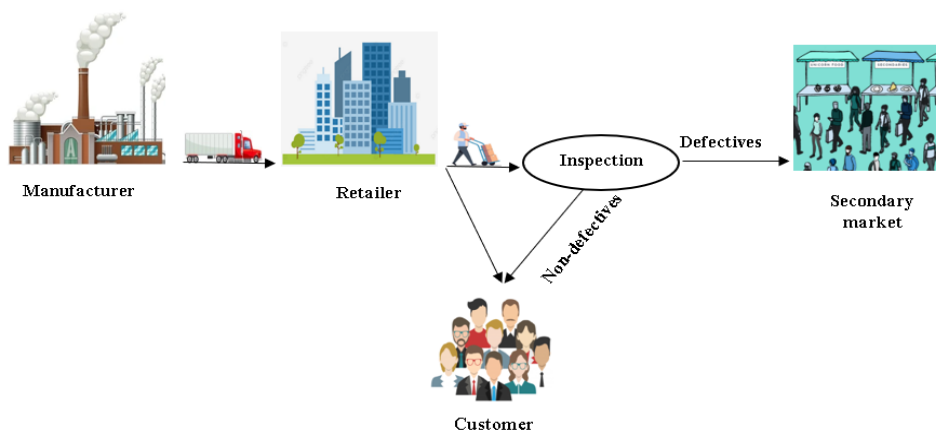


Figure 6.1: Schematic diagram of inventory flow in the supply chain

The following basic assumptions are considered to develop the proposed model:

1. This study assumes an inventory model with a single manufacturer and a single retailer trading for a single type of item.
2. The retailer receives the order quantity Q in some unequal-sized batch shipments and successive batch sizes are in geometric progression with a common ratio λ , where $0 < \lambda \leq \frac{R}{D}$ (Latha et al., 2021; Sen et al., 2021).
3. The manufacturer delivers the products without any inspection, so each batch may contain some faulty items. The defective rate ρ is a random variable with mean M_ρ (finite). The production rate of expected good quality items is finite and higher than the customer demand rate i.e., $(1 - M_\rho)R > D$ (Wu et al., 2007).
4. On arrival of the i -th ($i = 1, 2, \dots, n$) shipment of $\lambda^{i-1}q$ units, $\delta\lambda^{i-1}q$ units undergo a rigorous inspection process. After the screening, z_i defective items are identified and are sold in a secondary market at a $z\%$ discounted price. Remaining $(\lambda^{i-1}q - z_i)$ items contain $(\lambda^{i-1}\rho q - z_i)$ uninspected imperfect items that cause a penalty cost for the retailer (Sarkar et al., 2022b).
5. To avoid shortages during the inspection process, it is assumed that the number of perfect items in a batch is enough to satisfy the customer demand occurred at the time of screening i.e., $(1 - \rho)\lambda^{i-1}q \geq \frac{\delta\lambda^{i-1}qD}{y}$ for $i = 1, 2, \dots, n$ (Rad et al., 2014).
6. Successive batches arrive whenever the shortage quantity of the retailer's inventory reaches to the level b . For simplicity, it is assumed that shortages are satisfied from the next shipment before screening and those items are considered to be faultless. Shortages at the retailer's end are assumed to be completely backordered.
7. The inspection process is assumed to be error-free.
8. The manufacturer reduces the set up cost through investment defined by the function $S(A) = Se^{-rA}$, which is continuous, convex and decreasing with respect to the initial set up cost A (Dey et al., 2019b).

9. Customer demand is assumed to be linearly dependent on retail price (p), greening level (g) of the product and promotional effort (θ) i.e.,
 $D(p, g, \theta) = \alpha - \beta p + \gamma \theta + \eta g$, where $\gamma, \beta, \eta > 0$ are positive constants satisfying $\alpha + \gamma \theta + \eta g > \beta p$.
10. The retailer enhances the market demand by making a promotional effort (θ) and a greening effort (g). The promotional effort cost is $\zeta(\theta - 1)^2$, which is continuous, increasing and convex with respect of the promotional effort level θ ($\theta > 1$). The greening investment of the retailer μg^2 is convex and increasing with respect to the greening level g .
11. The retailer only pays for non-defective items and the manufacturer provides a warranty cost (c_w) for each defective item (Mukherjee et al., 2019; Sarkar and Giri, 2020b).
12. The holding cost of the retailer is greater than that of the manufacturer i.e., $h_r > h_m$. The cost of transport to move the items from manufacturer's warehouse to the retailer's warehouse is dependent on the size of the shipment, i.e., the greater the load, the higher the transportation cost.
13. A variable carbon emissions cost occurs at the manufacturer's end due to production and transportation of the products.

6.3 Notations

The following notations are used for developing the proposed model:

- R : Production rate (units/unit time)
 w : Unit wholesale price (\$/unit)
 D : Demand rate at the retailer (units/unit time)
 Q : Order quantity of the retailer (units)
 α : Basic market demand (units/unit time)
 β : Price sensitivity coefficient of the customer demand
 γ : Promotional effort sensitivity coefficient of the customer demand
 η : Greening effort sensitivity coefficient of the customer demand
 c_m : Unit production cost (\$/unit)

δ	: Proportion of quantity inspected in each shipment
ρ	: Defective rate, follows a Beta distribution
M_ρ	: Mean rate of defective
S	: Initial setup cost of the manufacturer (\$/set up)
λ	: Proportional increase in successive batch sizes ($1 < \lambda \leq \frac{R}{D}$)
c_p	: Unit carbon emission cost due to production(\$/unit)
c_f	: Unit carbon emission cost due to transportation(\$/unit)
f	: Unit transportation cost(\$/unit)
c_w	: Unit warranty cost for defective items (\$/unit)
h_m	: Unit holding cost of the manufacturer (\$/unit/unit time)
h_r	: Unit holding cost of the retailer (\$/unit/unit time)
z	: Fraction of retail price for defective items(\$/unit)
K	: Ordering cost of the retailer (\$/order)
I_b	: Unit inspection cost of the retailer (\$/unit)
I_u	: Unit penalty cost for the uninspected defective items (\$/unit)
ψ_r	: Interest rate of the retailer
ψ_m	: Interest rate of the manufacturer
$\phi(0 \leq \phi \leq 1)$: Bargaining power of the retailer over the manufacturer
c_b	: Unit shortage cost of the retailer (\$/unit/unit time)
ξ	: Promotional effort cost parameter (\$)
μ	: Greening effort cost parameter (\$)
z_i	: Number of imperfect items among inspected $\delta\lambda^{i-1}q$ items ($i = 1, 2, \dots, n$)
y	: Inspection rate of the retailer (unit/unit time)
T	: Cycle length (time)
AEP_m	: Average expected profit of the manufacturer (\$/unit time)
AEP_r	: Average expected profit of the retailer (\$/unit time)
AEP_s	: Average expected profit of the supply chain (\$/unit time)

**Decision
variables**

n	: Number of batch shipments (positive integer)
A	: Investment for set up cost reduction(\$)
q	: Initial batch size (units)
p	: Unit retail price (\$/unit)
b	: Shortage quantity at the retailer's end (units)
θ	: Promotional effort of the retailer
g	: Greening effort of the retailer

6.4 Model Development

This chapter considers a single-manufacturer single-retailer supply chain model for an imperfect production system. The retailer places an order of size Q and the manufacturer transfers it in n unequal-sized batches viz. $q, \lambda q, \lambda^2 q, \dots, \lambda^{n-1} q$. The production system of the manufacturer is erroneous and generates defective items. The manufacturer delivers the products without any quality assurance. On reaching of each delivery, the retailer sends a δ proportion of the lot for an immediate inspection. After screening of $\lambda^{i-1} q$ units ($i = 1, 2, \dots, n$), z_i units are found to be defective and those are sold at $z\%$ price discount in a secondary market. The leftover $\lambda^{i-1} q - z_i$ units enter into the main inventory system of the retailer. The number of imperfect items (z_i) in a sub-lot of $\lambda^{i-1} q$ units is a random variable and follows a hypergeometric distribution with parameters q, ρ and δ . For a given ρ , the probability mass function of z_i is

$$Pr(z_i|\rho) = \frac{C_{z_i}^{\rho\lambda^{i-1}q} C_{\delta\lambda^{i-1}q-z_i}^{(1-\rho)\lambda^{i-1}q}}{C_{\delta\lambda^{i-1}q}^{\lambda^{i-1}q}} \text{ where } 0 \leq z_i \leq \min \{ \delta\lambda^{i-1}q, \rho\lambda^{i-1}q \} \text{ for } i = 1, 2, \dots, n \quad (6.1)$$

Therefore,

$$E(z_1 + z_2 + z_3 + \dots + z_n) = \frac{(\lambda^n - 1)\delta q M_\rho}{(\lambda - 1)} \quad (6.2)$$

$$E(z_1 + \lambda z_2 + \lambda^2 z_3 + \dots + \lambda^{n-1} z_n) = \frac{(\lambda^{2n} - 1)\delta q M_\rho}{(\lambda^2 - 1)} \quad (6.3)$$

$$E(z_1^2 + z_2^2 + z_3^2 + \dots + z_n^2) = \sum_{i=0}^{n-1} \left(\frac{\delta(1-\delta)M_\rho(1-M_\rho)q^2\lambda^{2i}}{\lambda^i q - 1} + \delta^2 q^2 M_\rho^2 \lambda^{2i} \right) \quad (6.4)$$

Successive deliveries arrive at the retailer when the backorder level reaches to the level b . The inventory flow between the manufacturer and the retailer is illustrated in Figures 6.2 and 6.3.

A. Manufacturer's profit

Since the total order quantity of the retailer is $Q = \frac{(\lambda^n - 1)q}{(\lambda - 1)}$, therefore, the production cost of the manufacturer is $\frac{(\lambda^n - 1)q c_m}{(\lambda - 1)}$.

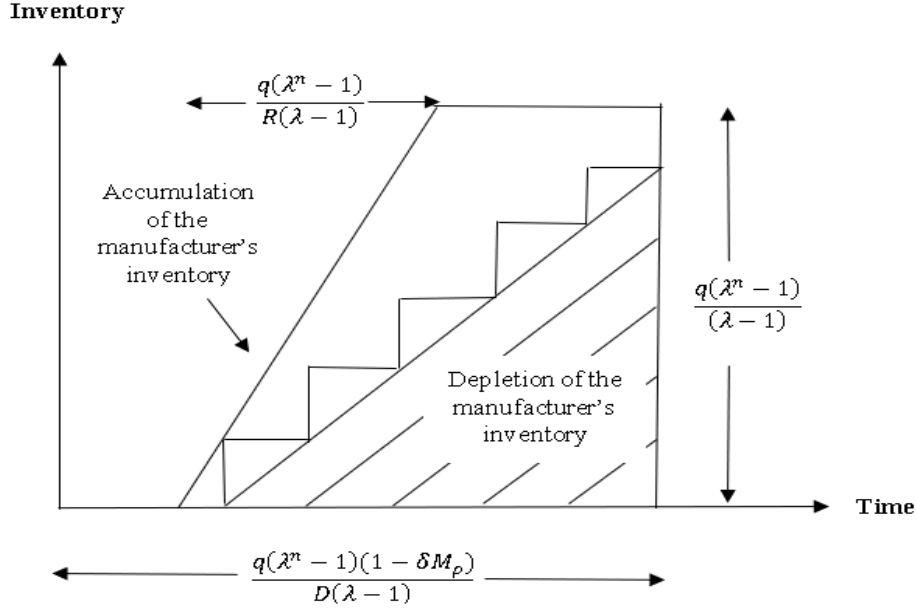


Figure 6.2: Accumulation and depletion of the manufacturer's inventory

Instead of a fixed set up cost, the manufacturer embraces a continuous investment for reducing the set up cost. Therefore, the combined set up cost and investment cost = $AT + Se^{-rA}$.

$$\text{Variable transportation cost} = \frac{(\lambda^n - 1)qf}{(\lambda - 1)}.$$

$$\text{Warranty cost payable for defective items} = \frac{(\lambda^n - 1)q\rho c_w}{(\lambda - 1)}.$$

$$\text{Variable carbon emission costs due to production and transportation} = \frac{(\lambda^n - 1)q(c_p + c_f)}{(\lambda - 1)}.$$

$$\text{Inventory holding cost} = h_m q^2 \left[\frac{\lambda(\lambda^{n-1} - 1)(\lambda^n - 1)}{D(\lambda + 1)(\lambda - 1)^2} + \frac{(\lambda^n - 1)(2\lambda - \lambda^n - 1)}{2R(\lambda - 1)^2} \right].$$

Therefore, the total profit of the manufacturer (TP_m)

$$\begin{aligned} &= \left[\frac{(\lambda^n - 1)q - (\lambda - 1)(z_1 + z_2 + \dots + z_n)}{(\lambda - 1)} \right] w - \frac{(\lambda^n - 1)qc_m}{(\lambda - 1)} - AT - Se^{-rA} \\ &\quad - \frac{(\lambda^n - 1)qf}{(\lambda - 1)} - \frac{(\lambda^n - 1)q\rho c_w}{(\lambda - 1)} - \frac{(\lambda^n - 1)q(c_p + c_f)}{(\lambda - 1)} \\ &\quad - h_m q^2 \left[\frac{\lambda(\lambda^{n-1} - 1)(\lambda^n - 1)}{D(\lambda + 1)(\lambda - 1)^2} + \frac{(\lambda^n - 1)(2\lambda - \lambda^n - 1)}{2R(\lambda - 1)^2} \right] \end{aligned}$$

$$\text{Expected cycle length } E(T) = \frac{(\lambda^n - 1)q - (\lambda - 1)E(z_1 + z_2 + \dots + z_n)}{(\lambda - 1)} = \frac{q(\lambda^n - 1)(1 - \delta M_\rho)}{D(\lambda - 1)}$$

$$= \frac{1}{2D} \left[\frac{(\lambda^{2n} - 1)q^2}{\lambda^2 - 1} - \frac{2qb(\lambda^n - 1)}{(\lambda - 1)} + nb^2 - 2q(z_1 + \lambda z_2 + \dots + \lambda^{n-1}z_n) + 2b(z_1 + z_2 + \dots + z_n) + (z_1^2 + z_2^2 + \dots + z_n^2) \right] + \frac{\delta q}{y} (z_1 + \lambda z_2 + \dots + \lambda^{n-1}z_n)$$

Therefore, the total profit of the retailer (TP_r)

$$= (p - w) \left[\frac{(\lambda^n - 1)q - (\lambda - 1)(z_1 + z_2 + \dots + z_n)}{(\lambda - 1)} \right] + zp(z_1 + z_2 + \dots + z_n) - K - \frac{nc_b b^2}{2D} - \frac{(\lambda^n - 1)I_b \delta q}{(\lambda - 1)} - I_u \left[\frac{(\lambda^n - 1)\rho q - (\lambda - 1)(z_1 + z_2 + \dots + z_n)}{(\lambda - 1)} \right] - \frac{h_r}{2D} \left[\frac{(\lambda^{2n} - 1)q^2}{\lambda^2 - 1} - \frac{2qb(\lambda^n - 1)}{(\lambda - 1)} + nb^2 - 2q(z_1 + \lambda z_2 + \dots + \lambda^{n-1}z_n) + 2b(z_1 + z_2 + \dots + z_n) + (z_1^2 + z_2^2 + \dots + z_n^2) + \frac{\delta q}{y} (z_1 + \lambda z_2 + \dots + \lambda^{n-1}z_n) \right] - \xi(\theta - 1)^2 T - \mu g^2 T$$

Hence the average expected profit of the retailer $AEP_r(q, p, b, g, \theta)$

$$= pD - wD + \frac{\delta zpM_\rho D}{(1 - \delta M_\rho)} - \frac{KD(\lambda - 1)}{q(\lambda^n - 1)(1 - \delta M_\rho)} - \frac{\delta I_b D}{(1 - \delta M_\rho)} - \frac{(1 - \delta)I_u M_\rho D}{(1 - \delta M_\rho)} - \frac{nc_b b^2 (\lambda - 1)}{2q(\lambda^n - 1)(1 - \delta M_\rho)} - \frac{h_r (\lambda - 1)}{2q(\lambda^n - 1)(1 - \delta M_\rho)} \left[\frac{(\lambda^{2n} - 1)q^2}{\lambda^2 - 1} - \frac{2qb(\lambda^n - 1)}{(\lambda - 1)} + nb^2 + \frac{2b\delta q M_\rho (\lambda^n - 1)}{(\lambda - 1)} - \frac{2(\lambda^{2n} - 1)\delta M_\rho q^2}{(\lambda^2 - 1)} + \sum_{i=0}^{n-1} \left(\frac{\delta(1 - \delta)M_\rho(1 - M_\rho)q^2 \lambda^{2i}}{\lambda^i q - 1} + \delta^2 q^2 M_\rho^2 \lambda^{2i} \right) \right] - \frac{h_r \delta^2 q M_\rho D (\lambda^n + 1)}{y(\lambda + 1)(1 - \delta M_\rho)} - \mu g^2 - \xi(\theta - 1)^2 \quad (6.6)$$

6.4.1 Decentralized Model

In the decentralized scenario, both the manufacturer and the retailer maximize their own profits individually. In this study, we consider a leader-follower Stackelberg gaming approach where the retailer acts as the leader and the manufacturer as the follower. First, the manufacturer provides his decisions on the number of shipments and set up cost. Then, based on these reactions, the retailer optimizes his profit with respect to the selling price, batch size, backorder quantity and the promotional effort and greening level.

For convenience, the following notations are used:

$$X_1 = w + \frac{K(\lambda - 1)}{q(\lambda^n - 1)(1 - \delta M_\rho)} + \frac{\delta I_b}{(1 - \delta M_\rho)} + \frac{(1 - \delta)I_u M_\rho}{(1 - \delta M_\rho)} + \frac{h_r \delta^2 q M_\rho (\lambda^n + 1)}{y(\lambda + 1)(1 - \delta M_\rho)} \quad (6.7)$$

$$X_2 = \sum_{i=0}^{n-1} \left\{ \frac{\delta(1-\delta)M_\rho(1-M_\rho)\lambda^{2i}}{\lambda^{iq}-1} + \delta^2 M_\rho^2 \lambda^{2i} \right\} \quad (6.8)$$

$$X_3 = \frac{KD(\lambda-1)}{(\lambda^n-1)(1-\delta M_\rho)} + \frac{nc_b b^2(\lambda-1)}{2(\lambda^n-1)(1-\delta M_\rho)} + \frac{h_r n b^2(\lambda-1)}{2(\lambda^n-1)(1-\delta M_\rho)} \quad (6.9)$$

$$X_4 = \frac{h_r(\lambda^n+1)(1-2\delta M_\rho)}{2(\lambda+1)(1-\delta M_\rho)} + \frac{h_r X_2(\lambda-1)}{2(\lambda^n-1)(1-\delta M_\rho)} + \frac{h_r M_\rho D \delta^2(\lambda^n+1)}{y(\lambda+1)(1-\delta M_\rho)} \quad (6.10)$$

$$X_5 = \frac{(c_m+f)}{(1-\delta M_\rho)} + \frac{S e^{-rA}(\lambda-1)}{q(\lambda^n-1)(1-\delta M_\rho)} + \frac{(c_p+c_f)}{(1-\delta M_\rho)} + \frac{c_w M_\rho}{(1-\delta M_\rho)} + \frac{\delta I_b}{(1-\delta M_\rho)} \\ + \frac{h_m q(2\lambda-\lambda^n-1)}{2R(\lambda-1)(1-\delta M_\rho)} + \frac{K(\lambda-1)}{q(\lambda^n-1)(1-\delta M_\rho)} + \frac{(1-\delta)I_u M_\rho}{(1-\delta M_\rho)} + \frac{h_r \delta^2 q M_\rho(\lambda^n+1)}{y(\lambda+1)(1-\delta M_\rho)} \quad (6.11)$$

$$X_6 = \frac{S e^{-rA} D(\lambda-1)}{q(\lambda^n-1)(1-\delta M_\rho)} + X_3 \quad (6.12)$$

$$X_7 = h_m \left[\frac{\lambda(\lambda^{n-1}-1)}{(\lambda^2-1)} + \frac{(2\lambda-\lambda^n-1)D}{2R(\lambda-1)(1-\delta M_\rho)} \right] + X_4 \quad (6.13)$$

$$X_8 = \left(1 + \frac{\delta z M_\rho}{1-\delta M_\rho} \right) \quad (6.14)$$

$$X_9 = \frac{rSD(\lambda-1)e^{-rA}}{q(\lambda^n-1)(1-\delta M_\rho)} \quad (6.15)$$

$$X_{10} = \frac{n(h_r+c_b)(\lambda-1)}{q(\lambda^n-1)(1-\delta M_\rho)} \quad (6.16)$$

Proposition 6.1 The average expected profit function of the manufacturer (AEP_m) is concave with respect to A for given value of n and the optimal set up cost investment is $A^* = \frac{1}{r} \log \left[\frac{rSD(\lambda-1)}{q(\lambda^n-1)(1-\delta M_\rho)} \right]$.

Proof. Differentiating equation (6.5) with respect to A , we get

$$\frac{\partial AEP_m}{\partial A} = -1 + \frac{rSD(\lambda-1)e^{-rA}}{q(\lambda^n-1)(1-\delta M_\rho)} \quad (6.17)$$

$$\frac{\partial^2 AEP_m}{\partial A^2} = -\frac{r^2SD(\lambda-1)e^{-rA}}{q(\lambda^n-1)(1-\delta M_\rho)} \quad (6.18)$$

From (6.18), it is clear that, $\frac{\partial^2 AEP_m}{\partial A^2} < 0$. This indicates that, AEP_m is concave with respect to A for known value of n and it has a unique optimal solution. By solving $\frac{\partial AEP_m}{\partial A} = 0$, the optimal investment is obtained as $A^* = \frac{1}{r} \log \left[\frac{rSD(\lambda-1)}{q(\lambda^n-1)(1-\delta M_\rho)} \right]$.

Proposition 6.2 The average expected profit function of the manufacturer (AEP_m) is concave with respect to n for given value of A if

$$2RSD(\lambda - 1)(\lambda^2 - 1)(\lambda^n + 1)e^{-rA} > h_m q^2 (\lambda^n - 1)^3 [D(\lambda + 1) - 2R(1 - \delta M_\rho)]$$

and the optimal number of batch shipment is

$$n^* = \frac{\log \left[1 + \sqrt{\frac{2RD(\lambda-1)(\lambda^2-1)Se^{-rA}}{h_m q^2 [2R(1-\delta M_\rho) - D(\lambda+1)]}} \right]}{\log \lambda}.$$

Proof. Taking n as real and differentiating (6.5) partially with respect to n , we get

$$\frac{\partial AEP_m}{\partial n} = \frac{SD(\lambda - 1)e^{-rA}\lambda^n \log \lambda}{q(\lambda^n - 1)^2(1 - \delta M_\rho)} - \frac{h_m q \lambda^n \log \lambda}{(\lambda^2 - 1)} + \frac{h_m q D \lambda^n \log \lambda}{2R(\lambda - 1)(1 - \delta M_\rho)} \quad (6.19)$$

$$\frac{\partial^2 AEP_m}{\partial n^2} = -\frac{SD(\lambda - 1)e^{-rA}\lambda^n (\log \lambda)^2 (\lambda^n + 1)}{q(\lambda^n - 1)^3(1 - \delta M_\rho)} - \frac{h_m q \lambda^n (\log \lambda)^2}{(\lambda^2 - 1)} + \frac{h_m q D \lambda^n (\log \lambda)^2}{2R(\lambda - 1)(1 - \delta M_\rho)} \quad (6.20)$$

Equation (6.20) shows that $\frac{\partial^2 AEP_m}{\partial n^2} < 0$ only if

$$2RSD(\lambda - 1)(\lambda^2 - 1)(\lambda^n + 1)e^{-rA} > h_m q^2 (\lambda^n - 1)^3 [D(\lambda + 1) - 2R(1 - \delta M_\rho)].$$

Then, from the first order optimality condition, the optimal (real) value of n can be obtained as

$$n^* = \frac{\log \left[1 + \sqrt{\frac{2RD(\lambda-1)(\lambda^2-1)Se^{-rA}}{h_m q^2 [2R(1-\delta M_\rho) - D(\lambda+1)]}} \right]}{\log \lambda}$$

Consequently, the optimal (integer) number of batch shipments can be determined in the following manner:

$$n^{opt} = \begin{cases} \lfloor n^* \rfloor, & \text{if } AEP_m(\lfloor n^* \rfloor) > AEP_m(\lceil n^* \rceil) \\ \lceil n^* \rceil, & \text{if } AEP_m(\lfloor n^* \rfloor) < AEP_m(\lceil n^* \rceil) \end{cases} \quad (6.21)$$

Now, taking into account the manufacturer's response (A^* , n^*), the retailer determines his optimal decisions for the batch size (q), retail price (p), shortage quantity (b), promotional (θ) and greening effort (g). Substitution of the values of A^* and n^* in equation (6.6) results in a higher order non-linear complex profit function of the

retailer, whose concavity could not be proved analytically. We will therefore check it numerically.

6.4.2 Centralized Model

In the centralized model structure, all the supply chain members act as a single business entity. Rather than focusing on their individual benefits, they jointly enhance the entire supply chain's profitability. Even though an ideal coordination scheme is very difficult to execute in real life business scenario, the integrated decision making approach is advantageous for achieving higher system profitability and providing better customer services than the decentralized scenario. In the centralized model, both the manufacturer and the retailer collaboratively set the optimal decisions on the number of batch shipments, set up cost investment, batch size, retail price, back-order level, promotional effort, and greening level. The average expected supply chain profit in this case becomes

$$\begin{aligned}
 & AEP_s(n, A, q, p, b, g, \theta) \\
 = & pD + \frac{\delta zpM_\rho D}{(1 - \delta M_\rho)} - \frac{c_m D}{(1 - \delta M_\rho)} - A - \frac{Se^{-rA}D(\lambda - 1)}{q(\lambda^n - 1)(1 - \delta M_\rho)} - \frac{(c_p + c_f)D}{(1 - \delta M_\rho)} - \frac{fD}{(1 - \delta M_\rho)} \\
 & - \frac{c_w M_\rho D}{(1 - \delta M_\rho)} - h_m q \left[\frac{\lambda(\lambda^{n-1} - 1)}{(\lambda^2 - 1)} + \frac{(2\lambda - \lambda^n - 1)D}{2R(\lambda - 1)(1 - \delta M_\rho)} \right] - \frac{KD(\lambda - 1)}{q(\lambda^n - 1)(1 - \delta M_\rho)} \\
 & - \frac{nc_b b^2(\lambda - 1)}{2q(\lambda^n - 1)(1 - \delta M_\rho)} - \mu g^2 - \frac{h_r(\lambda - 1)}{2q(\lambda^n - 1)(1 - \delta M_\rho)} \left[\frac{(\lambda^{2n} - 1)q^2}{(\lambda^2 - 1)} - \frac{2qb(\lambda^n - 1)}{(\lambda - 1)} + nb^2 \right. \\
 & \left. - \frac{2(\lambda^{2n} - 1)\delta M_\rho q^2}{(\lambda^2 - 1)} + \frac{2b\delta q M_\rho(\lambda^n - 1)}{(\lambda - 1)} + \sum_{i=0}^{n-1} \left(\frac{\delta(1 - \delta)M_\rho(1 - M_\rho)q^2\lambda^{2i}}{\lambda^i q - 1} + \delta^2 q^2 M_\rho^2 \lambda^{2i} \right) \right] \\
 & - \frac{h_r \delta^2 q M_\rho D(\lambda^n + 1)}{y(\lambda + 1)(1 - \delta M_\rho)} - \frac{\delta I_b D}{(1 - \delta M_\rho)} - \xi(\theta - 1)^2 - \frac{(1 - \delta)I_u M_\rho D}{(1 - \delta M_\rho)} \tag{6.22}
 \end{aligned}$$

Proposition 6.3 The average expected profit of the entire supply chain (AEP_s) is concave in p , θ and g for given values of n , q , b , and A if $[1 - \delta M_\rho(1 - z)](\xi\eta^2 + \mu\gamma^2) < 4\mu\beta\xi(1 - \delta M_\rho)$. The optimal values of p , θ and g are

$$p^* = p^c = \frac{(\alpha + \gamma\theta)[1 - (1 - z)\delta M_\rho] + X_5\beta(1 - \delta M_\rho)}{2\beta[1 - (1 - z)\delta M_\rho]} \tag{6.23}$$

$$\theta^* = \theta^c = 1 + \frac{\gamma}{2\xi} \left(p + \frac{\delta zp M_\rho}{1 - \delta M_\rho} - X_5 \right) \quad (6.24)$$

$$\text{and } g^* = g^c = \frac{\eta}{2\mu} \left(p + \frac{\delta zp M_\rho}{1 - \delta M_\rho} - X_5 \right) \quad (6.25)$$

Proof. Partial differentiation of (6.22) with respect to p , θ and g give

$$\frac{\partial AEP_s}{\partial p} = \left(1 + \frac{\delta z M_\rho}{1 - \delta M_\rho} \right) D - \beta p \left(1 + \frac{\delta z M_\rho}{1 - \delta M_\rho} \right) + X_5 \beta \quad (6.26)$$

$$\frac{\partial^2 AEP_s}{\partial p^2} = -2\beta \left(1 + \frac{\delta z M_\rho}{1 - \delta M_\rho} \right) \quad (6.27)$$

$$\frac{\partial AEP_s}{\partial \theta} = \left(p + \frac{\delta zp M_\rho}{1 - \delta M_\rho} - X_5 \right) \gamma - 2\xi(\theta - 1) \quad (6.28)$$

$$\frac{\partial^2 AEP_s}{\partial \theta^2} = -2\xi \quad (6.29)$$

$$\frac{\partial AEP_s}{\partial g} = \left(p + \frac{\delta zp M_\rho}{1 - \delta M_\rho} - X_5 \right) \eta - 2\mu g \quad (6.30)$$

$$\frac{\partial^2 AEP_s}{\partial g^2} = -2\mu \quad (6.31)$$

$$\frac{\partial^2 AEP_s}{\partial \theta \partial p} = \frac{\partial^2 AEP_s}{\partial p \partial \theta} = \mu X_8 \quad (6.32)$$

$$\frac{\partial^2 AEP_s}{\partial \theta \partial g} = \frac{\partial^2 AEP_s}{\partial g \partial \theta} = 0 \quad (6.33)$$

$$\frac{\partial^2 AEP_s}{\partial g \partial p} = \frac{\partial^2 AEP_s}{\partial p \partial g} = \eta X_8 \quad (6.34)$$

For given n , q , b and A , the Hessian matrix associated with the profit function AEP_s is

$$H_1 = \begin{pmatrix} \frac{\partial^2 AEP_s}{\partial \theta^2} & \frac{\partial^2 AEP_s}{\partial \theta \partial g} & \frac{\partial^2 AEP_s}{\partial \theta \partial p} \\ \frac{\partial^2 AEP_s}{\partial g \partial \theta} & \frac{\partial^2 AEP_s}{\partial g^2} & \frac{\partial^2 AEP_s}{\partial g \partial p} \\ \frac{\partial^2 AEP_s}{\partial p \partial \theta} & \frac{\partial^2 AEP_s}{\partial p \partial g} & \frac{\partial^2 AEP_s}{\partial p^2} \end{pmatrix} = \begin{pmatrix} -2\xi & 0 & \gamma X_8 \\ 0 & -2\mu & \eta X_8 \\ \gamma X_8 & \eta X_8 & -2\beta X_8 \end{pmatrix} \quad (6.35)$$

For concavity of AEP_s , H_1 should be negative definite.

Now, the first order principal minor = $|H_{11}| = -2\xi < 0$ as $\xi > 0$.

The second order principal minor = $|H_{12}| = 4\xi\mu > 0$ since $\xi, \mu > 0$.

The third order principal minor $|H_1| = |H_{13}| = 2X_8^2(\xi\eta^2 + \mu\gamma^2) - 8\mu\beta\xi X_8 < 0$ if

$$[1 - \delta M_\rho(1 - z)](\xi\eta^2 + \mu\gamma^2) < 4\mu\beta\xi(1 - \delta M_\rho).$$

Therefore, if this condition is satisfied, the average expected supply chain profit function is jointly concave in p , θ and g for given values of n , q , b , and A . Then the optimal values can be determined by using the first order optimality conditions i.e., by solving $\frac{\partial AEP_s}{\partial p} = 0$, $\frac{\partial AEP_s}{\partial \theta} = 0$, and $\frac{\partial AEP_s}{\partial g} = 0$ for p , θ and g , respectively. These equations provide the optimal values of p , θ and g as in equations (6.23), (6.24) and (6.25).

Proposition 6.4 The average expected profit of the entire supply chain (AEP_s) is concave with respect to q , b , and A for given values of n , p , θ and g if $q(rb^2X_{10} + X_9) < 2rX_6$.

The optimal values of q , b , and A are

$$q^* = q^c = \sqrt{\frac{X_6}{X_7}} \quad (6.36)$$

$$b^* = b^c = \frac{h_r q (\lambda^n - 1) (1 - \delta M_\rho)}{(h_r + c_b) n (\lambda - 1)} \quad (6.37)$$

$$\text{and } A^* = A^c = \frac{1}{r} \log \left[\frac{rSD(\lambda - 1)}{q(\lambda^n - 1)(1 - \delta M_\rho)} \right] \quad (6.38)$$

Proof. Differentiating (6.22) partially with respect to q , b and A , we get

$$\frac{\partial AEP_s}{\partial q} = \frac{X_6}{q^2} - X_7 \quad (6.39)$$

$$\frac{\partial^2 AEP_s}{\partial q^2} = -\frac{2X_6}{q^3} \quad (6.40)$$

$$\frac{\partial AEP_s}{\partial b} = -\frac{nc_b b (\lambda - 1)}{q(\lambda^n - 1)(1 - \delta M_\rho)} + \frac{h_r}{(1 - \delta M_\rho)} - \frac{nbh_r (\lambda - 1)}{q(\lambda^n - 1)(1 - \delta M_\rho)} - \frac{h_r \delta M_\rho}{(1 - \delta M_\rho)} \quad (6.41)$$

$$\frac{\partial^2 AEP_s}{\partial b^2} = -\frac{(h_r + c_b) n (\lambda - 1)}{q(\lambda^n - 1)(1 - \delta M_\rho)} \quad (6.42)$$

$$\frac{\partial AEP_s}{\partial A} = -1 + \frac{rSD(\lambda - 1)e^{-rA}}{q(\lambda^n - 1)(1 - \delta M_\rho)} \quad (6.43)$$

$$\frac{\partial^2 AEP_s}{\partial A^2} = -\frac{r^2 SD(\lambda - 1)e^{-rA}}{q(\lambda^n - 1)(1 - \delta M_\rho)} \quad (6.44)$$

$$\frac{\partial^2 AEP_s}{\partial A \partial b} = \frac{\partial^2 AEP_s}{\partial b \partial A} = 0 \quad (6.45)$$

$$\frac{\partial^2 AEP_s}{\partial A \partial q} = \frac{\partial^2 AEP_s}{\partial q \partial A} = \frac{X_9}{q} \quad (6.46)$$

$$\frac{\partial^2 AEP_s}{\partial q \partial b} = \frac{\partial^2 AEP_s}{\partial b \partial q} = \frac{bX_{10}}{q} \quad (6.47)$$

For given n, p, θ and g , the Hessian matrix associated with AEP_s is

$$H_2 = \begin{pmatrix} \frac{\partial^2 AEP_s}{\partial A^2} & \frac{\partial^2 AEP_s}{\partial A \partial b} & \frac{\partial^2 AEP_s}{\partial A \partial q} \\ \frac{\partial^2 AEP_s}{\partial b \partial A} & \frac{\partial^2 AEP_s}{\partial b^2} & \frac{\partial^2 AEP_s}{\partial b \partial q} \\ \frac{\partial^2 AEP_s}{\partial q \partial A} & \frac{\partial^2 AEP_s}{\partial q \partial b} & \frac{\partial^2 AEP_s}{\partial q^2} \end{pmatrix} = \begin{pmatrix} -rX_9 & 0 & \frac{X_9}{q} \\ 0 & -X_{10} & \frac{bX_{10}}{q} \\ \frac{X_9}{q} & \frac{bX_{10}}{q} & -\frac{2X_6}{q^3} \end{pmatrix} \quad (6.48)$$

Now, the first order principal minor $|H_{21}| = -rX_9 < 0$ as $r, X_9 > 0$.

The second order principal minor $|H_{22}| = rX_9X_{10} > 0$ as $r, X_9, X_{10} > 0$.

The third order principal minor $|H_2| = |H_{23}| = -\frac{2rX_6X_9X_{10}}{q^3} + \frac{X_9X_{10}(rb^2X_{10}+X_9)}{q^2} < 0$ if $q(rb^2X_{10} + X_9) < 2rX_6$.

Therefore, if the above condition holds, then the Hessian matrix H_2 is negative definite i.e., the average expected supply chain profit is jointly concave in q, b and A , for given values of n, p, θ and g . Then unique optimal values of q, b , and A obtained by using the first order optimality conditions are given in equations (6.36), (6.37) and (6.38).

Henceforth, for $A^* = A^c, q^* = q^c, p^* = p^c, b^* = b^c, g^* = g^c, \theta^* = \theta^c$, the average expected supply chain profit function AEP_s becomes a function of n only. For the existence of an unique optimal number of batch shipments, the average expected profit function of the whole supply chain should be concave in n . But, the presence of highly non-linear terms in the average expected profit function of the supply chain, makes it difficult to prove the concavity of the supply chain profit function with respect to n .

6.4.3 Coordinated Model

When each supply chain member optimizes its own objective, the consequences can adversely affect the overall performance of the supply chain. On the other hand, the joint decision-making policy may not be beneficial to each member and may hinder their individual profitability. In this regard, coordination mechanisms are indispensable to motivate the players to undertake global optimal decisions while maintain-

ing their own profitability. In this chapter, we consider a trade credit coordination mechanism and a wholesale price discount contract between the manufacturer and the retailer.

6.4.3.1 Trade credit mechanism

Under the trade credit coordination mechanism, the manufacturer offers the retailer a credit period to pay, and in return, the retailer has to adopt optimal inventory, pricing, greening, and promotional strategies. In this way, they can reach their anticipated profit level and boost the market demand by providing a more environment-friendly product at cheaper rate and exerting higher promotional efforts. Moreover, the manufacturer sets the number of shipments, according to the centralized structure. This coordination scheme will also be lucrative for the retailer as he can earn interest through this delay in payment mechanism.

If the manufacturer allows a credit period of τ to the retailer, then the retailer gains an interest $\psi_r \tau w D^c$ and the manufacturer incurs a interest loss of $\psi_m \tau w D^c$, where ψ_r and ψ_m are the interest rates of the retailer and the manufacturer, respectively.

The average expected profit of the manufacturer $AEP_m^{co}(n^c, A^c, \tau)$ is

$$\begin{aligned}
&= wD^c - \frac{c_m D^c}{(1 - \delta M_\rho)} - A^c - \frac{S e^{-rA^c} D(\lambda - 1)}{q^c(\lambda^n - 1)(1 - \delta M_\rho)} - \frac{(c_p + c_f)D^c}{(1 - \delta M_\rho)} \\
&- \frac{fD^c}{(1 - \delta M_\rho)} - \frac{c_w M_\rho D^c}{(1 - \delta M_\rho)} - h_m q^c \left[\frac{\lambda(\lambda^{n^c} - 1)}{(\lambda^2 - 1)} + \frac{(2\lambda - \lambda^{n^c} - 1)D^c}{2R(\lambda - 1)(1 - \delta M_\rho)} \right] \\
&- \psi_m \tau w D^c \tag{6.49}
\end{aligned}$$

and the average expected profit of the retailer $AEP_r^{co}(q^c, p^c, b^c, g^c, \theta^c, \tau)$ is

$$\begin{aligned}
&= p^c D^c - wD^c + \frac{\delta z p^c M_\rho D^c}{(1 - \delta M_\rho)} - \frac{KD^c(\lambda - 1)}{q^c(\lambda^{n^c} - 1)(1 - \delta M_\rho)} - \frac{\delta I_b D^c}{(1 - \delta M_\rho)} - \frac{(1 - \delta)I_u M_\rho D^c}{(1 - \delta M_\rho)} \\
&- \frac{n^c c_b b^c (\lambda - 1)}{2q^c(\lambda^{n^c} - 1)(1 - \delta M_\rho)} - \frac{h_r(\lambda - 1)}{2q^c(\lambda^{n^c} - 1)(1 - \delta M_\rho)} \left[\frac{(\lambda^{2n^c} - 1)q^{c^2}}{(\lambda^2 - 1)} - \frac{2q^c b^c (\lambda^{n^c} - 1)}{(\lambda - 1)} \right. \\
&+ n^c b^c - \frac{2(\lambda^{2n^c} - 1)\delta M_\rho q^{c^2}}{(\lambda^2 - 1)} + \frac{2b^c \delta q^c M_\rho (\lambda^{n^c} - 1)}{(\lambda - 1)} + \sum_{i=0}^{n^c-1} \left(\frac{\delta(1 - \delta)M_\rho(1 - M_\rho)q^{c^2}\lambda^{2i}}{\lambda^i q^c - 1} \right. \\
&\left. \left. + \delta^2 q^{c^2} M_\rho^2 \lambda^{2i} \right) \right] - \frac{h_r \delta^2 q^c M_\rho D^c (\lambda^{n^c} + 1)}{y(\lambda + 1)(1 - \delta M_\rho)} - \xi(\theta^c - 1)^2 - \mu g^{c^2} + \psi_r \tau w D^c \tag{6.50}
\end{aligned}$$

The delay in payment contract will be acceptable to the retailer only if it can generate a higher or equal profit level for him compared to the decentralized structure.

Therefore, the necessary condition for participation of the retailer in this contract is

$$AEP_r^{co}(q^c, p^c, b^c, g^c, \theta^c, \tau) \geq AEP_r^d(q^d, p^d, b^d, g^d, \theta^d) \quad (6.51)$$

Using equations (6.6) and (6.50), the above inequality gives

$$\tau \geq \frac{\nabla^d - \nabla^c}{\psi_r w D^c} = \tau_{min} \quad (6.52)$$

$$\begin{aligned} \text{where } \nabla &= pD - wD + \frac{\delta z p M_\rho D}{(1 - \delta M_\rho)} - \frac{KD(\lambda - 1)}{q(\lambda^n - 1)(1 - \delta M_\rho)} - \frac{\delta I_b D}{(1 - \delta M_\rho)} - \frac{(1 - \delta) I_u M_\rho D}{(1 - \delta M_\rho)} \\ &- \frac{nc_b b^2 (\lambda - 1)}{2q(\lambda^n - 1)(1 - \delta M_\rho)} - \mu g^2 - \xi(\theta - 1)^2 - \frac{h_r(\lambda - 1)}{2q(\lambda^n - 1)(1 - \delta M_\rho)} \left[\frac{(\lambda^{2n} - 1)q^2}{(\lambda^2 - 1)} \right. \\ &- \frac{2qb(\lambda^n - 1)}{(\lambda - 1)} + nb^2 - \frac{2(\lambda^{2n} - 1)\delta M_\rho q^2}{(\lambda^2 - 1)} + \frac{2b\delta q M_\rho (\lambda^n - 1)}{(\lambda - 1)} \\ &\left. + \sum_{i=0}^{n-1} \left(\frac{\delta(1 - \delta)M_\rho(1 - M_\rho)q^2\lambda^{2i}}{\lambda^i q - 1} + \delta^2 q^2 M_\rho^2 \lambda^{2i} \right) \right] - \frac{h_r \delta^2 q M_\rho D (\lambda^n + 1)}{y(\lambda + 1)(1 - \delta M_\rho)} \end{aligned}$$

Although the manufacturer plays a leading role in implementing the coordination scheme, still he would ensure his own profit level. Therefore, he will provide a credit period to the retailer up to a certain limit in such a way that it does not lead to less profit for him as compared to the decentralized scenario. Hence, the necessary condition for the manufacturer's involvement in the coordination mechanism is

$$AEP_m^{co}(n^c, A^c, \tau) \geq AEP_m^d(n^d, A^d) \quad (6.53)$$

Using (6.5) and (6.49), the above inequality provides

$$\tau \leq \frac{v^c - v^d}{\psi_m w D^c} = \tau_{max} \quad (6.54)$$

$$\begin{aligned} \text{where } v &= wD - \frac{c_m D}{(1 - \delta M_\rho)} - A - \frac{Se^{-rA}D(\lambda - 1)}{q(\lambda^n - 1)(1 - \delta M_\rho)} - \frac{(c_p + c_f)D}{(1 - \delta M_\rho)} - \frac{fD}{(1 - \delta M_\rho)} \\ &- \frac{c_w M_\rho D}{(1 - \delta M_\rho)} - h_m q \left[\frac{\lambda(\lambda^{n-1} - 1)}{(\lambda^2 - 1)} + \frac{(2\lambda - \lambda^n - 1)D}{2R(\lambda - 1)(1 - \delta M_\rho)} \right] \end{aligned} \quad (6.55)$$

It is noteworthy that, the retailer will participate in the trade credit contract only if $\tau \geq \tau_{min}$ and the manufacturer will incentivize the retailer towards the coordination only if $\tau \leq \tau_{max}$. Therefore, the trade credit mechanism appears to be a win-win for both supply chain members if $0 < \tau_{min} \leq \tau \leq \tau_{max}$. For any value of τ in this region, the proposed trade credit mechanism ensures higher profit level for each

supply chain player and is efficient enough to encourage them to adopt global optimal decisions. Whenever the retailer agrees to the credit period τ_{min} , his profitability in the coordinated structure remains the same as the decentralized one. The surplus profit goes into the manufacturer's account. Likewise, whenever the manufacturer allows a credit period τ_{max} to the retailer, it generates the same profit level for him as the decentralized scenario. In such circumstance, the retailer enjoys the entire benefit generated by the coordination mechanism. For a fair profit-sharing between the upstream and downstream members, they can share the surplus, according to their bargaining power over each other. If ϕ be the retailer's bargaining power over the manufacturer, then the optimal trade credit period is given by

$$\tau_{opt} = (1 - \phi)\tau_{min} + \phi\tau_{max} = (1 - \phi) \left[\frac{\nabla^d - \nabla^c}{\psi_r w D^c} \right] + \phi \left[\frac{v^c - v^d}{\psi_m w D^c} \right] \quad (6.56)$$

However, under the proposed trade credit mechanism, the average expected profit of the whole supply chain is

$$\begin{aligned} & AEP_s^{co}(n^c, A^c, q^c, p^c, b^c, g^c, \theta^c, \tau) \\ = & p^c D^c + \frac{\delta z p^c M_\rho D^c}{(1 - \delta M_\rho)} - \frac{c_m D^c}{(1 - \delta M_\rho)} - A^c - \frac{S e^{-r A^c} D (\lambda - 1)}{q^c (\lambda^{n^c} - 1) (1 - \delta M_\rho)} - \frac{(c_p + c_f) D}{(1 - \delta M_\rho)} \\ & - \frac{c_w M_\rho D^c}{(1 - \delta M_\rho)} - h_m q^c \left[\frac{\lambda (\lambda^{n^c} - 1)}{(\lambda^2 - 1)} + \frac{(2\lambda - \lambda^{n^c} - 1) D}{2R(\lambda - 1)(1 - \delta M_\rho)} \right] - \frac{K D^c (\lambda - 1)}{q^c (\lambda^{n^c} - 1) (1 - \delta M_\rho)} \\ & - \frac{n^c c_b b^c (\lambda - 1)}{2q^c (\lambda^{n^c} - 1) (1 - \delta M_\rho)} - \frac{h_r (\lambda - 1)}{2q^c (\lambda^{n^c} - 1) (1 - \delta M_\rho)} \left[\frac{(\lambda^{2n^c} - 1) q^{c2}}{(\lambda^2 - 1)} - \frac{2q^c b^c (\lambda^{n^c} - 1)}{(\lambda - 1)} \right. \\ & \left. + n^c b^c - \frac{2(\lambda^{2n^c} - 1) \delta M_\rho q^{c2}}{(\lambda^2 - 1)} + \frac{2b^c \delta q^c M_\rho (\lambda^{n^c} - 1)}{(\lambda - 1)} + \sum_{i=0}^{n^c-1} \left(\frac{\delta(1 - \delta) M_\rho (1 - M_\rho) q^{c2} \lambda^{2i}}{\lambda^i q^c - 1} \right. \right. \\ & \left. \left. + \delta^2 q^{c2} M_\rho^2 \lambda^{2i} \right) \right] - \frac{h_r \delta^2 q^c M_\rho D^c (\lambda^{n^c} + 1)}{y(\lambda + 1)(1 - \delta M_\rho)} - \frac{\delta I_b D^c}{(1 - \delta M_\rho)} - \frac{(1 - \delta) I_u M_\rho D^c}{(1 - \delta M_\rho)} - \zeta (\theta^c - 1)^2 \\ & - \frac{f D^c}{(1 - \delta M_\rho)} + (\psi_r - \psi_m) \tau w D^c \end{aligned} \quad (6.57)$$

Therefore, from Equations (6.22) and (6.57), we get

$$AEP_s^{co}(n^c, A^c, q^c, p^c, b^c, g^c, \theta^c, \tau) - AEP_s^c(n^c, A^c, q^c, p^c, b^c, g^c, \theta^c) = (\psi_r - \psi_m) \tau w D^c \quad (6.58)$$

Depending on the interest rates of the manufacturer and the retailer, three different scenarios can arise:

(i) $\psi_r > \psi_m$: In this case, from the above equation (6.58), it is apparent that the proposed trade credit mechanism exceeds the centralized profit level and generates higher profitability for the entire supply chain. Thus, whenever the retailer's interest rate is higher than the manufacturer's, the trade credit contract coordinates the channel and effectively improves the profit margins of both upstream and downstream players as well as the whole supply chain.

(ii) $\psi_r = \psi_m$: In this case, equation (6.58) reflects that, $AEP_s^{co}(n^c, A^c, q^c, p^c, b^c, g^c, \theta^c, \tau) = AEP_s^c(n^c, A^c, q^c, p^c, b^c, g^c, \theta^c)$. Therefore, when the interest rates are same at both the manufacturer and the retailer, the trade credit mechanism successfully reaches the anticipated centralized supply chain profit and coordination can be achieved.

(iii) $\psi_r < \psi_m$: In this situation, the interest earned by the retailer is less than the interest lost by the manufacturer. Then, equation (6.58) clearly shows that, $AEP_s^{co}(n^c, A^c, q^c, p^c, b^c, g^c, \theta^c, \tau) < AEP_s^c(n^c, A^c, q^c, p^c, b^c, g^c, \theta^c)$ i.e., the proposed trade credit mechanism eventually fails to achieve channel coordination. Here we propose a wholesale price discount contract as an alternative coordination scheme.

6.4.3.2 Wholesale price discount contract

Integrated decision making allows companies to experience competitive advantages and reaps more revenue than decentralized decision making. But, it needlessly compels all supply chain members to adhere the globally optimal decisions that may not be appropriate for every supply chain entity. In this context, supply chain agents often take part in various effective coordination mechanisms. Wholesale price discount contract is one of these fruitful schemes. Under this contract, the manufacturer offers a discount on the wholesale price to the retailer and in exchange the retailer is asked to set his optimal decisions in line with the centralized structure. Supposedly, the price discount contract offered to the retailer is

$$w = \begin{cases} w, & \text{if } q < q^{c*} \\ w(1 - \chi), & \text{if } q \geq q^{c*} \end{cases} \quad (6.59)$$

Entering into this contract, the average expected profit of the retailer becomes

$$\begin{aligned}
& AEP_r^{co}(q^c, p^c, b^c, g^c, \theta^c, \chi) \\
= & p^c D^c - w(1 - \chi)D^c + \frac{\delta z p^c M_\rho D^c}{(1 - \delta M_\rho)} - \frac{KD^c(\lambda - 1)}{q^c(\lambda^{n^c} - 1)(1 - \delta M_\rho)} - \frac{(1 - \delta)I_u M_\rho D^c}{(1 - \delta M_\rho)} \\
& - \frac{\delta I_b D^c}{(1 - \delta M_\rho)} - \frac{n^c c_b b^c (\lambda - 1)}{2q^c(\lambda^{n^c} - 1)(1 - \delta M_\rho)} - \frac{h_r(\lambda - 1)}{2q^c(\lambda^{n^c} - 1)(1 - \delta M_\rho)} \left[\frac{(\lambda^{2n^c} - 1)q^{c2}}{(\lambda^2 - 1)} + n^c b^c \right. \\
& - \frac{2q^c b^c (\lambda^{n^c} - 1)}{(\lambda - 1)} - \frac{2(\lambda^{2n^c} - 1)\delta M_\rho q^{c2}}{(\lambda^2 - 1)} + \frac{2b^c \delta q^c M_\rho (\lambda^{n^c} - 1)}{(\lambda - 1)} + \sum_{i=0}^{n^c-1} \left(\delta^2 q^{c2} M_\rho^2 \lambda^{2i} \right. \\
& \left. \left. + \frac{\delta(1 - \delta)M_\rho(1 - M_\rho)q^{c2}\lambda^{2i}}{\lambda^i q^c - 1} \right) \right] - \frac{h_r \delta^2 q^c M_\rho D^c (\lambda^{n^c} + 1)}{y(\lambda + 1)(1 - \delta M_\rho)} - \xi(\theta^c - 1)^2 - \mu g^{c2} \quad (6.60)
\end{aligned}$$

and the average expected profit of the manufacturer is

$$\begin{aligned}
AEP_m^{co}(n^c, A^c, \chi) = & w(1 - \chi)D^c - \frac{c_m D^c}{(1 - \delta M_\rho)} - A^c - \frac{Se^{-rA^c} D(\lambda - 1)}{q^c(\lambda^n - 1)(1 - \delta M_\rho)} - \frac{(c_p + c_f)D^c}{(1 - \delta M_\rho)} \\
& - \frac{fD^c}{(1 - \delta M_\rho)} - \frac{c_w M_\rho D^c}{(1 - \delta M_\rho)} - h_m q^c \left[\frac{\lambda(\lambda^{n^c-1} - 1)}{(\lambda^2 - 1)} + \frac{(2\lambda - \lambda^{n^c} - 1)D^c}{2R(\lambda - 1)(1 - \delta M_\rho)} \right] \quad (6.61)
\end{aligned}$$

The retailer will involve into this wholesale price discount contract only if he can earn a higher profit than in the decentralized model. Consequently, the required condition for the retailer to engage in the price discount mechanism is

$$AEP_r^{co}(q^c, p^c, b^c, g^c, \theta^c, \chi) \geq AEP_r^d(q^d, p^d, b^d, g^d, \theta^d) \quad (6.62)$$

Now, using equations (6.6) and (6.60), the above inequality implies that

$$\chi \geq \frac{(\sigma^d - \sigma^c) - w(D^d - D^c)}{wD^c} = \chi_{min} \quad (6.63)$$

$$\begin{aligned}
\text{where } \sigma = & pD + \frac{\delta z p M_\rho D}{(1 - \delta M_\rho)} - \frac{KD(\lambda - 1)}{q(\lambda^n - 1)(1 - \delta M_\rho)} - \frac{\delta I_b D}{(1 - \delta M_\rho)} - \frac{(1 - \delta)I_u M_\rho D}{(1 - \delta M_\rho)} \\
& - \frac{n c_b b^2 (\lambda - 1)}{2q(\lambda^n - 1)(1 - \delta M_\rho)} - \xi(\theta - 1)^2 - \frac{h_r(\lambda - 1)}{2q(\lambda^n - 1)(1 - \delta M_\rho)} \left[\frac{(\lambda^{2n} - 1)q^2}{(\lambda^2 - 1)} \right. \\
& - \frac{2qb(\lambda^n - 1)}{(\lambda - 1)} + nb^2 - \frac{2(\lambda^{2n} - 1)\delta M_\rho q^2}{(\lambda^2 - 1)} + \frac{2b\delta q M_\rho (\lambda^n - 1)}{(\lambda - 1)} - \mu g^2 \\
& \left. + \sum_{i=0}^{n-1} \left(\frac{\delta(1 - \delta)M_\rho(1 - M_\rho)q^2\lambda^{2i}}{\lambda^i q - 1} + \delta^2 q^2 M_\rho^2 \lambda^{2i} \right) \right] - \frac{h_r \delta^2 q M_\rho D (\lambda^n + 1)}{y(\lambda + 1)(1 - \delta M_\rho)}
\end{aligned}$$

The manufacturer will provide the product at a lower price to the retailer only if this contract assures his higher profitability than the decentralized scenario despite this price discount. Therefore, the necessary criteria to be fulfilled for the engagement of the manufacturer in this contract is

$$AEP_m^{co}(n^c, A^c, \chi) \geq AEP_m^d(n^d, A^d) \quad (6.64)$$

Now, using equations (6.5) and (6.61), from the above inequality it appears

$$\chi \leq \frac{(\kappa^d - \kappa^c) - w(D^d - D^c)}{wD^c} = \chi_{max} \quad (6.65)$$

$$\begin{aligned} \text{where } \kappa = & \frac{c_m D}{(1 - \delta M_\rho)} + A + \frac{Se^{-rA} D(\lambda - 1)}{q(\lambda^n - 1)(1 - \delta M_\rho)} + \frac{(c_p + c_f)D}{(1 - \delta M_\rho)} + \frac{fD}{(1 - \delta M_\rho)} \\ & + \frac{c_w M_\rho D}{(1 - \delta M_\rho)} + h_m q \left[\frac{\lambda(\lambda^{n-1} - 1)}{(\lambda^2 - 1)} + \frac{(2\lambda - \lambda^n - 1)D}{2R(\lambda - 1)(1 - \delta M_\rho)} \right] \end{aligned}$$

It is noticeable that, the retailer requires a minimum price discount χ_{min} and on the other hand, the manufacturer can provide at most χ_{max} price discount. So, for all values of χ lying in $[\chi_{min}, \chi_{max}]$, the proposed wholesale price discount contract effectively coordinates the supply chain and appears to be beneficial for supply chain players. They can experience a higher profit margin by adopting this price discount coordination mechanism. Furthermore, the additional revenue obtained through the wholesale price contract is

$$\Delta AEP = AEP_s^{co}(n^c, A^c, q^c, p^c, b^c, g^c, \theta^c, \chi) - AEP_s^d(n^d, A^d, q^d, p^d, b^d, g^c, \theta^d).$$

The manufacturer and the retailer share this extra profits through bargaining. Let ϕ be the bargaining power of the retailer over the manufacturer. Then the bargaining power of the manufacturer is $(1 - \phi)$. Accordingly, the average expected profit of the retailer under this contract is

$$AEP_r^{co}(q^c, p^c, b^c, g^c, \theta^c, \chi) = AEP_r^d(q^d, p^d, b^d, g^d, \theta^d) + \phi \Delta AEP \quad (6.66)$$

From the above equation (6.66), the exact price discount ratio jointly decided by the manufacturer and the retailer can be derived. It goes without saying that, the rest of the surplus is in the hands of the manufacturer.

6.5 Numerical Analysis

In this section, we examine and analyze the performance of the proposed model under different scenarios through numerical examples. The following set of parameter values are considered and the optimal outcomes are shown in Table 6.1. Similar to Wu et al., 2007, it is assumed that, the defective rate ρ follows a Beta distribution with parameter values $s = 1$ and $t = 9$. The probability density function of ρ is $h(\rho) = 9(1 - \rho)^8$ with $0 < \rho < 1$. Hence, $E(\rho) = M_\rho = \frac{s}{(s+t)} = 0.1$.

Example 1. $R = 10000; S = 1000; K = 300; f = 10; c_p = 1.8; c_f = 2.5; c_m = 18; c_w = 10; I_b = 1.8; I_u = 12; y = 175200; \alpha = 5000; \beta = 9; \gamma = 7; \eta = 6; \mu = 50; \lambda = 1.9; r = 0.01; M_\rho = 0.1; \delta = 0.1; h_m = 0.5; h_r = 0.6; c_b = 0.7; w = 220; \xi = 25; z = 0.75; \psi_r = 0.38; \psi_m = 0.35; \phi = 0.4$. (with appropriate units)

It is observed from Table 6.1 that the integrated decision-making provides about 13% more surplus in the profitability of the entire supply chain than in the decentralized scenario. In the centralized case, the product reaches the end-customer at a 23% lower price than the decentralized case and the retailer can exert a 36% higher promotional effort level to persuade the customer to purchase their product. The result shows that the customer demand increases significantly. The retailer orders more quantity and maximum allowable backorder quantity also increases. It is also observed that the set up cost investment of the manufacturer is reduced by about 22%. As a consequence, increased consumer demand and cost reduction collectively enhance the profitability of the whole supply chain considerably. Since $\psi_r > \psi_m$ i.e., the retailer can earn higher interest rate than the manufacturer, both supply chain players agree to adopt the delay in payment contract. Table 6.1 shows that, the minimum credit period required to convince the retailer to accept the delay in payment contract is 150 days and the manufacturer can provide a maximum credit period of 327 days without compromising his own profit. Whenever the credit period is 150 days, the average profit of the retailer appears to be equal with that of the decentralized model and the manufacturer achieves the same profit as the decentralized case when the credit period is 327 days. In the feasible interval [150 days, 327 days], a win-win situation arises i.e., for any value of τ lying within this interval, the delay in payment contract is found to be beneficial for both the retailer and the manufacturer. Optimal trade credit period is obtained as 221 days.

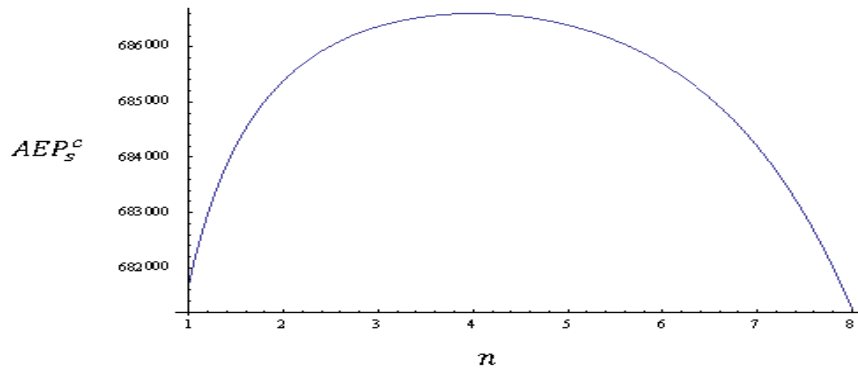


Figure 6.4: Concavity of the average supply chain profit versus n

Optimal results in Table 6.1 confirm that the delay in payment contract is lucrative not only to the individual supply chain members but also to the entire supply chain. Under this coordinated scenario, the average profits of the manufacturer and the retailer are increased about 16% and 13%, respectively compared to their decentralized profits. By adopting this coordination mechanism, the supply chain players achieve even more system profit than the centralized profit.

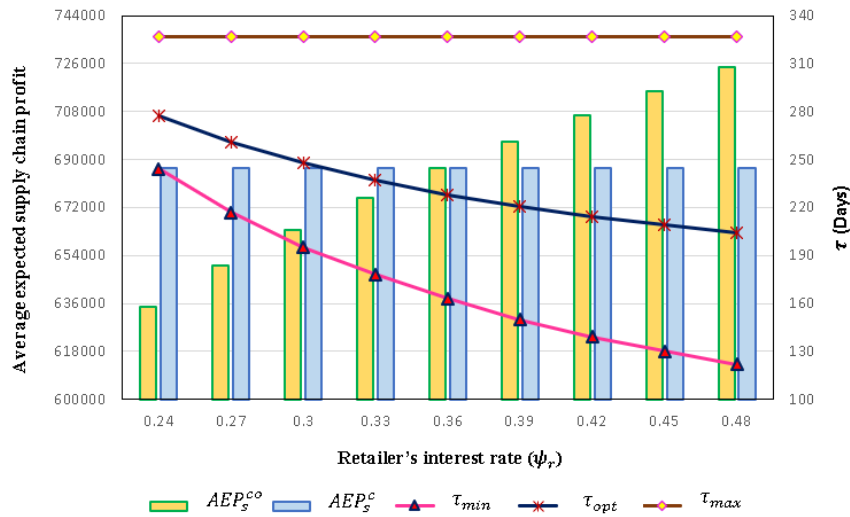


Figure 6.5: Supply chain profit versus retailer's interest rate (ψ_r) and credit period (τ)

Figure 6.5 exhibits the change in the credit period and the supply chain profitability along with the change of ψ_r . It is observed that, as soon as the interest rate earned at the retailer increases, the minimum credit period τ_{min} decreases i.e., the retailer agrees to accept the delay in payment mechanism for a shorter trade credit period.

Since it has no effect on the optimal decisions of the manufacturer, the maximum allowable credit period τ_{max} remains unchanged.

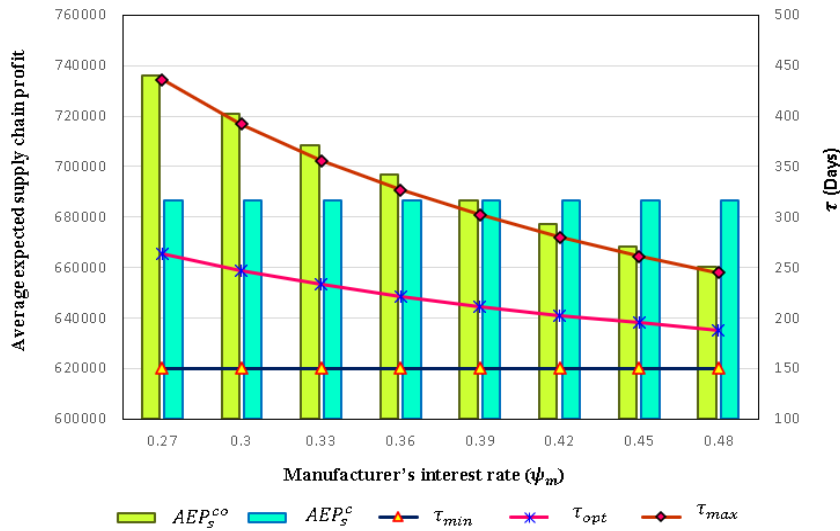


Figure 6.6: Supply chain profit versus manufacturer's interest rate (ψ_m) and credit period (τ)

Consequently, the optimal trade credit period τ_{opt} begins to decrease. It can be noticed that, at $\psi_r = 0.36$, the delay in payment contract results same system profit as the centralized ones. But for $\psi_r < 0.36$ and $\tau < 163$ days, the supply chain profit under the delay in payment mechanism is lesser than the centralized profit. So, in this case, the delay in payment contract fails to coordinate the supply chain. It is evident from Figure 6.5 that, for $\psi_r > 0.36$ and $\tau_{min} < 163$ days, the delay in payment mechanism generates higher supply chain profit compared to the centralized case. Figure 6.6 depicts the changes in supply chain profit levels and trade credit period with respect to the interest rate at the manufacturer. It should be noted that whenever the manufacturer's interest rate increases, the manufacturer may offer the retailer a longer credit period. So, the value of τ_{max} gradually increases with the increment of ψ_m while τ_{min} remains constant. As a result, the optimal trade credit period τ_{opt} also increases. At $\psi_m = 0.39$, the coordinated profit of the whole supply chain is equal to its decentralized profit. But when $\psi_m > 0.39$, although the proposed coordination scheme gives higher system profit within the feasible region than the decentralized model case, it fails to reach the centralized profit level. Thus, for $\psi_m < 0.39$ and $\tau_{opt} > 211$ days, the proposed coordination mechanism provides more system profit than the centralized scenario.

Table 6.1: Optimal results for different models

	Example 1	Example 2	Example 3	Example 4
Decentralized Model				
n^{d*}	5	7	5	7
A^{d*}	307.60	197.04	259.72	197.04
q^{d*}	32.80	26.45	49.08	26.45
p^{d*}	401.75	424.47	504.42	424.47
b^{d*}	72.45	67.85	104.47	67.85
g^{d*}	11	19.84	48.41	19.84
θ^{d*}	26.65	20.84	57.47	20.84
AP_m^{d*}	317756	192270	322248	192270
AP_r^{d*}	277183	184411	262422	184411
AP_s^{d*}	594939	376681	584670	376681
Centralized Model				
n^{c*}	4	6	5	6
A^{c*}	238.42	195.85	213.66	195.85
q^{c*}	179.40	51.96	83.75	51.96
p^{c*}	313	332.52	404.33	332.52
b^{c*}	273.97	112.21	237.25	112.21
g^{c*}	17.31	30.19	78.15	30.19
θ^{c*}	41.38	31.19	92.17	31.19
AP_m^{c*}	-	-	-	-
AP_r^{c*}	-	-	-	-
AP_s^{c*}	686602	426973	683893	426973
Coordinated Model				
n^{co*}	4	6	5	6
A^{co*}	238.42	195.85	213.66	195.85
q^{co*}	179.40	51.96	83.75	51.96
p^{co*}	313	332.52	404.33	332.52
b^{co*}	273.97	112.21	237.25	112.21
g^{co*}	17.31	30.19	78.15	30.19
θ^{co*}	41.38	31.19	92.17	31.19
$\tau_{min}(day)$	150	178	-	178
$\tau_{max}(day)$	327	357	-	357
$\tau_{opt}(day)$	221	285	-	267
$\chi_{min}(\%)$	-	-	16	15
$\chi_{max}(\%)$	-	-	32	29
$\chi_{mean}(\%)$	-	-	24	22
AP_m^{co*}	376952	212387	371859	217416 [†]
AP_r^{co*}	319936	214586	312034	209557 [†]
AP_s^{co*}	696888	426973	683893	426973 [†]

† represents both the trade credit and price discount mechanism

Special cases:

Case I: All the items are of perfect quality

If all the items are non-defective then $E(\rho) = Var(\rho) = 0$. Optimal results under this scenario are presented in Table 6.2. It is noteworthy that the average expected profit of the whole supply chain under decentralized scenario decreases when the products are all of perfect quality, which validates the findings of existing literature. In the decentralized model, it is found from Table 6.2 that, although the profit level of the manufacturer increases, the retailer’s profit level decreases than our proposed model with imperfect quality products. This happens because the increment of the retail price of the product and reduction in the promotional effort level of the retailer jointly reduce the customer demand. A contrary situation can be noticed in the centralized and coordinated models which is in striking contrast with some existing literature. In the centralized and coordinated models of perfect quality items, customer demand and order quantity have increased compared to our proposed model. This may be one of the major reasons behind this kind of discrepancy in optimal results. Outcomes of Table 6.2 suggest that defective rate of the product should be reduced to achieve a better profit level.

Table 6.2: Optimal results for different models when $M_\rho = 0$

Model	n^*	A^*	q^*	p^*	b^*	g^*	θ^*	τ_{min}	τ_{max}	τ_{opt}	AEP_m^*	AEP_r^*	AEP_s^*
Decentralized	5	306.75	32.67	401.91	72.90	10.89	26.40	-	-	-	319071	273997	593068
Centralized	4	238.40	178.26	311.85	274.98	17.24	41.22	-	-	-	-	-	686555
Coordinated	4	238.40	178.26	311.85	274.98	17.24	41.22	153	332	224	379446	317601	697047

Case II: Items are sold after full inspection

Whenever $\delta = 1$ i.e., all the items undergo a rigorous inspection before selling, then it is evident from the optimal results given in Table 6.3 that, the average expected supply chain profit increases with full inspection compared to the sampling inspection policy in each model case. This is in line with the existing literature (Sarkar et al., 2022b). But a contrary situation arises for higher inspection cost. In Example 1, the retailer’s inspection cost is \$1.8, so for this lower cost, full inspection appears to be more lucrative for the entire supply chain. However, for higher values of I_b , we can experience a radically different environment from the early findings. Figure 6.7

exhibits a comparative study of the sampling inspection and full inspection policy for different values of inspection cost. From Figure 6.7, it appears that, the average supply chain profit decreases relatively rapidly with the raising inspection cost in case of full inspection strategy. Therefore, it is noteworthy that, the acceptability of the sampling inspection versus full inspection policies depends upon a threshold value of the inspection cost which gives legitimacy to the findings of Bose and Guha, 2021.

Table 6.3: Optimal results for different models when $\delta = 1$

Model	n^*	A^*	q^*	p^*	b^*	g^*	θ^*	τ_{min}	τ_{max}	τ_{opt}	AEP_m^*	AEP_r^*	AEP_s^*
DM	5	317.01	34.29	396.16	69.13	12.41	29.97	-	-	-	329315	326642	655957
CM	4	240.05	194.92	315.29	270.61	18.68	44.59	-	-	-	-	-	739553
COM	4	240.05	194.92	315.29	270.61	18.68	44.59	136	297	200	383296	365628	748924

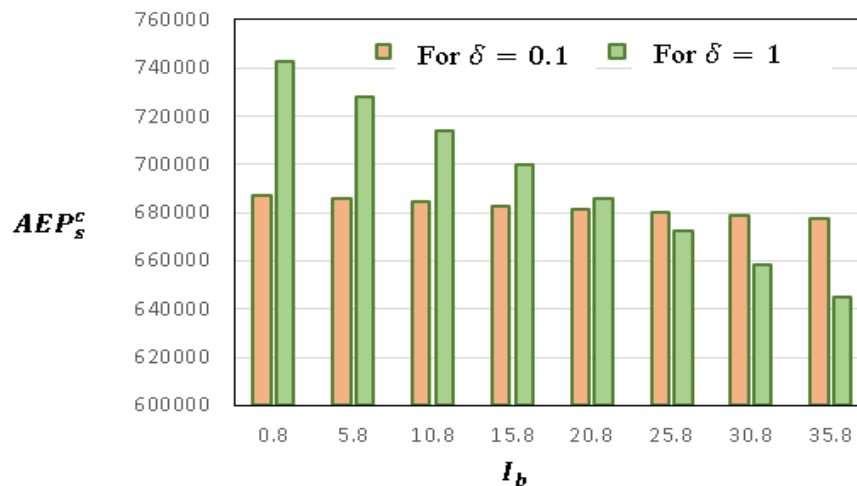


Figure 6.7: Average expected supply chain profit for different values of I_b in sampling inspection vs. full inspection

Case III: Items are sold without inspection

Whenever the retailer does not perform any inspection and sells the products to the end customers directly at a regular price, the optimal results are presented in Table 6.4. It is found that, the model with no inspection results in the lowest supply chain profit compared to the other cases. Furthermore, the model with sample inspection appears to be more advantageous than the model with no inspection. Moreover, when $E(\rho) = 0.1$, the sample inspection policy results in only 0.77% higher profit

than that of the model with no inspection. But, for $E(\rho) = 0.5$, the sample inspection policy provides a 4% higher profit. Therefore, it may be noted that, whenever the percentage of defective items increases in a batch, the profit deviation between sample inspection and no inspection increases expeditiously.

Table 6.4: Optimal results for different models when $\delta = 0$

Model	n^*	A^*	q^*	p^*	b^*	g^*	θ^*	τ_{min}	τ_{max}	τ_{opt}	AEP_m^*	AEP_r^*	AEP_s^*
DM	5	306.54	32.64	402.38	72.83	10.85	26.32	-	-	-	316472	272334	588806
CM	4	238.25	177.85	312.78	274.35	17.17	41.06	-	-	-	-	-	681343
COM	4	238.25	177.85	312.78	274.35	17.17	41.06	152	330	223	376233	315495	691728

Example 2. $R = 7000; S = 800; K = 200; f = 15; c_p = 3; c_f = 4; c_m = 10; c_w = 7; I_b = 1.6; I_u = 10; y = 175200; \alpha = 3000; \beta = 5; \gamma = 3; \eta = 2; \mu = 10; \lambda = 1.7; r = 0.01; M_\rho = 0.1; \delta = 0.2; h_m = 0.3; h_r = 0.8; c_b = 1.2; w = 200; \xi = 15; z = 0.6; \psi_r = 0.3; \psi_m = 0.3; \phi = 0.6$. (with appropriate units)

In this example, $\psi_m = \psi_r$ i.e., the interest rate earned at the retailer and the interest rate lost at the manufacturer are same. The optimal results given in Table 6.1 reflects that the trade credit mechanism effectively coordinates the model and attains the same supply chain profit as the centralized one. The supply chain members share the additional profit between them according to their bargaining power. Since, the retailer possess a higher bargaining power than the manufacturer, the profit growth of the retailer is more than that of the manufacturer.

Example 3. $R = 5000; S = 1000; K = 300; f = 18; c_p = 2.5; c_f = 3.8; c_m = 15; c_w = 10; I_b = 1.2; I_u = 9; y = 175200; \alpha = 4000; \beta = 6; \gamma = 4; \eta = 3; \lambda = 2; r = 0.01; M_\rho = 0.1; \delta = 0.1; h_m = 0.5; h_r = 0.6; c_b = 0.7; w = 280; \xi = 8; z = 0.5; \psi_r = 0.2; \psi_m = 0.25; \phi = 0.5$. (with appropriate units)

This example considers the case where $\psi_m > \psi_r$. It is found that the trade credit coordination scheme fails to reach the profit level of the supply chain as in the centralized structure. The retailer and the manufacturer prefer to adopt the wholesale price contract as an alternative coordination scheme. It is apparent from Table 6.1 that the wholesale price contract effectively achieves the centralized supply chain profit. The minimum payable price discount for the retailer to participate in the wholesale price contract is 16% and the manufacturer can offer a maximum 32%

price discount to the retailer. So, for any price discount χ lying in $[16\%, 32\%]$, the wholesale price discount contract coordinates the channel and proves to be beneficial for both supply chain members. The additional profit is distributed among the manufacturer and the retailer pursuant to their bargaining power. Since the retailer and the manufacturer both have the same bargaining power, χ is simply taken as the mean of $[16\%, 32\%]$ i.e., 24%. It is evident from Table 6.1 that the price discount contract improves the profitability of the manufacturer and the retailer by 15% and 19%, respectively compared to the decentralized structure.

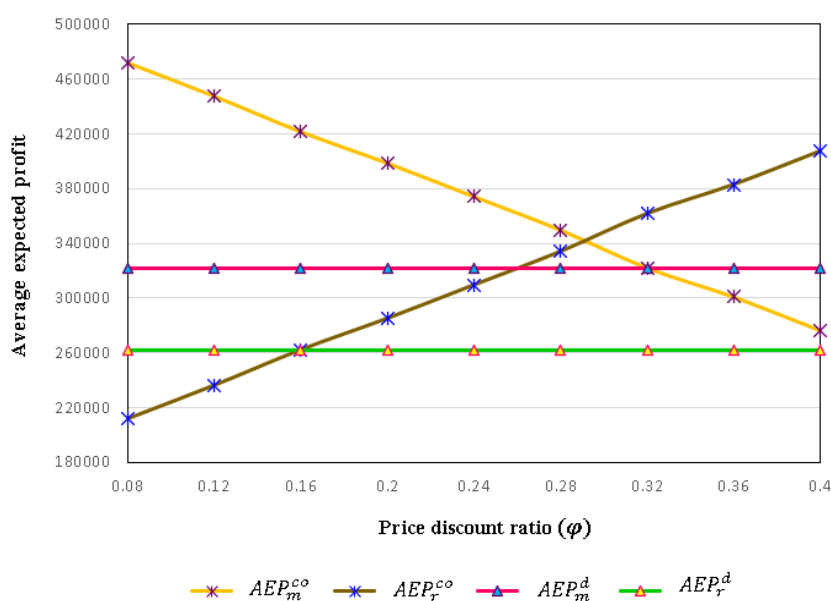


Figure 6.8: Average expected profit of the manufacturer and the retailer versus χ

Figure 6.8 flaunts the changes in the profitability of both supply chain members for the increasing price discount rate. As can be seen from Figure 6.8 that, along with the increment of price discount rate, the retailer's profit increases and the manufacturer's profit level gradually decreases. When the price discount rate is less than 16%, the manufacturer becomes gainer as he earns a higher profit than the decentralized scenario. However, the retailer will not participate in this coordination scheme as it leads to a lower profit level for him. At $\chi = 16\%$, the retailer's profit is same in both the decentralized and coordinated model. On the other hand, at $\chi = 32\%$, the price discount mechanism yields the same profit as the decentralized case for the manufacturer. Right after the price discount rate crosses the upper limit, the coordinated profit of the manufacturer begins to be lower than its decentralized profit. In

the feasible region $16\% \leq \chi \leq 32\%$, the retailer and the manufacturer both achieve a higher profit compared to their decentralized profits.

Example 4. $R = 7000; S = 800; K = 200; f = 15; c_p = 3; c_f = 4; c_m = 10; c_w = 7; I_b = 1.6; I_u = 10; y = 175200; \alpha = 3000; \beta = 5; \gamma = 3; \eta = 2; \lambda = 1.7; r = 0.01; M_\rho = 0.1; \delta = 0.2; h_m = 0.3; h_r = 0.8; c_b = 1.2; w = 200; \xi = 15; z = 0.6; \psi_r = 0.3; \psi_m = 0.3; \phi = 0.5$. (with appropriate units)

In this example, the interest rate at both the manufacturer and the retailer are same. Both the trade credit mechanism and wholesale price discount contract appear to be equivalent and yield the same supply chain profit. Table 6.1 shows that each of these coordination schemes enhances the profitability of the manufacturer and the retailer by 13% and 14%, respectively compared to the decentralized model. Henceforth, the supply chain members can choose any of these alternatives to coordinate the channel.

6.6 Sensitivity Analysis

In this section, a detailed sensitivity analysis has been performed with respect to some key parameters in Example 1 to analyze its impact on the performance of the entire supply chain. One parameter value is varied at a time, keeping remaining parameter values unchanged.

(i) Effect of price sensitivity β - The effect of price sensitivity on the profitability of the supply chain in different models is illustrated in Figure 6.9. It is observed that the average supply chain profit decreases drastically with increasing customer price sensitivity. As β increases, the retail price, promotional effort and greening level gradually decrease. This indicates that whenever the market is highly sensitive to price, retailers are forced to reduce the retail price of the product in order to survive in a competitive market. In highly price-sensitive market, customers mainly consider the price of the product. So, in such a situation, investing in promotional efforts or greening the product may not be so effective in influencing customers' purchasing decision.

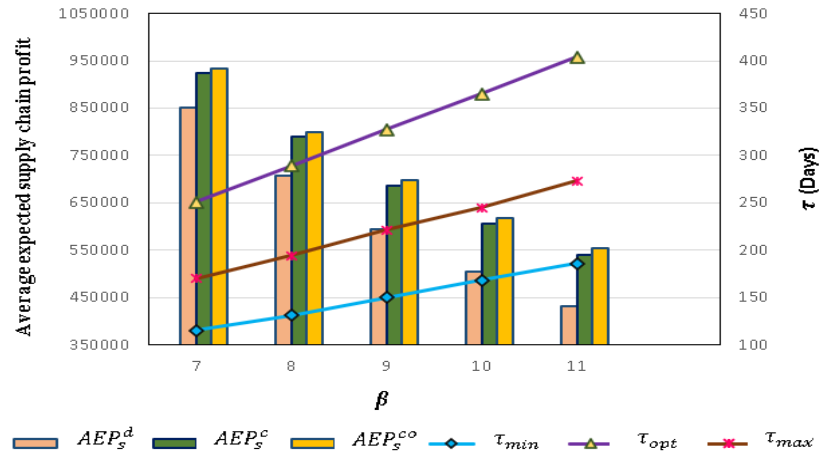


Figure 6.9: Average expected supply chain profit vs. price elasticity coefficient (β)

Moreover, in a low price-sensitive market, new companies may invest in promoting their product and adopt a greening investment strategy to drag customer's attention towards their product. They may even charge higher prices for more green products. Therefore, as it increases, the decreased selling price, promotional effort and product's greening level together reduce the customer demand as well as the entire supply chain's profitability significantly. For higher values of β , the trade credit coordination scheme seems to be more effective as it allows the supply chain members to attract customers by delivering the products at a lower price than the decentralized scenario. At the same time, by adopting this coordination mechanism, companies can raise promotional effort and greening level of the product even in a high price sensitive market. It is also seen that, as β increases, the minimum credit period at the retailer as well as the optimal credit period increases.

(ii) *Effect of promotional effort sensitivity γ and greening effort sensitivity η* - Figure 6.10 demonstrates the effect of γ on the profit level of the entire supply chain. It can be observed that when promotional efforts have a greater impact on the customer, the profit of the entire supply chain gradually increases. It is found that, when γ increases, the retailer not only increases its promotional efforts, but also increases the green level of the product. These factors left a positive impact on the customer demand. Also, using the customer's positive attitude towards the promotional effort, the retailer increases the selling price of the product. Therefore, the average profit of the retailer increases as γ increases.

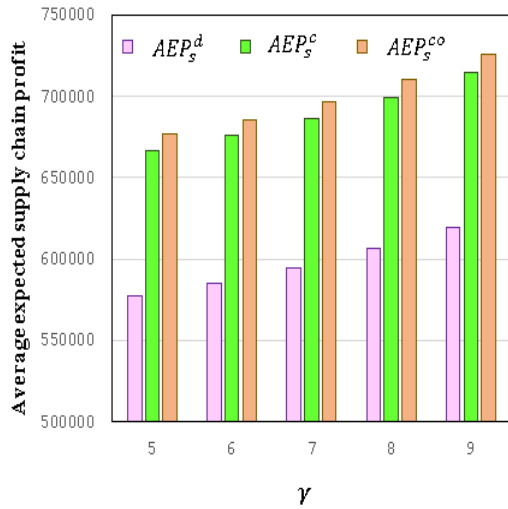


Figure 6.10: Average expected supply chain profit vs. promotional effort sensitivity coefficient (γ)

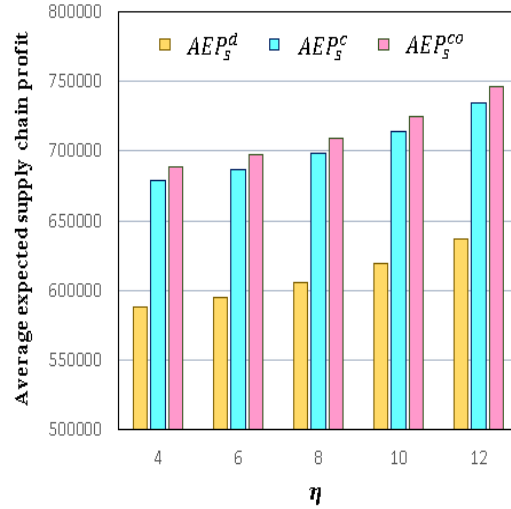


Figure 6.11: Average expected supply chain profit vs. greening effort sensitivity coefficient (η)

However, increase in γ also increases the set up cost of the manufacturer, but he compensates it from the increased customer demand and achieves higher profits. Although, the impact of γ on the credit periods is found to be negligible, under the coordination mechanism the retailer can exert higher promotional effort and improve the greening level of the product for the same value of γ than the decentralized scenario.

A similar impact is observed for increasing values of η . Figure 6.11 demonstrates how η affects on the supply chain profitability. Similarly, when η increases, it simultaneously improves both the promotion effort and the greening level of the product. Whenever customers show high environmental awareness and prefer to buy a more greener product, the greening effort by the retailer becomes lucrative for both members. It is noteworthy that the proposed coordination mechanism appears to be more conducive for the supply chain players to enhance their environmental responsibilities as the retailer can pursue higher greening efforts for the same value of η than the decentralized structure.

(iii) *Effect of discounted price z at the secondary market* - Figure 6.12 represents the effect of z on the supply chain profitability in three different model cases. It is noticeable that, with the increasing value of z , the supply chain profit level increases significantly in each model. A higher value of z indicates that the retailer may sell

the defective items at a higher price in the secondary market. Naturally, this increases the retailer's sales revenue. It is found that, with the increment of z , the manufacturer's profit margin also increases, albeit slightly. As a result, the members can compensate the loss incurred due to the defective items and the average supply chain profit gradually increases.

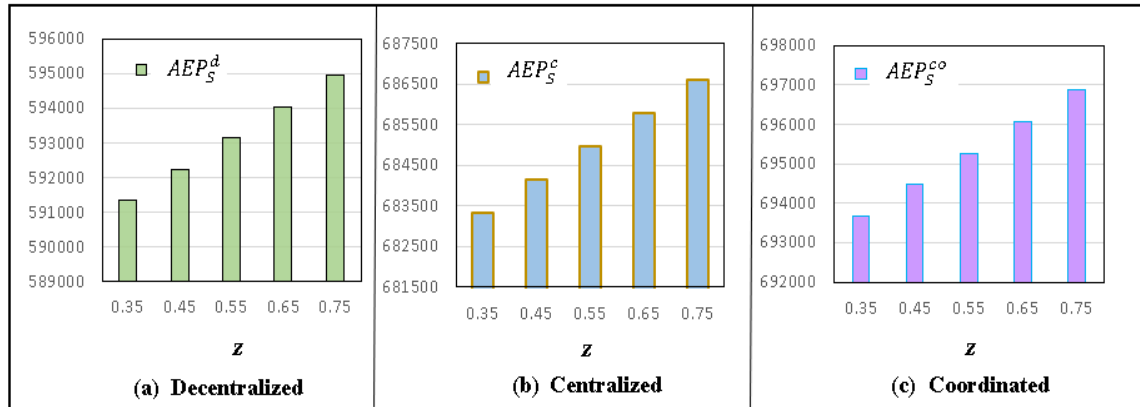


Figure 6.12: Average expected supply chain profit for different values of price discount ratio (z)

(iv) *Effect of inspection cost I_b* - The impact of unit inspection cost I_b on the optimal profitability of different models is depicted in Figure 6.13. It is worthy of note that the optimal profit level of the supply chain exhibits a contrasting behavior for sample inspection and full inspection policy. Whenever the inspection cost is very low, it is apparent from Figure 6.13 that the full inspection policy appears to be much more beneficial for the whole supply chain compared to the sample inspection. For $\delta = 0.1$, a very small fraction of the received lot undergoes through the inspection. As a consequence, the impact of higher inspection costs on supply chain profits in the case of sample inspection has proven to be almost negligible. In contrast, the inspection cost has a greater impact on the supply chain profitability in case of full inspection policy. If the entire lot goes through screening and the inspection cost gradually increases, the retailer has to bear a higher expense to conduct the inspection process, which leads to a rapid decline in the supply chain's profit margins. When the inspection cost I_b exceeds a threshold value, the full inspection policy is no longer beneficial and in such business scenarios, the retailers should adopt a sample inspection policy. This is in line with Bose and Guha, 2021. In the decentralized structure for $I_b > 25.7$, in the centralized model for $I_b > 22.5$, and in the coordinated model for $I_b > 21.6$, the sample inspection policy becomes more acceptable to decision makers.

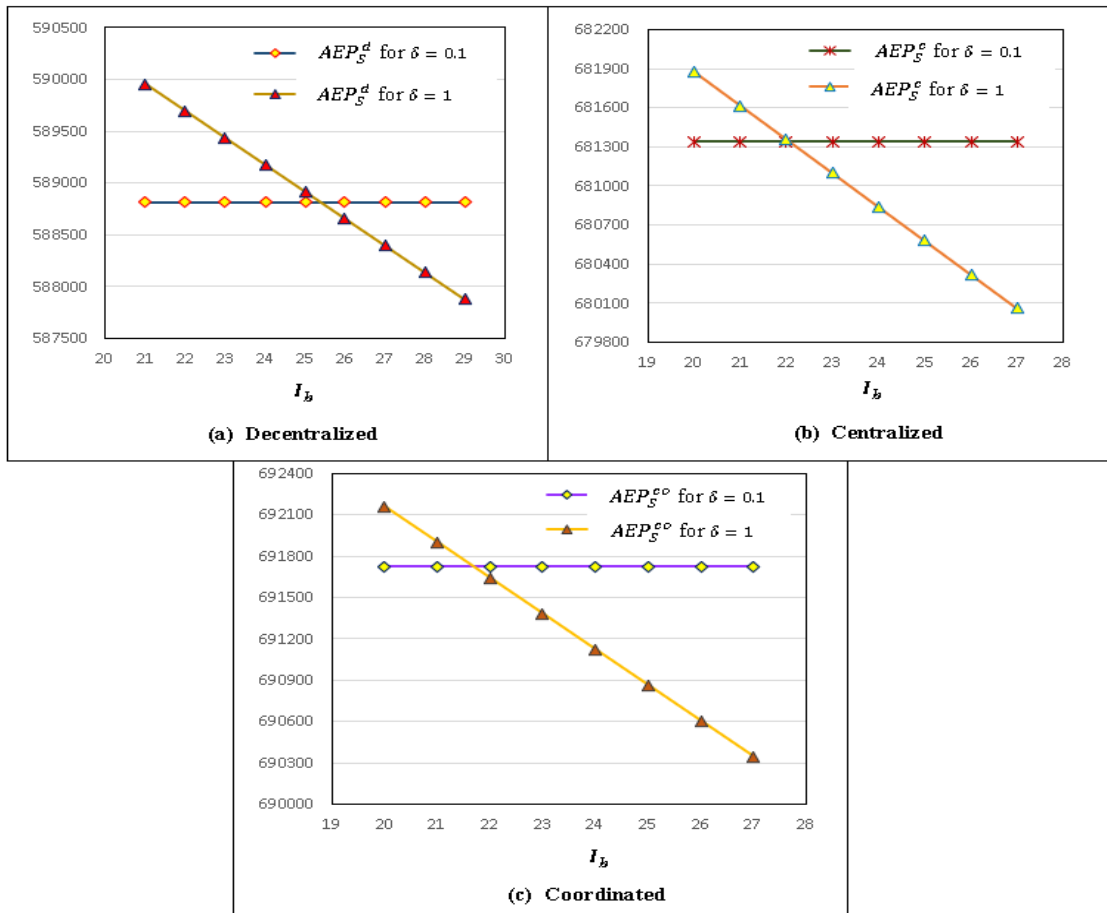


Figure 6.13: Average expected supply chain profit in (a) decentralized, (b) centralized and (c) coordinated model for different values of I_b in sampling inspection vs. full inspection

(v) *Effect of inspected fraction δ* - The effect of an increase in δ for different values of I_b is illustrated in Figure 6.14. It is observed that, while $I_b = 1.8$, along with the increase of δ the supply chain profit increases expeditiously in each model. In addition, the order quantity of the retailer also increases. This is quite natural as more the items inspected, the more defective items are detected. Whereby, the retailer places order for more quantities to meet the market demand. A completely reversed situation arises for $I_b = 51.8$ and $I_b = 101.8$. For these higher values of I_b , the supply chain profit decreases drastically along with increase in δ . Henceforth, it becomes clear that, the full inspection may not be always best fit for the optimal profitability. This finding contrasts strikingly with Sarkar et al., 2022b. The unit inspection cost and fraction of items sent for screening concurrently affect the profit level of the supply chain immensely.

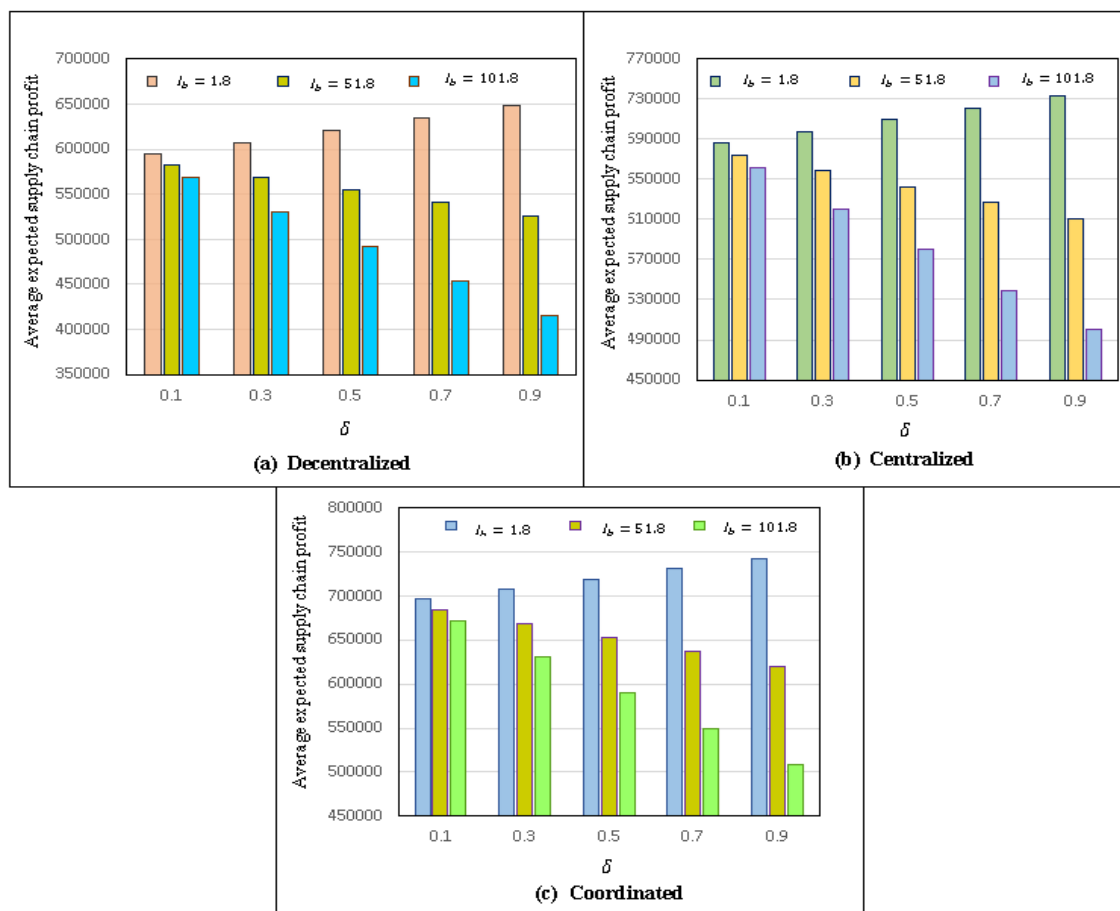


Figure 6.14: Average expected supply chain profit in (a) decentralized, (b) centralized and (c) coordinated model vs. δ

(vi) *Effect of unit penalty cost I_u* - Figure 6.15 demonstrates the effect of the unit penalty cost on the optimal profit level in both scenarios such as sample inspection and no inspection. It is evident from Figure 6.15 that penalty cost for each model case has a negative impact on the supply chain profit. However, in both cases, the profitability decreases with increase of I_u . Still, it is noticeable that, the sample inspection generates higher system profit than the model without any inspection whenever the inspection cost is low. Although the decadence of the profit curve persists with increasing penalty cost, a contrary situation arises for higher inspection cost. Herein the sample inspection policy fails to prove its effectiveness. However, when the penalty cost hikes, it spreads a negative impact on the market demand and the retailer should conduct a rigorous inspection before selling the items to retain goodwill in front of customers.

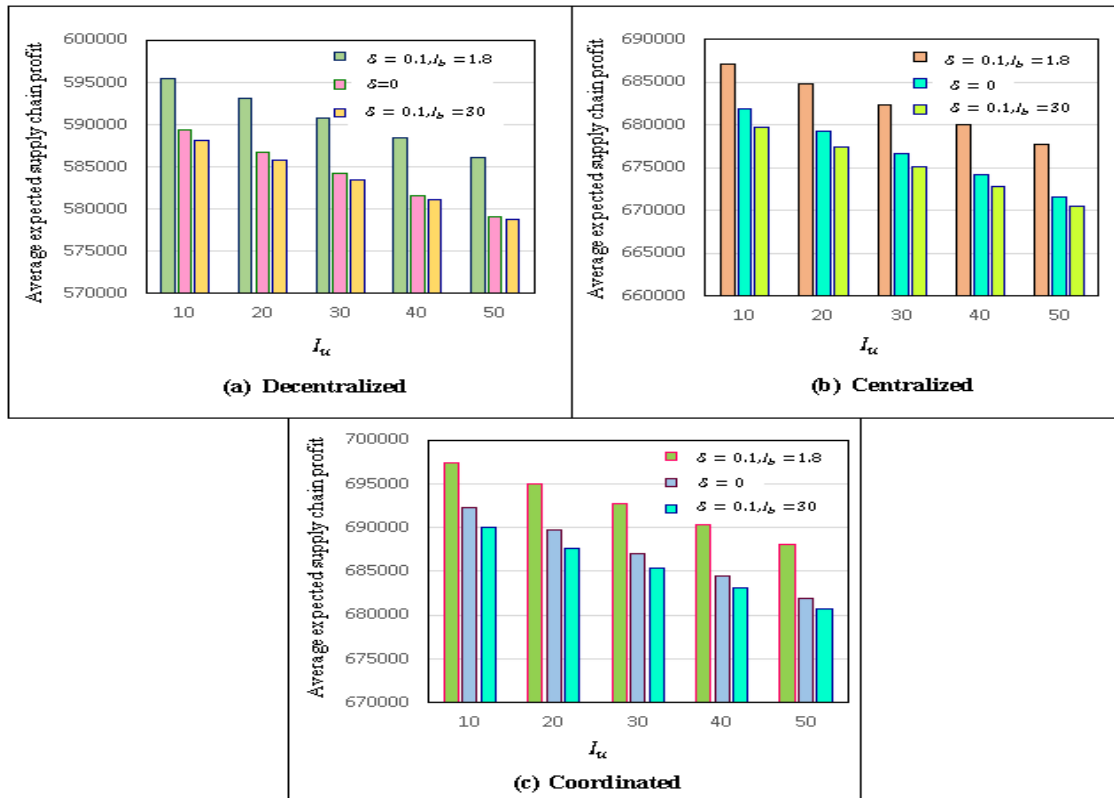


Figure 6.15: Average expected supply chain profit in (a) decentralized, (b) centralized and (c) coordinated model vs. I_u for sample inspection and no inspection

6.7 Managerial Insights

Nowadays, companies are encountering various challenging issues to balance the economical and environmental achievements at the same time in order to survive in the global competitive market. The numerical examples illustrated in this chapter present several managerial insights.

(i) In reality, however, companies are focusing on complying with and enacting current laws and regulations for environmental protection. But most companies are facing skyrocketing environmental costs and risking not being financially viable. At the same time, its emerging from various recent studies that, the people are becoming more and more environmentally conscious and showing a great interest in purchasing the eco-friendly products and services. Our results illustrate that although the retailer's greening effort increases its cost burden, but it improves significantly the profit levels of both players as well as their environmental performance. It also enhances the company's reputation for their eco-friendly business strategies. In a

highly environmentally-conscious market, it is desirable for business managers to prioritize environmental health in order to benefit both economically and socially.

(ii) To ensure steady business development, improve goodwill and mitigate the quality risks and costs, pre-delivery inspection of the items is indispensable for the companies. But, when there are lots of items in a batch, then conducting a 100% or full inspection is very expensive and time-consuming. In such circumstances, the sample inspection policy may be preferred as the best alternative instead of conducting a full inspection or no inspection at all. From the numerical results of this study, it is noteworthy that full inspection or sample inspection is not always the best fit in every situation. Inspection cost and uninspected penalty cost have a major impact on choosing the best inspection policy. Decision makers should tactfully choose the inspection strategies considering various crucial factors such as quality of item, inspection costs, inspection rate, penalty cost, lot size etc. They should keep in mind that the consequences of delivering a defective item can sometimes be quite fatal, for example the avionic systems.

(iii) Promotional effort is an important marketing strategy adopted by companies to persuade the customers to buy the products. Numerical outcomes of this article show that the promotional efforts initiated by the retailer have a positive impact on the profitability of the system. When the customers are more acquainted of promotions, companies should apply more promotional efforts to convince customers and make their products more enchanting and acceptable to the customers. New companies can create a recognizable brand image and make customers aware of the existence and quality of their product. Moreover, companies can promote their green initiatives to create environmental awareness among customers.

(iv) To lead in the competitive market nowadays, companies are striving to provide high quality products at lower prices. But in doing so, various entities may face financial losses. In this regard, coordination schemes have opened a new door for the companies. The findings of this article may encourage business managers to participate in such coordination mechanisms which effectively ameliorate overall performance of the system. Under such coordination schemes, retailers can raise their greening and promotional efforts, to a relatively higher extent than the individual optimization structure. In addition, the retail price of the product can be greatly reduced. All these factors collectively stimulate the market demand for the

product and instigate the consumers to follow environmental rules and regulations.

(v) Set up cost has a large impact on the production and a one time investment to reduce the huge set up cost could be a viable option for the companies.

(vi) It is desirable for the companies to reduce the selling price of a product to keep up with market demand while the customers are more concerned about the price of the product.

6.8 Concluding Remarks

This chapter considers a single-manufacturer single-retailer supply chain model with imperfect production system. The order quantity is transferred to the retailer following an unequal sized batch shipment policy. After receiving each batch, a fraction of items undergoes a rigorous inspection and the identified defective items are sold at a reduced price in a secondary market. The retailer bears a penalty cost for each unidentified imperfect item. Consumer demand depends on the retail price, promotional effort and greening effort levels. The manufacturer applies a continuous investment strategy for set up cost reduction. Models under decentralized and centralized structure are developed and further a trade credit mechanism and a wholesale price discount contract have been proposed to coordinate the channel. Through a detailed numerical analysis, this study confirms that these coordination incentives can successfully motivate supply chain players to participate in these schemes and remarkably improve the profit levels of each player as well as the whole system. Sensitivity analysis of some crucial parameters provides a thorough discussion on the acceptability of the sample inspection policy over the full inspection policy.

However, there are some limitations of our study. Firstly, the assumption of single-manufacturer single-retailer model for single item is quite limited. The present model can be extended to multi-supplier multi-retailer model considering multiple items to cover the real world business scenarios (Gharaei et al., 2019; Malleeswaran and Uthayakumar, 2022). Secondly, an error-free screening is considered in this study. So, it would be more realistic to consider errors in the inspection process

(Öztürk, 2020; Bose and Guha, 2021). The proposed sample inspection policy could be modified by introducing an acceptance criterion of the sample (Bose and Guha, 2021). In today's sophisticated business environment, there are so many substitutable products and alternatives. Thus, it is somewhat unrealistic to consider completely backlogging of shortages. So, considering a partially backlogged shortage would be more realistic approach (Sarkar and Giri, 2020a). We have assumed that the retailer makes both greening and promotional efforts. Therefore, it would be more interesting to consider that the manufacturer can share the cost burden of the retailer through a suitable cost-sharing contract (Zhou et al., 2016). The implementation of other appropriate coordination mechanisms such as revenue-sharing contract, quantity discount contract, buy back contracts, etc. would be another fruitful extension of this study.

CHAPTER 7

A single-manufacturer multi-retailer integrated inventory model with price dependent demand and stochastic lead time

7.1 Introduction

Lead time refers to the time interval between order placement and receiving the delivery of items. It plays a vital role in supply chain management. The assumption of deterministic lead time is not valid in most real world situations because of various reasons such as delays in production process, transit time, inspection, loading and unloading, and so on. So, it may be random in nature which affects the demand forecasting and makes the customer looking for alternatives. So, for an efficient management of supply chain and facilitate of economic success, it is essential to concentrate on such a sensible factor. Therefore, dealing with stochastic lead time is very fascinating and challenging to avoid a planned shortage at the retailer's end and to efficiently manage the phenomena of early arrival. In this regard, the stochastic nature of lead time is incorporated in this chapter*. In two-level supply chain system, an interesting strategic problem arises at the retailer's end in determining

*This chapter is based on the work published in *International Journal of Supply and Operations Management*, 7(4), 384-409.

optimal ordering quantity and the number of shipments decided by the manufacturer to transfer the ordered quantity of the retailer. Although we treated the lead time as constant or zero in the earlier chapters. However, it is rather impractical because there is almost always a delay between placing an order and having it delivered, which greatly affects customer satisfaction. Therefore, it has become an inevitable task for the business managers to consider the stochastic nature of the lead time for efficient management of business. Previous chapters, on the other hand, have exclusively looked at single-manufacturer single-retailer supply chain models. It would be preferable to take into account a multi-retailer situation for a better real-world implementation. Keeping in mind the importance of lead time and focusing on multi-retailer scenario, in this chapter, we develop a single-manufacturer multi-retailer supply chain model assuming that the lead time is a random variable following normal distribution and the customer demand is influenced by its selling price. In this chapter, our aim is to answer the following research questions:

- What will be the optimal pricing and the batch shipment policy for the retailers and the manufacturer in a price sensitive market ?
- How does the stochastic nature of lead time influence optimal inventory decisions in a single manufacturer multi-retailer model with price-dependent demand ?
- What is the impact of integration on the profitability and managerial decisions of the supply chain ?

7.2 Problem Definition

This chapter investigates a two-tier supply chain consisting of a single manufacturer and multiple retailers. Retailers' orders are fulfilled by the manufacturer in a number of equal-sized batches. Customer demand is influenced by the product's selling price. The lead time is assumed to be random and follows a normal distribution. Different situations, such as early or late batch arrival, have been studied. Model cases for both centralized and decentralized systems have been investigated. In the decentralized structure, a Stackelberg gaming strategy is utilized to determine the

optimal solution. Figure 7.1 exhibits the schematic diagram of the proposed model.

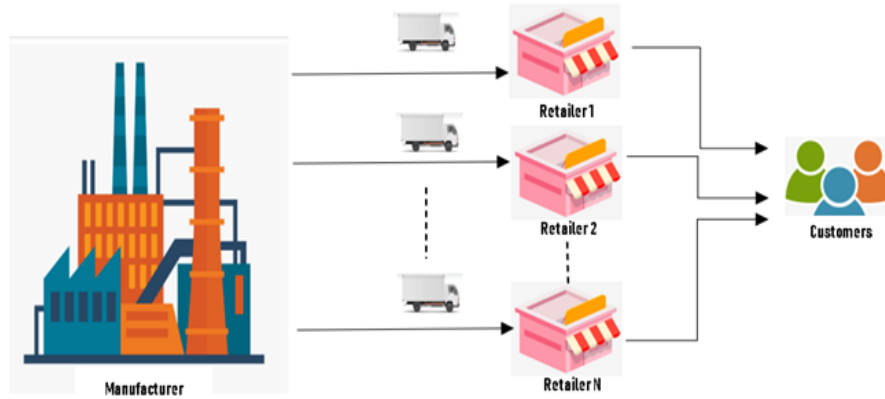


Figure 7.1: Logistics diagram of the proposed single-manufacturer multi-retailer supply chain model

7.3 Notations and Assumptions

Notations used for developing the proposed model are given below. The subscript ' i ' is used to indicate the i -th retailer ($i = 1, 2, \dots, N$).

- R : Production rate (units/unit time)
- A_m : Set-up cost (\$/set up)
- h_m : Manufacturer's holding cost ($h_m > h_i$) (\$/unit/unit time)
- F : Transportation cost (\$/shipment)
- w : Unit wholesale price (\$/unit)
- N : Number of retailers (positive integer)
- Q : Total order quantity [$= \sum_{i=1}^N Q_i$] (units)
- D : Total market demand [$= \sum_{i=1}^N D_i$] (units/unit time)
- L : Lead time, a stochastic random variable
- A_i : Ordering cost of the retailer (\$/order)
- h_i : Holding cost of the retailer (\$/unit/unit time)
- D_i : Demand rate at the retailer [$R > \sum_{i=1}^N D_i$] (units/unit time)
- a_i : Basic market demand (units/unit time)
- β_i : Consumer sensitivity coefficient to retail price

Q_i	: Order quantity of the retailer (units)
c_i	: Shortage cost at the retailer (\$/item/unit time)
r_i	: Reorder point of the retailer
l_i	: Lead time variable of the retailer
T_i	: Cycle length of the retailer (time)
σ_i	: Standard deviation of the lead time of the retailer
$f_L(\cdot)$: Probability density function of lead time
AEP_i	: Average expected profit of the retailer (\$/unit time)
AEP_m	: Average expected profit of the manufacturer (\$/unit time)
AEP_s	: Average expected profit of the supply chain (\$/unit time)

**Decision
variables**

n	: Number of batches delivered to each retailer (positive integer)
q_i	: Batch size of the i -th retailer (units)
p_i	: Unit retail price of the i -th retailer (\$/unit)

The basic assumptions made to formulate the proposed model are as follows:

1. The manufacturer produces a single type of product and meets the demand of multiple retailers.
2. The i -th retailer places his order of quantity Q_i . The manufacturer produces the total order quantity $\sum_{i=1}^N Q_i$ of all retailers in one set up at a production rate R , and then transfers the order quantity of the i -th retailer in n equal batches of size q_i such that $\frac{Q_i}{D_i} = \frac{Q}{D}$ for all $i = 1, 2, \dots, N$.
3. The consumer's demand rate depends linearly on the selling price of the product, i.e., the consumer demand rate at the i -th retailer is $D_i(p_i) = a_i - \beta_i p_i$, where a_i represents the basic consumer demand and β_i is a positive integer such that $a_i > \beta_i p_i$, for all $i = 1, 2, \dots, N$.
4. The production rate is constant and higher than the collective demand rates of all the retailers i.e., $R > \sum_{i=1}^N D_i$.
5. Shortages are allowed and are assumed to be completely backlogged at each retailer's end.
6. The i -th retailer places his next order when its stock level reaches a certain reorder point r_i .

7. The lead time is stochastic in nature and follows a normal distribution, and the lead time for all shipments are independent of each other.

7.4 Model Formulation

We assume that, the manufacturer transfers the order quantity Q_i of the i -th retailer in n equal batch shipments of size q_i . If the total order quantity of N retailers is Q , then we have $Q_i = nq_i$ and $Q = \sum_{i=1}^N Q_i$.

The i -th retailer places the next order when the stock reaches to a level r_i . The reorder level is determined so as to arrive the shipment to the retailer's end at or before the time of selling this r_i quantity at a demand rate D_i . The mean lead time is $\frac{r_i}{D_i}$. Due to various reasons, the batches may reach the retailer's end early or late. So, depending on the duration of lead time three cases may arise:

Case(i) When the batch q_i reaches to the retailer earlier i.e., $0 < l_i < \frac{r_i}{D_i}$.

In this case, the inventory holding area of the i -th retailer is determined from 7.2 (a) as

$$\begin{aligned}
 & \text{Area (ABCD + CEGH + EFG)} \\
 &= \left[\frac{1}{2} \times (AB + CD) \times BC \right] + (CE \times EG) + \left(\frac{1}{2} \times EF \times EG \right) \\
 &= \frac{r_i(q_i - D_i l_i)}{D_i} + \frac{1}{2}(q_i - D_i l_i) \frac{(q_i - D_i l_i)}{D_i} + (r_i - D_i l_i)l_i + \frac{1}{2}D_i l_i^2 \\
 &= \frac{1}{2} \left[\frac{q_i^2}{D_i} + 2q_i \left(\frac{r_i}{D_i} - l_i \right) \right] \quad \text{where } r_i = \frac{q_i D_i}{R}
 \end{aligned}$$

The order quantity of the i -th retailer Q_i for n shipments is given by

$$\frac{n}{2} \left[\frac{q_i^2}{D_i} + 2q_i \left(\frac{r_i}{D_i} - l_i \right) \right]$$

The expected inventory holding cost for the order quantity Q_i of the i -th retailer is

$$h_i \int_0^{\frac{r_i}{D_i}} \frac{n}{2} \left[\frac{q_i^2}{D_i} + 2q_i \left(\frac{r_i}{D_i} - l_i \right) \right] f_L(l_i) dl_i$$

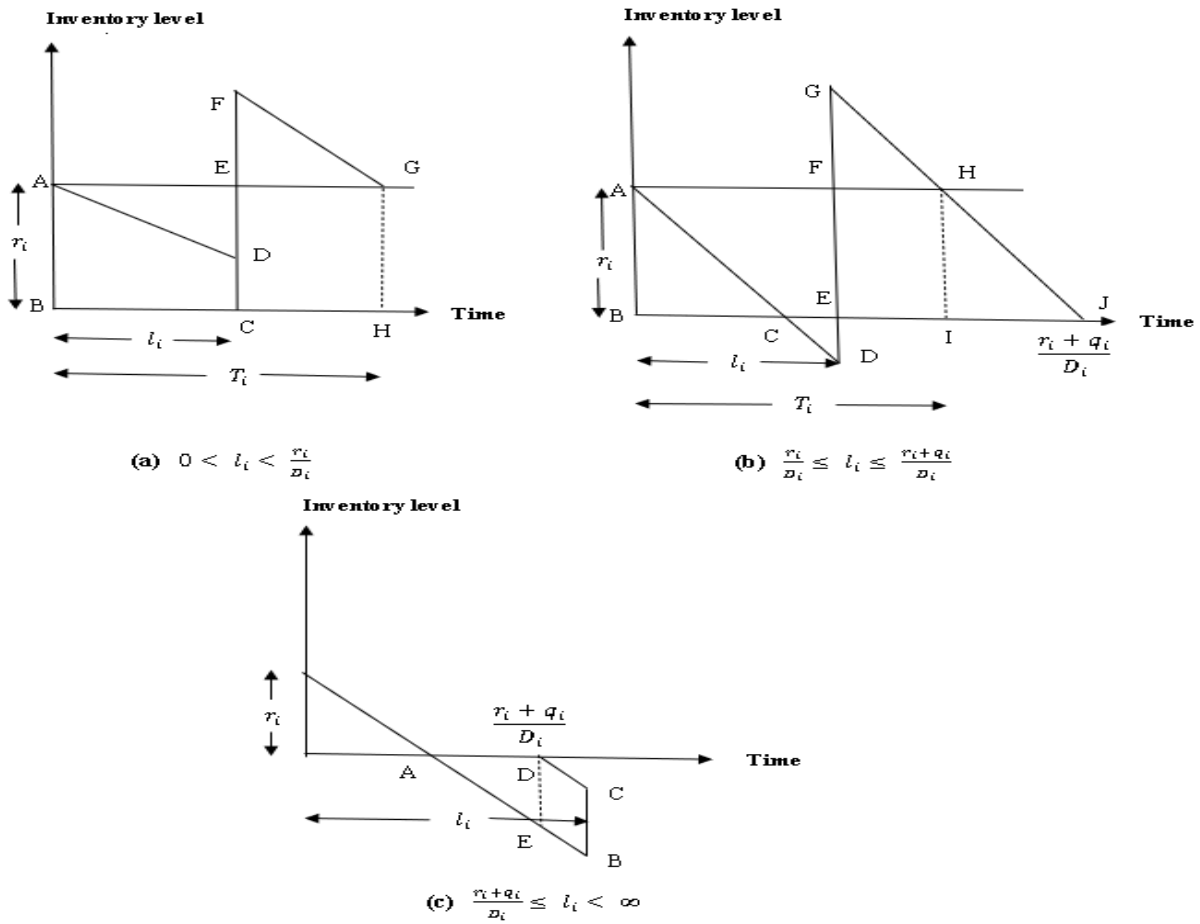


Figure 7.2: Inventory of i -th retailer under stochastic lead time

Case(ii) When the batch q_i reaches late to the i -th retailer and the lead time l_i lies in the range $\frac{r_i}{D_i} \leq l_i \leq \frac{r_i+q_i}{D_i}$.

In this case, both inventory and shortages occur at the retailer's end. From Figure 7.2(b), shortage area of the i -th retailer is obtained as $= \text{Area}(CDE) = \frac{(D_i l_i - r_i)^2}{2D_i}$.

So, the expected shortage cost at the i -th retailer for n batches is given by

$$\frac{nc_i}{2} \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{D_i} f_L(l_i) dl_i$$

Inventory holding area of the i -th retailer for the batch q_i is

$$\begin{aligned} & \text{Area}(ABC + EFHI + FGH) \\ &= \frac{r_i^2}{2D_i} + \frac{(q_i - D_i l_i)^2}{2D_i} + \frac{r_i(q_i - D_i l_i)}{D_i} \\ &= \frac{(q_i - D_i l_i + r_i)^2}{2D_i} \end{aligned}$$

Hence the expected holding cost at the i -th retailer for n shipments is obtained as

$$nh_i \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{2D_i} f_L(l_i) dl_i$$

It is assumed that, during this delay period, the batches remain in the manufacturer's stock house. So it causes an extra holding cost to the manufacturer. The extra inventory for this delayed delivery is $\sum_{i=1}^N \frac{nq_i(D_i l_i - r_i)}{D_i}$. Therefore, in this scenario, the extra holding cost paid by the manufacturer is

$$h_m \sum_{i=1}^N \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{nq_i(D_i l_i - r_i)}{D_i} f_L(l_i) dl_i$$

Case(iii) When the batch q_i arrives late to the retailer with lead time in the range $\frac{r_i+q_i}{D_i} \leq l_i < \infty$.

In this case, only shortages occur at the retailer's end and, from Figure 7.2(c), the shortage area for the batch q_i is obtained as

$$\begin{aligned} \text{Area}(ABCD) &= \frac{q_i^2}{2D_i} + \text{Area}(BCD) \\ &= \frac{q_i^2}{2D_i} + q_i \left(\frac{D_i l_i - q_i - r_i}{D_i} \right) \end{aligned}$$

So, the expected shortage cost of the i -th retailer for all batch shipments is

$$nc_i \int_{\frac{r_i+q_i}{D_i}}^{\infty} \left[\frac{q_i^2}{2D_i} + q_i \left(\frac{D_i l_i - q_i - r_i}{D_i} \right) \right] f_L(l_i) dl_i$$

Similarly as case(ii), the additional expected holding cost of the manufacturer is

$$h_m \sum_{i=1}^N \int_{\frac{r_i+q_i}{D_i}}^{\infty} \frac{nq_i(D_i l_i - r_i)}{D_i} f_L(l_i) dl_i$$

Combining all three cases, the expected holding cost at the i -th retailer for all batch shipments is given by

$$nh_i \int_0^{\frac{r_i}{D_i}} \frac{1}{2} \left[\frac{q_i^2}{D_i} + 2q_i \left(\frac{r_i}{D_i} - l_i \right) \right] f_L(l_i) dl_i + nh_i \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{2D_i} f_L(l_i) dl_i$$

and the expected shortage cost for all batch shipments is

$$\frac{nc_i}{2} \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{D_i} f_L(l_i) dl_i + nc_i \int_{\frac{r_i+q_i}{D_i}}^{\infty} \left[\frac{q_i^2}{2D_i} + q_i \left(\frac{D_i l_i - q_i - r_i}{D_i} \right) \right] f_L(l_i) dl_i$$

7.4.1 Decentralized Model

In the decentralized model scenario, the manufacturer and the retailers make their decisions independently in order to improve their own profits. Here we incorporate a Stackelberg gaming structure where the retailers act as the leader and the manufacturer is the follower. The manufacturer sets the number of shipments. Then, taking this response function into consideration, the retailers determine optimal retail prices of the product and batch sizes.

A. Retailer's profit function

The expected total profit of the i -th retailer is

$$p_i Q_i - w Q_i - nh_i \left[\int_0^{\frac{r_i}{D_i}} \frac{1}{2} \left[\frac{q_i^2}{D_i} + 2q_i \left(\frac{r_i}{D_i} - l_i \right) \right] f_L(l_i) dl_i + \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{2D_i} f_L(l_i) dl_i \right] \\ - A_i - nF - nc_i \left[\int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{2D_i} f_L(l_i) dl_i + \int_{\frac{r_i+q_i}{D_i}}^{\infty} \left[\frac{q_i^2}{2D_i} + q_i \left(\frac{D_i l_i - q_i - r_i}{D_i} \right) \right] f_L(l_i) dl_i \right]$$

Therefore, the average expected profit of the i -th retailer is obtained as

$$AEP_i(q_i, p_i) = p_i D_i - w D_i - \frac{(A_i + nF) D_i}{Q_i} - nh_i \left[\int_0^{\frac{r_i}{D_i}} \frac{1}{2} \left[\frac{q_i^2}{Q_i} + \frac{2}{n} (r_i - D_i l_i) \right] f_L(l_i) dl_i \right. \\ \left. + \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{2Q_i} f_L(l_i) dl_i \right] - nc_i \left[\int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{2Q_i} f_L(l_i) dl_i \right. \\ \left. + \int_{\frac{r_i+q_i}{D_i}}^{\infty} \left[\frac{q_i^2}{2Q_i} + q_i \left(\frac{D_i l_i - q_i - r_i}{Q_i} \right) \right] f_L(l_i) dl_i \right] \quad (7.1)$$

Proposition 7.1 The average expected profit function of the i -th retailer is concave in q_i for given p_i provided that

$$2(A_i + nF) D_i + n(h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} D_i^2 l_i^2 f_L(l_i) dl_i > 0. \quad (7.2)$$

Proof. Using $r_i = \frac{q_i D_i}{R}$, and differentiating equation (7.1) twice with respect to q_i ,

we obtain

$$\begin{aligned} \frac{\partial AEP_i}{\partial q_i} &= \frac{(A_i + nF)D_i}{nq_i^2} - \frac{h_i}{2} \int_0^{\frac{q_i}{R}} \left(\frac{R + 2D_i}{R} \right) f_L(l_i) dl_i - \frac{h_i}{2} \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \left(\frac{q_i^2(R + D_i)^2 - R^2 D_i^2 l_i^2}{q_i^2 R^2} \right) f_L(l_i) dl_i \\ &+ \frac{c_i}{2} \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \left(\frac{R^2 D_i^2 l_i^2 - q_i^2 D_i^2}{q_i^2 R^2} \right) f_L(l_i) dl_i + \frac{c_i}{2} \int_{\frac{q_i(R+D_i)}{RD_i}}^{\infty} \left(\frac{R + 2D_i}{R} \right) f_L(l_i) dl_i \end{aligned} \quad (7.3)$$

$$\frac{\partial^2 AEP_i}{\partial q_i^2} = \frac{-2(A_i + nF)D_i}{nq_i^3} - \frac{h_i}{2} \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{2D_i^2 l_i^2}{q_i^3} f_L(l_i) dl_i - \frac{c_i}{2} \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{2D_i^2 l_i^2}{q_i^3} f_L(l_i) dl_i \quad (7.4)$$

For given p_i , the average expected profit function of the i -th retailer is concave in q_i if $\frac{\partial^2 AEP_i}{\partial q_i^2} < 0$, which implies that $2(A_i + nF)D_i + n(h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} D_i^2 l_i^2 f_L(l_i) dl_i > 0$.

Proposition 7.2 The average expected profit function of the i -th retailer is concave in p_i for given q_i if

$$2\beta_i + (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i > 0. \quad (7.5)$$

Proof. After differentiating equation (7.1) with respect to p_i using $r_i = \frac{q_i D_i}{R}$, we obtain

$$\begin{aligned} \frac{\partial AEP_i}{\partial p_i} &= D_i + w\beta_i - \beta_i p_i + \frac{(A_i + nF)\beta_i}{Q_i} + h_i \int_0^{\frac{q_i}{R}} \frac{\beta_i (q_i - Rl_i)}{R} f_L(l_i) dl_i \\ &+ c_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i D_i (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i - c_i \int_{\frac{q_i(R+D_i)}{RD_i}}^{\infty} \frac{\beta_i (q_i - Rl_i)}{R} f_L(l_i) dl_i \\ &+ h_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i (q_i - Rl_i) [q_i R + D_i (q_i - Rl_i)]}{q_i R^2} f_L(l_i) dl_i \end{aligned} \quad (7.6)$$

$$\frac{\partial^2 AEP_i}{\partial p_i^2} = -2\beta_i - (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i \quad (7.7)$$

Since $\beta_i > 0$, $h_i > 0$ and $c_i > 0$, it implies that $\frac{\partial^2 AEP_i}{\partial p_i^2} < 0$ if

$$2\beta_i + (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i > 0.$$

B. Manufacturer's profit function

Total extra holding cost for the manufacturer, from both case(ii) and case(iii), is

$$h_m \sum_{i=1}^N \int_{\frac{r_i}{D_i}}^{\infty} \frac{nq_i(D_i l_i - r_i)}{D_i} f_L(l_i) dl_i$$

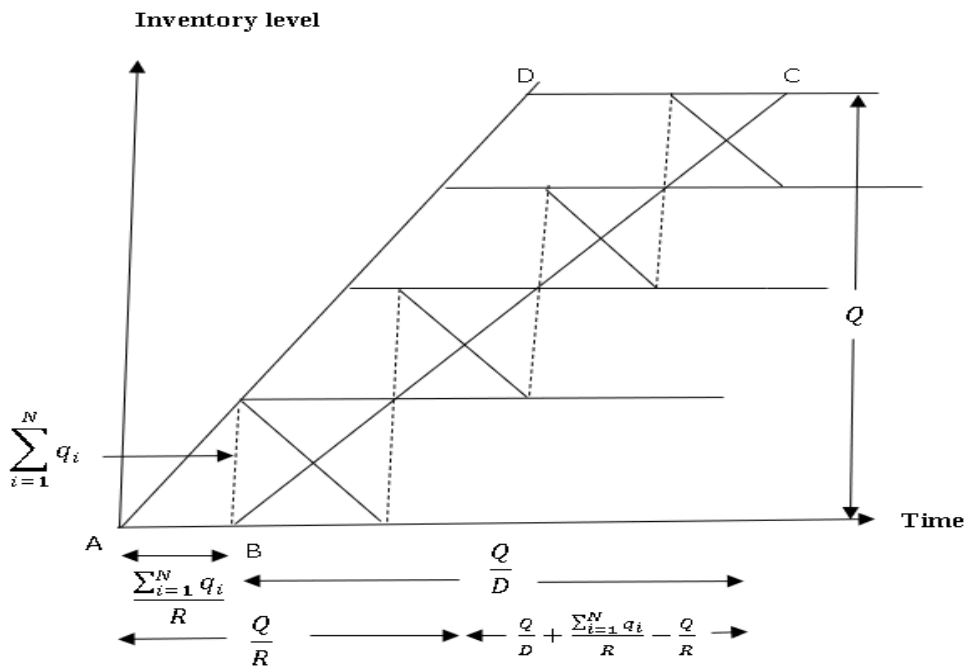


Figure 7.3: Joint inventory of the manufacturer and the retailers

In Figure 7.3, trapezium ABCD represents the joint inventory of the manufacturer-retailer system. Then, the average inventory of the system is

$$\begin{aligned} Area(ABCD) \times \frac{D}{Q} &= \frac{1}{2} \times (AB + CD) \times Q \times \frac{D}{Q} \\ &= \frac{1}{2} \left[\frac{\sum_{i=1}^N q_i}{R} + \left(\frac{Q}{D} + \frac{\sum_{i=1}^N q_i}{R} - \frac{Q}{R} \right) \right] D \\ &= \frac{D \sum_{i=1}^N q_i}{R} + \frac{Q}{2} \left(1 - \frac{D}{R} \right) \end{aligned}$$

The average holding area for N retailers

$$= \sum_{i=1}^N \left(\frac{q_i^2}{2D_i} \right) \left(\frac{D_i}{Q_i} \right) = \sum_{i=1}^N \frac{q_i^2}{2Q_i}$$

The average inventory holding area of the manufacturer

$$\begin{aligned}
 &= (\text{average joint inventory}) - (\text{average inventory of } N \text{ retailers}) \\
 &= \frac{D \sum_{i=1}^N q_i}{R} + \frac{Q}{2} \left(1 - \frac{D}{R}\right) - \sum_{i=1}^N \frac{q_i^2}{2Q_i}
 \end{aligned}$$

The average expected profit of the manufacturer is

$$\begin{aligned}
 AEP_m(n) &= wD - \frac{A_m D}{Q} - h_m \left[\frac{D \sum_{i=1}^N q_i}{R} + \frac{Q}{2} \left(1 - \frac{D}{R}\right) - \sum_{i=1}^N \frac{q_i^2}{2Q_i} \right] - h_m \sum_{i=1}^N \int_{\frac{r_i}{D_i}}^{\infty} (D_i l_i - r_i) f_L(l_i) dl_i \\
 &= wD - \frac{A_m D}{Q} - h_m \left[\frac{D \sum_{i=1}^N q_i}{R} + \frac{Q}{2} \left(1 - \frac{D}{R}\right) - \sum_{i=1}^N \frac{q_i^2}{2Q_i} \right] - \sum_{i=1}^N \frac{h_m D_i \sigma_i}{\sqrt{2\pi}} \quad (7.8)
 \end{aligned}$$

Proposition 7.3 The average expected profit function of the manufacturer is concave in n if $h_m s^2 < 2A_m D$. Then the optimal number of batch shipments is given by

$$n^* = \sqrt{\frac{R(2A_m D - h_m s^2)}{h_m s^2(R - D)}} \text{ where } s = \sum_{i=1}^N q_i. \quad (7.9)$$

Proof. Assuming n as real, not just an integer, we get from equation (7.8)

$$\frac{\partial AEP_m}{\partial n} = \frac{A_m D}{n^2 s} - \frac{h_m s}{2} + \frac{h_m s D}{2R} - \frac{h_m s}{2n^2} \quad (7.10)$$

$$\frac{\partial^2 AEP_m}{\partial n^2} = -\frac{2A_m D}{n^3 s} + \frac{h_m s}{n^3} \quad (7.11)$$

So, the manufacturer's average expected profit function AEP_m is concave in n if $\frac{\partial^2 AEP_m}{\partial n^2} < 0$, Which implies that $h_m s^2 < 2A_m D$.

If the above condition holds, then by using the first order optimality condition i.e., solving $\frac{\partial AEP_m}{\partial n} = 0$ for n , we get the optimal number of shipments as $n^* = \sqrt{\frac{R(2A_m D - h_m s^2)}{h_m s^2(R - D)}}$.

Solution Algorithm

Taking the best response from the manufacturer, the average expected profit of the i -th retailer can be optimized using the following solution algorithm. Here, we first assign some initial values to the decision variables of the remaining retailers to maximize the average expected profit of one retailer.

Step 1: Set $k = 1$.

Step 2: Set $i = 1$ and $q_j = q_j^{(k-1)}$, $p_j = p_j^{(k-1)}$ for all $j = i + 1, i + 2, \dots, N$.

Step 3: Optimize AEP_i taking n from the response function of the manufacturer and $q_j = q_j^{(k-1)}$, $p_j = p_j^{(k-1)}$ for all $j = i + 1, i + 2, \dots, N$. Set the optimal results as $q_i = q_i^{(k)}$ and $p_i = p_i^{(k)}$.

Step 4: Set $i = i + 1$.

Step 5: Optimize AEP_i taking n from the response function of the manufacturer and $q_j = q_j^{(k)}$, $p_j = p_j^{(k)}$ for $j = 1, 2, \dots, i - 1$ and $q_j = q_j^{(k-1)}$, $p_j = p_j^{(k-1)}$ for $j = i + 1, i + 2, \dots, N$. Set the optimal results as $q_i = q_i^{(k)}$ and $p_i = p_i^{(k)}$.

Step 6: Repeat steps 4 and 5 until $i = N$.

Step 7: Stop if $q_j^{(k)} = q_j^{(k-1)}$ and $p_j^{(k)} = p_j^{(k-1)}$ for all $j = 2, 3, \dots, N$ and consider $q_j^{(*)} = q_j^{(k)}$ and $p_j^{(*)} = p_j^{(k)}$ for all $j = 1, 2, 3, \dots, N$. Otherwise, set $k = k + 1$ and repeat steps 2 to 6.

Step 8: Evaluate the optimal value of n^* taking q_j^* and p_j^* for all $j = 1, 2, 3, \dots, N$.

Step 9: Using these results, calculate optimal values of AEP_m and AEP_s .

7.4.2 Centralized Model

Centralized decision making gives an efficient management of resource allocation through information sharing and better utilization of resources. Coordination between the supply chain members enhances the supply chain performance by reducing manufacturing cost, transportation costs, labour costs and lost-sale cost, and improving the product's availability to avoid stock outs. In the centralized structure, the manufacturer and all the retailers act as a single decision maker without worrying about their own profit levels. They jointly decide the optimal selling prices, number of batch shipments and batch sizes which lead to maximum average expected profit of the entire supply chain.

The average expected profit of the supply chain is

$$\begin{aligned}
& AEP_s(n, q_i, p_i) \\
&= \sum_{i=1}^N p_i D_i - \frac{A_m D}{Q} - h_m \left[\frac{D \sum_{i=1}^N q_i}{R} + \frac{Q}{2} \left(1 - \frac{D}{R} \right) - \sum_{i=1}^N \frac{q_i^2}{2Q_i} \right] - \sum_{i=1}^N \frac{h_m D_i \sigma_i}{\sqrt{2\pi}} - \sum_{i=1}^N \frac{(A_i + nF) D_i}{Q_i} \\
&\quad - \sum_{i=1}^N n h_i \left[\int_0^{\frac{r_i}{D_i}} \frac{1}{2} \left[\frac{q_i^2}{Q_i} + \frac{2}{n} (r_i - D_i l_i) \right] f_L(l_i) dl_i + \int_{\frac{r_i}{D_i}}^{\frac{r_i + q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{2Q_i} f_L(l_i) dl_i \right] \\
&\quad - \sum_{i=1}^N n c_i \left[\int_{\frac{r_i}{D_i}}^{\frac{r_i + q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{2Q_i} f_L(l_i) dl_i + \int_{\frac{r_i + q_i}{D_i}}^{\infty} \left[\frac{q_i^2}{2Q_i} + q_i \left(\frac{D_i l_i - q_i - r_i}{Q_i} \right) \right] f_L(l_i) dl_i \right] \quad (7.12)
\end{aligned}$$

Proposition 7.4 The average expected profit function (7.12) is concave in n for known q_i and p_i if $h_m s^2 < 2(A_m D + ms)$ and the optimal number of batch shipments is

$$n^* = \sqrt{\frac{R(2A_m D - h_m s^2 + 2ms)}{h_m s^2(R - D)}} \text{ where } s = \sum_{i=1}^N q_i \text{ and } m = \left(\sum_{i=1}^N \frac{A_i D_i}{q_i} \right) \quad (7.13)$$

Proof. Assuming n as real, not just an integer, we get from equation (7.12)

$$\frac{\partial AEP_s}{\partial n} = \frac{A_m D}{n^2 s} - \frac{h_m s}{2} + \frac{h_m s D}{2R} - \frac{h_m s}{2n^2} + \frac{m}{n^2} \quad (7.14)$$

$$\frac{\partial^2 AEP_s}{\partial n^2} = -\frac{2A_m D}{n^3 s} + \frac{h_m s}{n^3} - \frac{2m}{n^3} \quad (7.15)$$

The average expected profit function of the entire supply chain is concave in n if $\frac{\partial^2 AEP_s}{\partial n^2} < 0$, which implies that $h_m s^2 < 2(A_m D + ms)$. Now, if the above condition of concavity holds then one can derive the optimal number of batch shipments by using the first order optimality condition. Then, by solving $\frac{\partial AEP_s}{\partial n} = 0$ for n , we get the optimal number of shipments as $n^* = \sqrt{\frac{R(2A_m D - h_m s^2 + 2ms)}{h_m s^2(R - D)}}$.

Proposition 7.5 The average expected system profit function (7.12) will be concave in q_i for given n and p_i if

$$\frac{2A_m D}{s^3} + \frac{2(A_i + nF) D_i}{q_i^3} + n(h_i + c_i) \int_{\frac{r_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2 l_i^2}{q_i^3} f_L(l_i) dl_i > 0 \quad (7.16)$$

Proof. Using $r_i = \frac{q_i D_i}{R}$ and differentiating twice equation (7.12) with respect to q_i ,

we obtain

$$\begin{aligned} \frac{\partial AEP_s}{\partial q_i} &= \frac{A_m D}{ns^2} - \frac{h_m D}{R} + \frac{(A_i + nF)D_i}{nq_i^2} - \frac{h_i}{2} \int_0^{q_i/R} \left(\frac{R + 2D_i}{R} \right) f_L(l_i) dl_i - \frac{nh_m}{2} \left(1 - \frac{D}{R} \right) \\ &+ \frac{c_i}{2} \int_{q_i/R}^{q_i(R+D_i)/RD_i} \left(\frac{R^2 D_i^2 l_i^2 - q_i^2 D_i^2}{q_i^2 R^2} \right) f_L(l_i) dl_i + \frac{c_i}{2} \int_{q_i(R+D_i)/RD_i}^{\infty} \left(\frac{R + 2D_i}{R} \right) f_L(l_i) dl_i \\ &+ \frac{h_m}{2n} - \frac{h_i}{2} \int_{q_i/R}^{q_i(R+D_i)/RD_i} \frac{q_i^2 (R + D_i)^2 - R^2 D_i^2 l_i^2}{q_i^2 R^2} f_L(l_i) dl_i \end{aligned} \quad (7.17)$$

$$\frac{\partial^2 AEP_s}{\partial q_i^2} = -\frac{2A_m D}{ns^3} - \frac{2(A_i + nF)D_i}{nq_i^3} - \frac{h_i}{2} \int_{q_i/R}^{q_i(R+D_i)/RD_i} \frac{2D_i^2 l_i^2}{q_i^3} f_L(l_i) dl_i - \frac{c_i}{2} \int_{q_i/R}^{q_i(R+D_i)/RD_i} \frac{2D_i^2 l_i^2}{q_i^3} f_L(l_i) dl_i \quad (7.18)$$

Now, the average expected profit of the entire supply chain is concave in q_i if $\frac{\partial^2 AEP_s}{\partial q_i^2} < 0$, which gives $\frac{2A_m D}{s^3} + \frac{2(A_i + nF)D_i}{nq_i^3} + n(h_i + c_i) \int_{q_i/R}^{q_i(R+D_i)/RD_i} \frac{D_i^2 l_i^2}{q_i^3} f_L(l_i) dl_i > 0$.

Proposition 7.6 The average expected system profit function (7.12) is concave in p_i for given n and q_i if

$$2\beta_i + (h_i + c_i) \int_{q_i/R}^{q_i(R+D_i)/RD_i} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i > 0. \quad (7.19)$$

Proof. Using $r_i = \frac{q_i D_i}{R}$ and differentiating equation (7.12) with respect to p_i , we have

$$\begin{aligned} \frac{\partial AEP_s}{\partial p_i} &= D_i - \beta_i p_i + \frac{(A_i + nF)\beta_i}{Q_i} + h_i \int_{q_i/R}^{q_i(R+D_i)/RD_i} \frac{\beta_i (q_i - Rl_i) [q_i R + D_i (q_i - Rl_i)]}{q_i R^2} f_L(l_i) dl_i \\ &+ h_i \int_0^{q_i/R} \frac{\beta_i (q_i - Rl_i)}{R} f_L(l_i) dl_i + c_i \int_{q_i/R}^{q_i(R+D_i)/RD_i} \frac{\beta_i D_i (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i \\ &- c_i \int_{q_i(R+D_i)/RD_i}^{\infty} \frac{\beta_i (q_i - Rl_i)}{R} f_L(l_i) dl_i + \frac{A_m \beta_i}{Q} + \frac{h_m \beta_i s}{R} - \frac{h_m Q \beta_i}{2R} + \frac{h_m \beta_i \sigma_i}{\sqrt{2\pi}} \end{aligned} \quad (7.20)$$

$$\frac{\partial^2 AEP_s}{\partial p_i^2} = -2\beta_i - (h_i + c_i) \int_{q_i/R}^{q_i(R+D_i)/RD_i} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i \quad (7.21)$$

The average expected profit of entire system is concave in p_i if $\frac{\partial^2 AEP_s}{\partial p_i^2} < 0$ i.e., if

$$2\beta_i + (h_i + c_i) \int_{q_i/R}^{q_i(R+D_i)/RD_i} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i > 0.$$

Proposition 7.7 The average expected profit function (7.12) is jointly concave with respect to n , q_i and p_i if

- (i) $h_m s^2 < 2(A_m D + m s)$
- (ii) $2(h_i + c_i)X_4 X_5 > X_4^2 + 2\beta_i X_2 + (h_i + c_i)(X_2 X_3 + (h_i + c_i)X_5^2)$
- (iii) $(h_i + c_i)X_8 + 2\beta_i X_1 X_2 + X_1 X_4^2 + X_2 X_7^2 > (h_i + c_i)X_9 + 2X_4 X_6 X_7 + 2\beta_i X_6^2$

where

$$m = \sum_{i=1}^N \frac{A_i D_i}{q_i} \quad (7.22)$$

$$s = \sum_{i=1}^N q_i \quad (7.23)$$

$$X_1 = -\frac{2A_m D}{n^3 s} + \frac{h_m s}{n^3} - \frac{2m}{n^3} \quad (7.24)$$

$$X_2 = -\frac{2A_m D}{n s^3} - \frac{2(A_i + nF)D_i}{n q_i^3} - (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2 l_i^2}{q_i^3} f_L(l_i) dl_i \quad (7.25)$$

$$X_3 = \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2 (q_i - R l_i)^2}{q_i R^2} f_L(l_i) dl_i \quad (7.26)$$

$$X_4 = -\frac{(A_i + nF)\beta_i}{n q_i^2} - \frac{A_m \beta_i}{n s^2} + \frac{h_m \beta_i}{R} - \frac{n h_m \beta_i}{2R} + h_i \int_0^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i}{R} f_L(l_i) dl_i \\ - c_i \int_{\frac{q_i(R+D_i)}{RD_i}}^{\infty} \frac{\beta_i}{R} f_L(l_i) dl_i \quad (7.27)$$

$$X_5 = \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i D_i (R^2 l_i^2 - q_i^2)}{q_i^2 R^2} f_L(l_i) dl_i \quad (7.28)$$

$$X_6 = -\frac{A_m D}{n^2 s^2} - \frac{h_m}{2} \left(1 - \frac{D}{R}\right) - \frac{h_m}{2n^2} - \frac{A_i D_i}{n^2 q_i^2} \quad (7.29)$$

$$X_7 = -\frac{A_m \beta_i}{n^2 s} - \frac{h_m \beta_i s}{2R} - \frac{A_i \beta_i}{n^2 q_i} \quad (7.30)$$

$$X_8 = (X_1 X_2 X_3 + 2X_5 X_6 X_7 + (h_i + c_i) X_1 X_5^2) \quad (7.31)$$

$$X_9 = (2X_1 X_4 X_5 + X_3 X_6^2) \quad (7.32)$$

Proof. From equation (7.12), we have

$$\frac{\partial^2 AEP_s}{\partial n^2} = X_1 \quad (7.33)$$

$$\frac{\partial^2 AEP_s}{\partial q_i^2} = X_2 \quad (7.34)$$

$$\frac{\partial^2 AEP_s}{\partial p_i^2} = -2\beta_i - (h_i + c_i)X_3 \quad (7.35)$$

$$\frac{\partial^2 AEP_s}{\partial q_i \partial p_i} = \frac{\partial^2 AEP_s}{\partial p_i \partial q_i} = X_4 - (h_i + c_i)X_5 \quad (7.36)$$

$$\frac{\partial^2 AEP_s}{\partial n \partial q_i} = \frac{\partial^2 AEP_s}{\partial q_i \partial n} = X_6 \quad (7.37)$$

$$\frac{\partial^2 AEP_s}{\partial n \partial p_i} = \frac{\partial^2 AEP_s}{\partial p_i \partial n} = X_7 \quad (7.38)$$

The Hessian matrix associated with the average expected profit function $AEP_s(n, q_i, p_i)$ is obtained as

$$\begin{aligned} H &= \begin{pmatrix} \frac{\partial^2 AEP_s}{\partial n^2} & \frac{\partial^2 AEP_s}{\partial n \partial q_i} & \frac{\partial^2 AEP_s}{\partial n \partial p_i} \\ \frac{\partial^2 AEP_s}{\partial q_i \partial n} & \frac{\partial^2 AEP_s}{\partial q_i^2} & \frac{\partial^2 AEP_s}{\partial q_i \partial p_i} \\ \frac{\partial^2 AEP_s}{\partial p_i \partial n} & \frac{\partial^2 AEP_s}{\partial p_i \partial q_i} & \frac{\partial^2 AEP_s}{\partial p_i^2} \end{pmatrix} \\ &= \begin{pmatrix} X_1 & X_6 & X_7 \\ X_6 & X_2 & X_4 - (h_i + c_i)X_5 \\ X_7 & X_4 - (h_i + c_i)X_5 & -2\beta_i - (h_i + c_i)X_3 \end{pmatrix} \end{aligned} \quad (7.39)$$

$$\text{Now, } \frac{\partial^2 AEP_s}{\partial n^2} = X_1 < 0 \text{ if } h_m s^2 < 2(A_m D + ms). \quad (7.40)$$

The second order minor $= |H_2| = \begin{vmatrix} X_2 & X_4 - (h_i + c_i)X_5 \\ X_4 - (h_i + c_i)X_5 & -2\beta_i - (h_i + c_i)X_3 \end{vmatrix} > 0$ if

$$2(h_i + c_i)X_4 X_5 > X_4^2 + 2\beta_i X_2 + (h_i + c_i)(X_2 X_3 + (h_i + c_i)X_5^2) \quad (7.41)$$

and $|H| < 0$ only if

$$(h_i + c_i)X_8 + 2\beta_i X_1 X_2 + X_1 X_4^2 + X_2 X_7^2 > (h_i + c_i)X_9 + 2X_4 X_6 X_7 + 2\beta_i X_6^2 \quad (7.42)$$

Therefore, the Hessian matrix H is negative definite if the above conditions (7.40), (7.41) and (7.42) are satisfied. It ensures the concavity of the average expected profit

of the entire supply chain with respect to n , q_i and p_i and existence of unique optimal solution.

7.5 Numerical Analysis

To analyze the proposed model numerically, an example is taken with the following set of parameter-values:

$R = 2500$; $A_m = 500$; $w = 80$; $\sigma_1 = 0.12$; $a_1 = 1000$; $a_2 = 1000$; $F = 10$; $\sigma_2 = 0.13$; $h_1 = 5$; $h_2 = 4.8$; $h_m = 3.5$; $\beta_1 = 3.5$; $c_1 = 6$; $c_2 = 6$; $A_1 = 50$; $\beta_2 = 4.5$; $A_2 = 45$; $N = 2$. (with appropriate units)

The p. d. f. of the lead time of i -th retailer is assumed as $f_L(l_i) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{\frac{-1}{2\sigma_i^2} (l_i - \frac{r_i}{D_i})^2}$.

Here we have studied a system consisting of a single manufacturer and two retailers. The concavity of the profit function of the whole supply chain system with respect to its decision variables is observed for the chosen parameter-values. Figure 7.4 exhibits the concavity of the average expected profit function of the entire supply chain with respect to n .

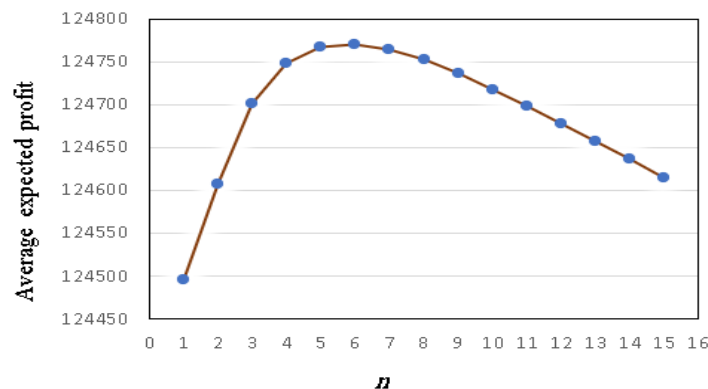


Figure 7.4: Concavity of the average expected profit of the supply chain w.r.t. n

The nature of the profit functions of both the retailers has been found to be concave for different values of (q_1, p_1) , and (q_2, p_2) in all situations. One instance for each retailer is reflected in Figures 7.5 and 7.6.

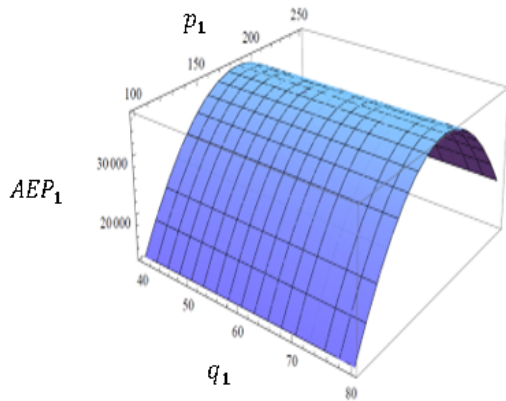


Figure 7.5: Concavity of the average expected profit function of the first retailer

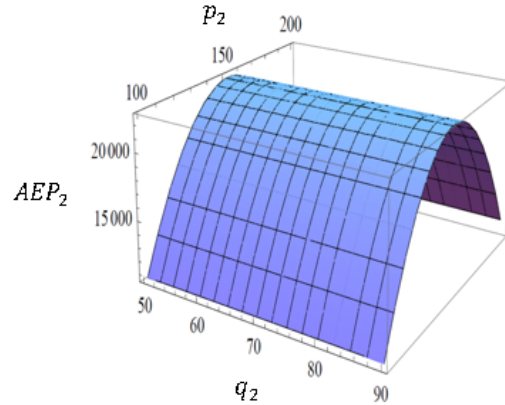


Figure 7.6: Concavity of the average expected profit function of the second retailer

Table 7.1: Optimal results for different models

Models	n^*	q_1^*	q_2^*	p_1^*	p_2^*	AEP_1^*	AEP_2^*	AEP_m^*	AEP_s^*
Decentralized	4	69.29	74.42	183.01	151.26	36821	22590	52805	112215
Centralized	6	60.07	63.49	143.33	111.59	-	-	-	124771

Using the prescribed solution algorithm, we find the optimal batch sizes as well as the order quantities of the retailers, selling price of the product and the number of batch shipments in the decentralized model. Table 7.1 shows the optimal results of both the decentralized and centralized models. It can be observed from Table 7.1 that the profit in the centralized model is more than the profit in the decentralized model. In our proposed model, we assume that, the customer demand is linearly dependent on its retail price. In the centralized decision making scenario, the manufacturer and all the retailers act as a single business manager and jointly make their optimal decisions in order to achieve highest system profit. So, the retailers can provide the product to the customers at a cheaper price than that of the decentralized case. That's why lower priced product increases the consumer demand significantly. Then the retailers order more quantity from the manufacturer. As a result, integration between the manufacturer and the retailers increases the total system profit significantly. We observe that the optimal ordering quantity of the first retailer is increased by 83.26 units and for the second retailer, it is increased by 83.26 units in the centralized model. Moreover, the retail prices of the product also decrease for both the retailers. This results in an increase of profit for the entire supply chain.

7.6 Sensitivity Analysis

The impacts of several important parameters on the average expected profit of the supply chain are analyzed in this section. We change one parameter-value at a time, keeping other parameter-values fixed. The results of sensitivity analysis are presented in Table 7.2. From Table 7.2 we have the following observations:

(i) The average expected profit of the entire supply chain is highly sensitive with respect to the parameters β_1 and β_2 . It can be noticed that, both the retailers reduce their selling prices of the product when the values of β_1 and β_2 increase. As higher values of β_1 and β_2 indicate that the customers prefer to purchase cheaper products, so in order to keep the consumer demand intact, the retailers reduce their selling prices. Also, the ordered quantities of both the retailers decrease as β_1 and β_2 increase. Consequently, the average expected profits of all individual supply chain players as well as the entire supply chain decrease gradually as β_1 and β_2 increase.

(ii) The impacts of the basic market demands a_1 and a_2 on the average expected profit of the supply chain can be realised from the optimal results given in Table 7.2. Whenever the customer demand increases, the retailers place orders for more items from the manufacturer. Also, with the increment of the basic market demand, the retailers get the opportunity to raise the retail prices and earn more revenue. This leads the supply chain system to a better profit level.

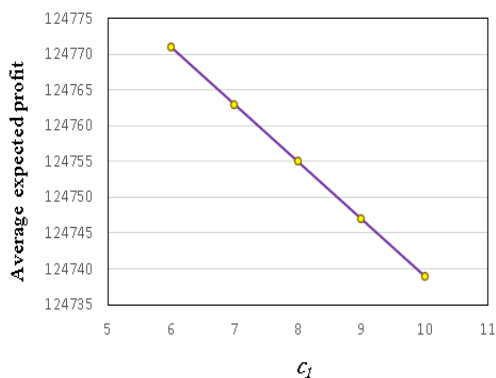


Figure 7.7: Average expected profit of the supply chain vs. c_1

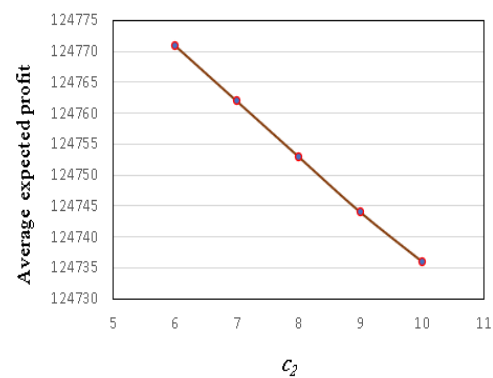


Figure 7.8: Average expected profit of the supply chain vs. c_2

(iii) From Figures 7.7 and 7.8, it can be seen that the average expected profit decreases while shortage costs c_1 and c_2 increase. It is quite natural that, when the shortage cost increases, the retailers increase their order quantities to avoid shortage. But the retail prices remain almost the same for all values of shortage cost.

Table 7.2: Optimal results for different values of β_1, β_2, a_1 and a_2

Parameter	Value	n^*	q_1^*	q_2^*	p_1^*	p_2^*	AEP_s^*
β_1	3.1	6	59.4274	62.3693	161.782	111.597	133999
	3.3	6	60.0768	63.4912	151.984	111.585	129100
	3.5	6	60.0696	63.4904	143.326	111.585	124771
	3.7	6	60.0624	63.4896	135.604	111.585	120910
	3.9	6	60.0552	63.4889	128.674	111.585	117445
β_2	4.1	6	59.4274	62.3693	161.782	111.597	133999
	4.3	6	60.0768	63.4912	151.984	111.585	129100
	4.5	6	60.0696	63.4904	143.326	111.585	124771
	4.7	6	60.0624	63.4896	135.604	111.585	120910
	4.9	6	60.0552	63.4889	128.674	111.585	117445
a_1	800	6	52.1983	62.7725	114.803	111.625	99154
	900	6	56.1819	63.1056	129.063	111.604	111247
	1000	6	60.0696	63.4904	143.326	111.585	124771
	1100	6	63.8784	63.9203	157.59	111.566	139725
	1200	6	67.6216	63.3906	171.856	111.548	156110
a_2	800	6	59.6716	55.0633	143.366	89.411	104870
	900	6	59.8473	59.3196	143.345	100.497	114264
	1000	6	60.0696	63.4904	143.326	111.585	124771
	1100	6	60.3313	67.5938	143.307	122.675	136391
	1200	6	67.6272	71.6443	143.289	133.766	149125

(iv) From Figures 7.9, we notice that the transportation cost affects the optimal number shipments significantly. As the transportation cost increases, the optimal number of batch shipments decreases. The average expected profit of the supply chain also decreases with increased transportation cost.

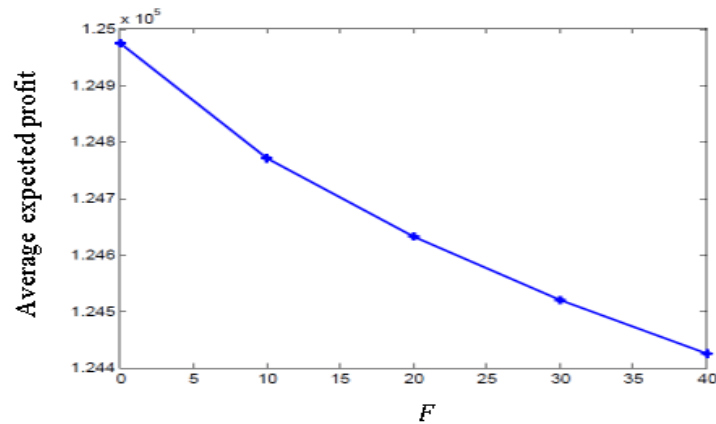


Figure 7.9: Average expected profit of the supply chain vs. F

(v) The standard deviations of the lead times of both the retailers do not affect the profit of the supply chain considerably. When the standard deviations σ_1 and σ_2 increase approximately by 36% and 33%, respectively, the average system profit decreases approximately by 4%.

7.7 Concluding Remarks

In this chapter, we have considered a two-level supply chain model consisting of a single manufacturer and multiple retailers under price-sensitive customer demand and stochastic lead time. The manufacturer produces a single product at a constant production rate and transfers it to the retailers in some equal batch shipments. Due to the stochastic nature of lead time the batches may reach early or late to the retailers. Depending on the length of lead time duration, three possible cases take into consideration at the retailers' end. Both the decentralized and centralized models are formulated. A solution algorithm is suggested to find the optimal solution of the decentralized model with a Stackelberg gaming structure. It is found that cooperation between the manufacturer and the retailers becomes more profitable for the

entire supply chain whenever the consumer demand is price dependent and lead time of delivering the ordered quantity follows a normal distribution. By collaborating with each other, the supply chain players can sell the product at a cheaper price to the end customers and enhance the market demand. This leads to a significant increment of the profitability of the whole supply chain.

Our proposed model is well-connected to the real world business environment. The present study would be helpful to companies to make the efficient management of supply chains in stochastic environments. The proposed model provides effective pricing strategies in a highly price-sensitive market. Companies can adopt integrated decision making to achieve the highest system profit and make their products more acceptable to the end customer at a lower price. Also, they can achieve higher customer service levels and survive in today's competitive market by efficiently managing the stochastic nature of lead time.

In our proposed model, we have considered that one manufacturer is trading with multiple retailers for a single product. Our model can be further extended considering multiple items. One can also implement any suitable coordination scheme between the manufacturer and all the retailers to improve their profits in the decentralized scenario. Furthermore, our proposed model can be enriched by considering more realistic assumptions, such as combined equal and unequal sized batch shipments, quality dependent demand and multiple manufacturers trading with multiple retailers.

Coordination of a single-manufacturer multi-retailer supply chain with price and green sensitive demand under stochastic lead time

8.1 Introduction

The growing importance of environmental protection and pollution reduction has been felt all over the world in recent years. Green supply chain management aims to prevent pollution while also producing environmentally friendly products. It involves many activities including green manufacturing, green packaging, green distribution, remanufacturing and waste management. The previous chapter dealt with only price sensitivity of customers and focused on only the decentralized and centralized scenarios. In this chapter*, we have extended this model by incorporating the environmental awareness of the customers. Furthermore, we demonstrate cooperation between the manufacturer and the retailers by a price discount mechanism.

In this chapter, we incorporate a price discount mechanism with green initiatives in a single-manufacturer multi-retailer supply chain model under stochastic lead time.

*This chapter is based on the work published in *Decision Making: Applications in Management and Engineering*, 6(1), 679 - 715.

Afterwards, we examine whether the proposed price discount contract is able to coordinate the supply chain or not. Finally, we look at the influence of the price discount contract on supply chain members' profitability and determine the conditions under which they accept the price discount contract. Our primary goal is to find answers to the following research questions:

- What will be the optimal strategies of the manufacturer and retailers when the market demand is price and green sensitive?
- What is the impact of a price discount contract on the optimal decisions of the supply chain?
- Is the price discount mechanism capable of coordinating the supply chain?
- What is the effect of greening investment on the profitability of the supply chain?

8.2 Problem Definition

This chapter considers a green supply chain model where the single manufacturer deals with multiple retailers for a single product. The manufacturer produces the items at a fixed production rate in a single set up and delivers the order quantities of the retailers with an equal sized batch shipment policy. Due to various unavoidable circumstances such as late start in production, varying transportation time, loading, unloading, etc., the batches may arrive early or late at the retailers. To deal with this type of delivery uncertainty, lead time is treated as a stochastic random variable which follows a normal distribution. Customer demand is assumed to be affected by the retail price and environmental performance of the product. The manufacturer adopts a green investment strategy to maintain his environmental responsibility as well as stimulate the customer demand in an eco-conscious market. In both decentralized and centralized settings, the manufacturer's and all retailers' optimal pricing and inventory strategies are derived. Following that, a wholesale price discount contract is implemented between the manufacturer and the retailers to coordinate the supply chain.

8.3 Notations and Assumptions

The following notations are used for developing the proposed model and The subscript i is used to indicate the ' i '-th retailer ($i = 1, 2, \dots, N$).

R	: Production rate (units/unit time)
A_m	: Set-up cost of the manufacturer (\$/set up)
h_m	: Manufacturer's holding cost (\$/unit /unit time)
F	: Transportation cost (\$/shipment)
w	: Unit wholesale price (\$/unit)
I	: Greening investment parameter (\$)
N	: Number of retailers (Positive integer)
Q	: Total order quantity [= $\sum_{i=1}^N Q_i$] (units)
L	: Lead time, a random variable with p.d.f. $f_L(\cdot)$
D	: Total market demand [= $\sum_{i=1}^N D_i$] (units /unit time)
A_i	: Retailer's ordering cost (\$ /order)
h_i	: Holding cost of the retailer (\$ /unit /unit time)
D_i	: Demand rate [$R > \sum_{i=1}^N D_i$] (unit /unit time)
a_i	: Basic market demand (units /unit time)
α_i	: Consumer sensitivity coefficient to greening level
β_i	: Consumer sensitivity coefficient to retail price
Q_i	: Order quantity (units)
c_i	: Shortage cost (\$ /unit /unit time)
r_i	: Reorder point (units)
σ_i	: Standard deviation of the lead time
AEP_m	: Average expected profit of the manufacturer (\$/unit time)
AEP_i	: Average expected profit of the retailer (\$/unit time)
AEP_s	: Average expected profit of the supply chain (\$/unit time)
$(\cdot)^d$: Decision variable in decentralized policy
$(\cdot)^c$: Decision variable in centralized policy
$(\cdot)^{co}$: Decision variable in coordinated mechanism

Decision variables

n	: Number of batches delivered to each retailer (positive integer)
θ	: Greening improvement level
q_i	: Batch size of the retailer (units)
p_i	: Unit retail price of the retailer (\$ /unit)
ϕ	: Price discount ratio, $\phi \in (0, 1)$

The basic assumptions for developing the proposed model are as follows:

1. A single manufacturer produces a single item and meets the demand of multiple retailers (Sarkar et al., 2018).
2. The manufacturer transfers the products to the retailers in a number of equal sized batches (Sarkar et al., 2020).
3. The retailers face a consumer demand dependent on the selling price and greenness of the product (Ghosh and Shah, 2015). We assume that the demand rate of the i -th retailer is a linear function of retail price and greening level of the product given by $D_i(p_i, \theta) = a_i - \beta_i p_i + \alpha_i \theta$, where a_i is the basic market demand, β_i and α_i are positive integers such that $a_i + \alpha_i \theta > \beta_i p_i$ for all $i = 1, 2, \dots, N$.
4. The manufacturer produces the product at a constant production rate R in one set-up and the production rate is greater than the sum of demands of all retailers i.e., $R > \sum_{i=1}^N D_i$ (Hoque, 2021).
5. Shortages are allowed and are assumed to be completely backlogged (Sarkar et al., 2018).
6. The i -th retailer places his next order when his inventory stock level reaches to a certain reorder level r_i (Hoque, 2013).
7. The lead time to meet the retailer's demand is a random variable which follows a normal distribution and the lead time for each shipment is independent of the others (Hoque, 2013).
8. Annual greening investment for the product is taken as $I\theta^2$, which is increasing and convex in the greening improvement level θ (Ghosh and Shah, 2015).

8.4 Model Formulation

We suppose that the manufacturer sells the produced items to N retailers. The manufacturer transfers the order quantity Q_i of the i -th retailer in n equal batches of size q_i . Total order quantity of N retailers is Q .

Therefore, $Q_i = nq_i$ and $Q = \sum_{i=1}^N Q_i$

The i -th retailer places the next order when the inventory stock reaches to a level r_i . The shipment is expected to arrive to the retailer's end at or before the time of selling this r_i quantity. Mean lead time is $\frac{r_i}{D_i}$. Due to various reasons, the batches may reach early or late. We assume that the lead time follows a normal distribution. Depending on the length of the lead time, three cases may arise:

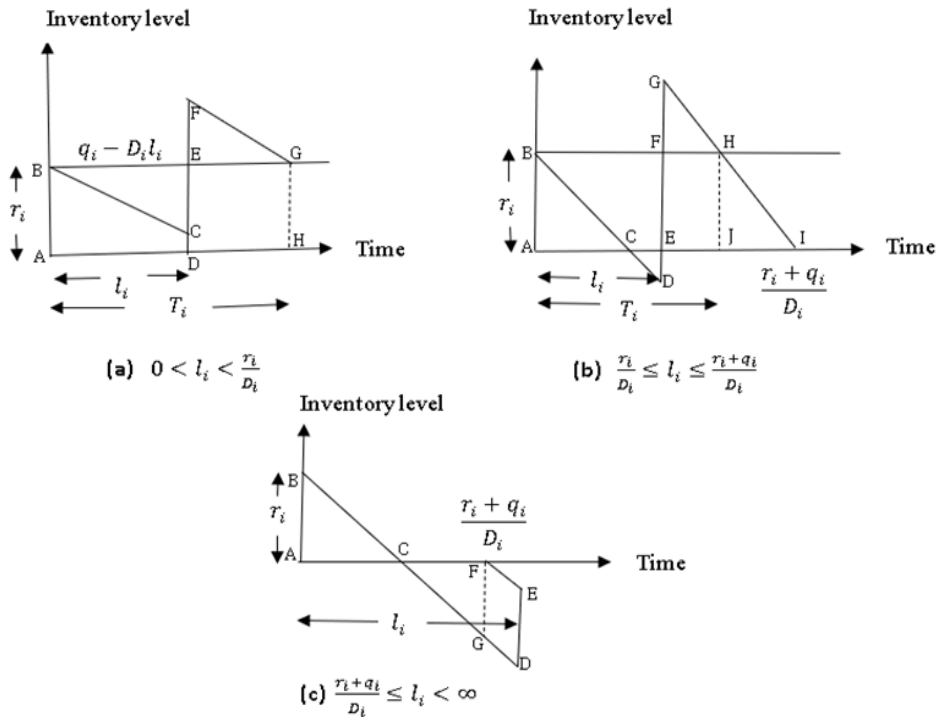


Figure 8.1: Inventory of i -th retailer under stochastic lead time

Case(i) When the batch q_i reaches to the retailer earlier i.e., $0 < l_i < \frac{r_i}{D_i}$.

In this case, similar to Hoque, 2013, the inventory holding area of the i -th retailer can be determined from Figure 8.1(a) as

$$\begin{aligned}
 & \text{Area}(ABCD + EFG + GHDE) \\
 &= \frac{1}{2}(r_i - D_i l_i + r_i)l_i + \frac{1}{2}(q_i - D_i l_i) \frac{(q_i - D_i l_i)}{D_i} + \frac{r_i(q_i - D_i l_i)}{D_i} \\
 &= \frac{1}{2} \left[\frac{q_i^2}{D_i} + 2q_i \left(\frac{r_i}{D_i} - l_i \right) \right] \quad \text{where } r_i = \frac{q_i D_i}{R}
 \end{aligned}$$

Then the order quantity Q_i of the i -th retailer is given by

$$\frac{n}{2} \left[\frac{q_i^2}{D_i} + 2q_i \left(\frac{r_i}{D_i} - l_i \right) \right]$$

The holding cost refers to the investment in storing the unsold products. The expected inventory holding cost for the order quantity Q_i of the i -th retailer is

$$h_i \int_0^{\frac{r_i}{D_i}} \frac{n}{2} \left[\frac{q_i^2}{D_i} + 2q_i \left(\frac{r_i}{D_i} - l_i \right) \right] f_L(l_i) dl_i$$

Case(ii) When the batch q_i reaches late to the i -th retailer and the lead time l_i lies in the range $\frac{r_i}{D_i} \leq l_i \leq \frac{r_i+q_i}{D_i}$.

In this case, shortages occur at the retailer's end. From Figure 8.1(b), shortage area at the i -th retailer is obtained as

$$\text{Area}(CDE) = \frac{1}{2D_i} (D_i l_i - r_i)^2.$$

So, the expected shortage cost of i -th retailer for n batches is given by

$$\frac{nc_i}{2} \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{D_i} f_L(l_i) dl_i$$

Inventory holding area of the i -th retailer for the batch q_i is

$$\begin{aligned} & \text{Area}(ABC + FGH + EFHJ) \\ &= \frac{r_i^2}{2D_i} + \frac{(q_i - D_i l_i)^2}{2D_i} + \frac{r_i(q_i - D_i l_i)}{D_i} \\ &= \frac{(q_i - D_i l_i + r_i)^2}{2D_i} \end{aligned}$$

Hence the expected inventory holding cost of i -th retailer for n shipments is

$$nh_i \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{2D_i} f_L(l_i) dl_i$$

It is assumed that, during this delay period, the batches remain in the manufacturer's stockhouse. So, it causes an extra holding cost to the manufacturer. The

extra inventory for this delayed delivery is $\sum_{i=1}^N \frac{nq_i(D_i l_i - r_i)}{D_i}$.

So, in this case, the additional inventory holding cost for the manufacturer is

$$h_m \sum_{i=1}^N \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{nq_i(D_i l_i - r_i)}{D_i} f_L(l_i) dl_i$$

Case(iii) When the batch q_i arrives late to the retailer with lead time in the range $\frac{r_i+q_i}{D_i} \leq l_i < \infty$.

In this case, shortages occur at the retailer's end and, from Figure 8.1(c), the shortage area for the batch q_i is obtained as

$$\begin{aligned} \text{Area}(CDEF) &= \frac{q_i^2}{2D_i} + \text{Area}(DEFG) \\ &= \frac{q_i^2}{2D_i} + q_i \left(\frac{D_i l_i - q_i - r_i}{D_i} \right) \end{aligned}$$

So, the expected shortage cost of the i -th retailer for all batch shipments is

$$nc_i \int_{\frac{r_i+q_i}{D_i}}^{\infty} \left[\frac{q_i^2}{2D_i} + q_i \left(\frac{D_i l_i - q_i - r_i}{D_i} \right) \right] f_L(l_i) dl_i$$

Similar to case(ii), the additional expected inventory holding cost for the manufacturer is

$$h_m \sum_{i=1}^N \int_{\frac{r_i+q_i}{D_i}}^{\infty} \frac{nq_i(D_i l_i - r_i)}{D_i} f_L(l_i) dl_i$$

Combining all three cases, the expected holding cost of the i -th retailer for all batches is given by

$$nh_i \int_0^{\frac{r_i}{D_i}} \frac{1}{2} \left[\frac{q_i^2}{D_i} + 2q_i \left(\frac{r_i}{D_i} - l_i \right) \right] f_L(l_i) dl_i + nh_i \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{2D_i} f_L(l_i) dl_i$$

and the expected shortage cost for all batch shipments is

$$nc_i \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{2D_i} f_L(l_i) dl_i + nc_i \int_{\frac{r_i+q_i}{D_i}}^{\infty} \left[\frac{q_i^2}{2D_i} + q_i \left(\frac{D_i l_i - q_i - r_i}{D_i} \right) \right] f_L(l_i) dl_i$$

8.4.1 Decentralized Model

In the decentralized model, the manufacturer and the retailers independently take their decisions in order to maximize their own profits. Here we consider the retailers to be the Stackelberg leader and the manufacturer as the follower. The manufacturer sets the number of shipments and greening level of the products. Then taking these responses into consideration, the retailers decide their optimal retail price and batch sizes.

A. Average expected profit of the manufacturer

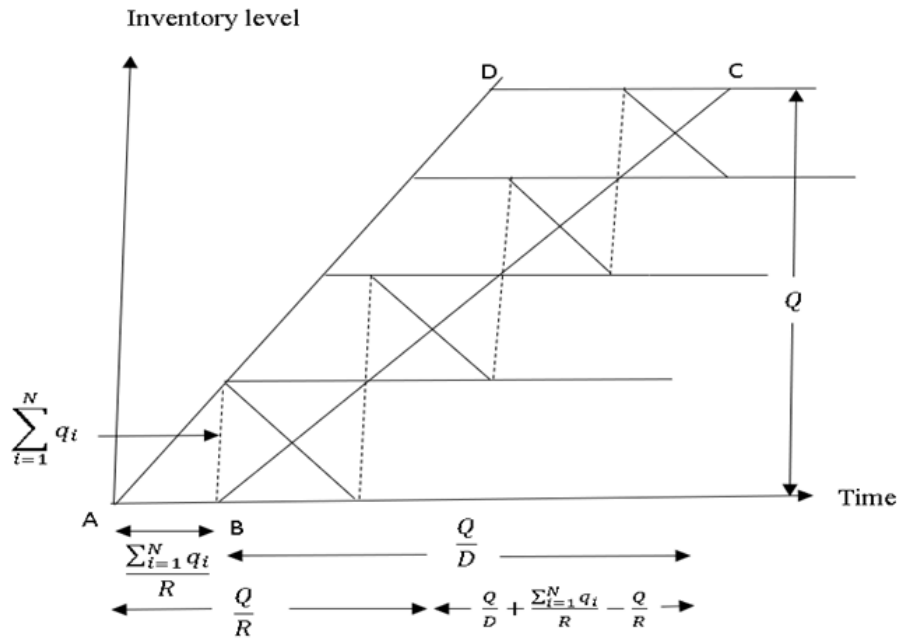


Figure 8.2: Joint inventory of the manufacturer and the retailers

The manufacturer's total extra holding cost from cases(ii) and (iii) is

$$h_m \sum_{i=1}^N \int_{\frac{r_i}{D_i}}^{\infty} \frac{nq_i(D_i l_i - r_i)}{D_i} f_L(l_i) dl_i$$

In Figure 8.2, the trapezium ABCD represents the joint inventory of the manufacturer and retailers. The average inventory of the manufacturer-retailer system is

$$\begin{aligned} \text{Area}(ABCD) \times \frac{D}{Q} &= \frac{1}{2} \times (AB + CD) \times Q \times \frac{D}{Q} \\ &= \frac{1}{2} \left[\frac{\sum_{i=1}^N q_i}{R} + \left(\frac{Q}{D} + \frac{\sum_{i=1}^N q_i}{R} - \frac{Q}{R} \right) \right] D \\ &= \frac{D \sum_{i=1}^N q_i}{R} + \frac{Q}{2} \left(1 - \frac{D}{R} \right) \end{aligned}$$

Average inventory holding of N retailers is

$$\sum_{i=1}^N \left(\frac{q_i^2}{2D_i} \right) \left(\frac{D_i}{Q_i} \right) = \sum_{i=1}^N \frac{q_i^2}{2Q_i}$$

Therefore, the average inventory holding of the manufacturer is

$$\frac{D \sum_{i=1}^N q_i}{R} + \frac{Q}{2} \left(1 - \frac{D}{R} \right) - \sum_{i=1}^N \frac{q_i^2}{2Q_i}$$

The set up cost incorporates the costs of materials and labours to get ready the machinery system for processing the new production lot of goods. It plays an important role in start-up of a new business and smooth running of it. The i -th retailer places an order of quantity Q_i . The manufacturer produces the total order quantity $Q = \sum_{i=1}^N Q_i$. The cycle length of the manufacturer is $\frac{Q}{D}$. Therefore, the average set up cost is $\frac{A_m D}{Q}$.

Investment for greening supports the environmentally-conscious business practices. In this case, the manufacturer's average greening investment is $I\theta^2$.

The average expected profit of the manufacturer is

$$\begin{aligned} AEP_m(n, \theta) &= wD - \frac{A_m D}{Q} - h_m \left[\frac{D \sum_{i=1}^N q_i}{R} + \frac{Q}{2} \left(1 - \frac{D}{R} \right) - \sum_{i=1}^N \frac{q_i^2}{2Q_i} \right] - I\theta^2 \\ &\quad - h_m \sum_{i=1}^N \int_{\frac{r_i}{D_i}}^{\infty} (D_i l_i - r_i) f_L(l_i) dl_i \\ &= wD - \frac{A_m D}{Q} - h_m \left[\frac{D \sum_{i=1}^N q_i}{R} + \frac{Q}{2} \left(1 - \frac{D}{R} \right) - \sum_{i=1}^N \frac{q_i^2}{2Q_i} \right] - \sum_{i=1}^N \frac{h_m D_i \sigma_i}{\sqrt{2\pi}} - I\theta^2 \end{aligned} \quad (8.1)$$

From equation (8.1), we have

$$\frac{\partial AEP_m}{\partial n} = \frac{A_m D}{n^2 s} - \frac{h_m s}{2} + \frac{h_m s D}{2R} - \frac{h_m s}{2n^2} \quad (8.2)$$

$$\frac{\partial AEP_m}{\partial \theta} = \left[w - \frac{A_m}{Q} - \frac{h_m s}{R} + \frac{h_m Q}{2R} - \left(\sum_{i=1}^N \frac{h_m \sigma_i}{\sqrt{2\pi}} \right) \right] u - 2I\theta \quad (8.3)$$

$$\frac{\partial^2 AEP_m}{\partial n^2} = -\frac{2A_m D}{n^3 s} + \frac{h_m s}{n^3} \quad (8.4)$$

$$\frac{\partial^2 AEP_m}{\partial \theta^2} = -2I \quad (8.5)$$

$$\frac{\partial^2 AEP_m}{\partial n \partial \theta} = \frac{A_m u}{n^2 s} + \frac{h_m s u}{2R} \quad (8.6)$$

$$\frac{\partial^2 AEP_m}{\partial \theta \partial n} = \frac{A_m u}{n^2 s} + \frac{h_m s u}{2R}, \text{ where } s = \sum_{i=1}^N q_i \text{ and } u = \sum_{i=1}^N \alpha_i \quad (8.7)$$

Proposition 8.1 The average expected profit function of the manufacturer is jointly concave in n and θ if $8IR^2ns(2A_mD_s - h_ms^2) > (2A_mRu + h_mun^2s^2)^2$.

Proof. Considering n as real, the Hessian matrix is

$$H = \begin{pmatrix} \frac{\partial^2 AEP_m}{\partial \theta^2} & \frac{\partial^2 AEP_m}{\partial \theta \partial n} \\ \frac{\partial^2 AEP_m}{\partial n \partial \theta} & \frac{\partial^2 AEP_m}{\partial n^2} \end{pmatrix} = \begin{pmatrix} -2I & \frac{A_m u}{n^2 s} + \frac{h_m s u}{2R} \\ \frac{A_m u}{n^2 s} + \frac{h_m s u}{2R} & -\frac{2A_m D}{n^3 s} + \frac{h_m s}{n^3} \end{pmatrix} \quad (8.8)$$

Here, $\frac{\partial^2 AEP_m}{\partial \theta^2} = -2I < 0$. So, the expected average profit function of the manufacturer will be concave in θ and n if $|H| > 0$. Substituting the values of the partial derivatives from the above and using the condition $|H| > 0$, we get after simplification, $8IR^2ns(2A_mD_s - h_ms^2) > (2A_mRu + h_mun^2s^2)^2$.

Proposition 8.2 At the equilibrium, the optimal number of shipments to each retailer, and the optimal greening level of the product are as follows:

$$n^* = \sqrt{\frac{R(2A_mD - h_ms^2)}{h_ms^2(R - D)}} \quad (8.9)$$

$$\theta^* = \frac{\left[w - \frac{A_m}{Q} - \frac{h_ms}{R} + \frac{h_mQ}{2R} - \left(\sum_{i=1}^N \frac{h_m\sigma_i}{\sqrt{2\pi}} \right) \right] u}{2I} \quad (8.10)$$

Proof. At the equilibrium, we have

$$\frac{\partial AEP_m}{\partial n} = \frac{A_mD}{n^2 s} - \frac{h_ms}{2} + \frac{h_msD}{2R} - \frac{h_ms}{2n^2} = 0 \quad (8.11)$$

$$\text{and } \frac{\partial AEP_m}{\partial \theta} = \left[w - \frac{A_m}{Q} - \frac{h_ms}{R} + \frac{h_mQ}{2R} - \left(\sum_{i=1}^N \frac{h_m\sigma_i}{\sqrt{2\pi}} \right) \right] u - 2I\theta = 0 \quad (8.12)$$

Solving equations (8.11) and (8.12), we get the optimal values of n and θ as given in equations (8.9) and (8.10) above.

For integer optimal value of n ,

$$n^{opt} = \begin{cases} \lfloor n^* \rfloor, & \text{if } AEP_m(\lfloor n^* \rfloor, \theta) \geq AEP_m(\lceil n^* \rceil, \theta) \\ \lceil n^* \rceil, & \text{if } AEP_m(\lfloor n^* \rfloor, \theta) \leq AEP_m(\lceil n^* \rceil, \theta) \end{cases} \quad (8.13)$$

Taking these response functions of the manufacturer, the retailers then set their batch sizes and retail prices.

B. Average expected profit of the i -th retailer

Since the expected cycle length for the i -th retailer is $\frac{Q_i}{D_i}$, therefore, the average ordering cost of the i -th retailer is given by $\frac{A_i D_i}{Q_i}$.

From manufacturing to delivery to the end customer and even returns, transportation is essential to the entire production process. It is practically impossible for a logistics firm to conduct business efficiently without transportation. As the number of shipments increases, the transportation cost increases. Since the manufacturer delivers order quantity to the i -th retailer in n shipments and the expected cycle length for the i -th retailer is $\frac{Q_i}{D_i}$, therefore, the average variable transportation cost is $\frac{nFD_i}{Q_i}$.

The expected total profit of the i -th retailer is

$$p_i Q_i - w Q_i - A_i - nF - \frac{nh_i}{2} \left[\int_0^{\frac{r_i}{D_i}} \left[\frac{q_i^2}{D_i} + 2q_i \left(\frac{r_i}{D_i} - l_i \right) \right] f_L(l_i) dl_i + \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{D_i} f_L(l_i) dl_i \right] \\ - \frac{nc_i}{2} \left[\int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{D_i} f_L(l_i) dl_i + \int_{\frac{r_i+q_i}{D_i}}^{\infty} \left[\frac{q_i^2}{D_i} + 2q_i \left(\frac{D_i l_i - q_i - r_i}{D_i} \right) \right] f_L(l_i) dl_i \right]$$

Therefore, the average expected profit of the i -th retailer is obtained as

$$AEP_i(q_i, p_i) = p_i D_i - w D_i - \frac{(A_i + nF) D_i}{Q_i} - \frac{h_i}{2} \left[\int_0^{\frac{r_i}{D_i}} \left[q_i + 2(r_i - D_i l_i) \right] f_L(l_i) dl_i \right. \\ \left. + \int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{q_i} f_L(l_i) dl_i \right] - \frac{c_i}{2} \left[\int_{\frac{r_i}{D_i}}^{\frac{r_i+q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{q_i} f_L(l_i) dl_i \right. \\ \left. + \int_{\frac{r_i+q_i}{D_i}}^{\infty} \left[q_i + 2(D_i l_i - q_i - r_i) \right] f_L(l_i) dl_i \right] \quad (8.14)$$

Proposition 8.3 The average expected profit of the i -th retailer is concave in q_i for given p_i if $\frac{2(A_i+nF)D_i}{n} + (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} D_i^2 l_i^2 f_L(l_i) dl_i > 0$.

Proof. Differentiating equation (8.14) twice with respect to q_i , we obtain

$$\frac{\partial AEP_i}{\partial q_i} = \frac{(A_i + nF) D_i}{n q_i^2} - \frac{h_i}{2} \int_0^{\frac{q_i}{R}} \left(\frac{R + 2D_i}{R} \right) f_L(l_i) dl_i - \frac{h_i}{2} \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{q_i^2 (R + D_i)^2 - R^2 D_i^2 l_i^2}{q_i^2 R^2} f_L(l_i) dl_i \\ + \frac{c_i}{2} \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{R^2 D_i^2 l_i^2 - q_i^2 D_i^2}{q_i^2 R^2} f_L(l_i) dl_i + \frac{c_i}{2} \int_{\frac{q_i(R+D_i)}{RD_i}}^{\infty} \left(\frac{R + 2D_i}{R} \right) f_L(l_i) dl_i \quad (8.15)$$

$$\frac{\partial^2 AEP_i}{\partial q_i^2} = -\frac{2(A_i + nF) D_i}{n q_i^3} - h_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2 l_i^2}{q_i^3} f_L(l_i) dl_i - c_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2 l_i^2}{q_i^3} f_L(l_i) dl_i \quad (8.16)$$

For given p_i , the average expected profit function of the i -th retailer is concave in q_i if $\frac{\partial^2 AEP_i}{\partial q_i^2}$ is negative. This implies the condition $\frac{2(A_i+nF)D_i}{n} + (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} D_i^2 l_i^2 f_L(l_i) dl_i > 0$.

Proposition 8.4 The average expected profit of the i -th retailer is concave in p_i for given q_i if $2\beta_i + (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i > 0$.

Proof. Differentiating equation (8.14) with respect to p_i , we obtain

$$\begin{aligned} \frac{\partial AEP_i}{\partial p_i} &= D_i + w\beta_i - \beta_i p_i + h_i \int_0^{\frac{q_i}{R}} \frac{\beta_i (q_i - Rl_i)}{R} f_L(l_i) dl_i + c_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i \beta_i (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i \\ &+ h_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i (q_i - Rl_i) [q_i R + D_i (q_i - Rl_i)]}{q_i R^2} f_L(l_i) dl_i - c_i \int_{\frac{q_i(R+D_i)}{RD_i}}^{\infty} \frac{\beta_i (q_i - Rl_i)}{R} f_L(l_i) dl_i \\ &+ \frac{(A_i + nF)\beta_i}{Q_i} \end{aligned} \quad (8.17)$$

$$\frac{\partial^2 AEP_i}{\partial p_i^2} = -2\beta_i - (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i \quad (8.18)$$

Since β_i, h_i and c_i all are positive, therefore, it implies that $\frac{\partial^2 AEP_i}{\partial p_i^2} < 0$ only if

$$2\beta_i + (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i > 0.$$

Therefore, the average expected profit function AEP_i is concave in p_i for given q_i if the above condition satisfies.

Solution Algorithm

Taking the best response from the manufacturer, the average expected profit of the i -th retailer can be optimized using the following solution algorithm. To optimize the average expected profit of the i -th retailer, we consider initial guess values to the decision variables of the remaining $(N - 1)$ retailers.

Step 1: Set $k = 1$.

Step 2: Set $i = 1$ and $q_j = q_j^{(k-1)}, p_j = p_j^{(k-1)}$ for all $j = i + 1, i + 2, \dots, N$.

Step 3: Optimize AEP_i taking n and θ from the response functions of the manufacturer and $q_j = q_j^{(k-1)}, p_j = p_j^{(k-1)}$ for all $j = i + 1, i + 2, \dots, N$. Set the optimal results as $q_i = q_i^{(k)}$ and $p_i = p_i^{(k)}$.

Step 4: Set $i = i + 1$.

Step 5: Optimize AEP_i taking n and θ from the manufacturer's response functions and $q_j = q_j^{(k)}$, $p_j = p_j^{(k)}$ for $j = 1, 2, \dots, i - 1$ and $q_j = q_j^{(k-1)}$, $p_j = p_j^{(k-1)}$ for $j = i + 1, i + 2, \dots, N$. Set the optimal results as $q_i = q_i^{(k)}$ and $p_i = p_i^{(k)}$.

Step 6: Repeat steps 4 and 5 until $i = N$.

Step 7: Stop if $q_j^{(k)} = q_j^{(k-1)}$ and $p_j^{(k)} = p_j^{(k-1)}$ for all $j = 2, 3, \dots, N$ and consider $q_j^{(*)} = q_j^{(k)}$ and $p_j^{(*)} = p_j^{(k)}$ for all $j = 1, 2, 3, \dots, N$. Otherwise, set $k = k + 1$ and repeat steps 2 to 6.

Step 8: Evaluate the optimal values of n^* and θ^* taking q_j^* and p_j^* for all $j = 1, 2, \dots, N$.

Step 9: Using these results, calculate optimal values of AEP_m and AEP_s .

8.4.2 Centralized Model

In this scenario, the manufacturer and all the retailers of the supply chain act jointly as a single decision maker. They determine the optimal selling prices of the product, greening improvement level, number of shipments and batch sizes in order to maximize the entire system profit rather than focusing on their individual profits. The average expected profit of the supply chain is

$$\begin{aligned}
 AEP_s(n, \theta, q_i, p_i) = & \sum_{i=1}^N p_i D_i - \frac{A_m D}{Q} - h_m \left[\frac{D \sum_{i=1}^N q_i}{R} + \frac{Q}{2} \left(1 - \frac{D}{R} \right) - \sum_{i=1}^N \frac{q_i^2}{2Q_i} \right] - \sum_{i=1}^N \frac{h_m D_i \sigma_i}{\sqrt{2\pi}} - I\theta^2 \\
 & - \sum_{i=1}^N \left[\frac{(A_i + nF)D_i}{Q_i} + \frac{h_i}{2} \left[\int_0^{r_i} [q_i + 2(r_i - D_i l_i)] f_L(l_i) dl_i \right. \right. \\
 & + \int_{\frac{r_i}{D_i}}^{\frac{r_i + q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{q_i} f_L(l_i) dl_i \left. \left. + \frac{c_i}{2} \left[\int_{\frac{r_i}{D_i}}^{\frac{r_i + q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{q_i} f_L(l_i) dl_i \right. \right. \right. \\
 & \left. \left. + \int_{\frac{r_i + q_i}{D_i}}^{\infty} [q_i + 2(D_i l_i - q_i - r_i)] f_L(l_i) dl_i \right] \right] \quad (8.19)
 \end{aligned}$$

Proposition 8.5 The average expected profit of the system is concave in n for given θ , q_i , and p_i if $h_m s^2 < 2(A_m D + s g)$, where $g = \sum_{i=1}^N \frac{A_i D_i}{q_i}$ and the optimal number of shipments is given by

$$n^* = \sqrt{\frac{R(2A_m D - h_m s^2 + 2s g)}{h_m s^2(R - D)}} \quad (8.20)$$

Proof. Considering n as real, from equation (8.19), we derive the following partial derivatives

$$\frac{\partial AEP_S}{\partial n} = \frac{A_m D}{n^2 s} - \frac{h_m s}{2} + \frac{h_m s D}{2R} - \frac{h_m s}{2n^2} + \frac{g}{n^2} \quad (8.21)$$

$$\frac{\partial^2 AEP_S}{\partial n^2} = -\frac{2A_m D}{n^3 s} + \frac{h_m s}{n^3} - \frac{2g}{n^3} \text{ where } s = \sum_{i=1}^N q_i \quad (8.22)$$

The average expected profit of the system will be concave in n for given θ , q_i , and p_i , if $\frac{\partial^2 AEP_S}{\partial n^2} < 0$, which implies $h_m s^2 < 2(A_m D + sg)$.

If the above condition holds then the system profit function attains the maximum value with respect to n , and the optimal value of n can be obtained by using the first order optimality condition i.e., $\frac{\partial AEP_S}{\partial n} = 0$. Solving it for n , one can get the optimal number of shipments as $n^* = \sqrt{\frac{R(2A_m D - h_m s^2 + 2sg)}{h_m s^2(R - D)}}$.

For integer optimal value of n ,

$$n^{opt} = \begin{cases} \lfloor n^* \rfloor, & \text{if } AEP_S(\lfloor n^* \rfloor, \theta, q_i, p_i) \geq AEP_S(\lceil n^* \rceil, \theta, q_i, p_i) \\ \lceil n^* \rceil, & \text{if } AEP_S(\lfloor n^* \rfloor, \theta, q_i, p_i) \leq AEP_S(\lceil n^* \rceil, \theta, q_i, p_i) \end{cases} \quad (8.23)$$

Proposition 8.6 For given values of n , q_i and p_i , the average expected profit function of the supply chain is concave in θ if

$$2I + \sum_{i=1}^N (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{(q_i - Rl_i)^2 \alpha_i^2}{q_i R^2} f_L(l_i) dl_i > 0.$$

Proof. From equation (8.19), we get

$$\begin{aligned} \frac{\partial AEP_S}{\partial \theta} &= \sum_{i=1}^N p_i \alpha_i - \left[\frac{A_m}{Q} + \frac{h_m s}{R} - \frac{h_m Q}{2R} + \left(\sum_{i=1}^N \frac{h_m \sigma_i}{\sqrt{2\pi}} \right) \right] u - 2I\theta - \sum_{i=1}^N \left[\frac{(A_i + nF)\alpha_i}{Q_i} \right. \\ &+ h_i \int_0^{\frac{q_i}{R}} \frac{\alpha_i (q_i - Rl_i)}{R} f_L(l_i) dl_i + h_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{[q_i R + D_i (q_i - Rl_i)] \alpha_i (q_i - Rl_i)}{q_i R^2} f_L(l_i) dl_i \\ &\left. + c_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{(q_i - Rl_i)^2 \alpha_i D_i}{q_i R^2} f_L(l_i) dl_i - c_i \int_{\frac{q_i(R+D_i)}{RD_i}}^{\infty} \frac{\alpha_i (q_i - Rl_i)}{R} f_L(l_i) dl_i \right] \end{aligned} \quad (8.24)$$

$$\frac{\partial^2 AEP_S}{\partial \theta^2} = -2I - \sum_{i=1}^N (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{(q_i - Rl_i)^2 \alpha_i^2}{q_i R^2} f_L(l_i) dl_i \quad (8.25)$$

It can be easily seen that, $\frac{\partial^2 AEP_S}{\partial \theta^2} < 0$ if $2I + \sum_{i=1}^N (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{(q_i - Rl_i)^2 \alpha_i^2}{q_i R^2} f_L(l_i) dl_i > 0$. Therefore, we can conclude that the average expected profit function of the supply chain system is concave in θ , for given values of n , q_i and p_i , if this condition holds.

Proposition 8.7 The average expected profit function of the supply chain system is concave in q_i for given n , θ and p_i if $\frac{2A_m D}{s^3} + \frac{2(A_i + nF)D_i}{q_i^3} + n(h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2 l_i^2}{q_i^3} f_L(l_i) dl_i > 0$.

Proof. Differentiating equation (8.19) with respect to q_i , we get

$$\begin{aligned} \frac{\partial AEP_S}{\partial q_i} &= \frac{(A_i + nF)D_i}{nq_i^2} - \frac{h_i}{2} \int_0^{\frac{q_i}{R}} \frac{(R + 2D_i)}{R} f_L(l_i) dl_i + \frac{c_i}{2} \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \left(\frac{R^2 D_i^2 l_i^2 - q_i^2 D_i^2}{q_i^2 R^2} \right) f_L(l_i) dl_i \\ &\quad - \frac{h_i}{2} \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \left(\frac{q_i^2 (R + D_i)^2 - R^2 D_i^2 l_i^2}{q_i^2 R^2} \right) f_L(l_i) dl_i + \frac{c_i}{2} \int_{\frac{q_i(R+D_i)}{RD_i}}^{\infty} \frac{(R + 2D_i)}{R} f_L(l_i) dl_i \\ &\quad - \frac{h_m D}{R} + \frac{A_m D}{ns^2} - \frac{nh_m}{2} \left(1 - \frac{D}{R} \right) + \frac{h_m}{2n} \end{aligned} \quad (8.26)$$

$$\begin{aligned} \frac{\partial^2 AEP_S}{\partial q_i^2} &= -\frac{2A_m D}{ns^3} - \frac{2(A_i + nF)D_i}{nq_i^3} - h_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2 l_i^2}{q_i^3} f_L(l_i) dl_i \\ &\quad - c_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2 l_i^2}{q_i^3} f_L(l_i) dl_i \end{aligned} \quad (8.27)$$

Now, the average expected profit of the entire supply chain is concave in q_i , for given n , θ and p_i , if $\frac{\partial^2 AEP_S}{\partial q_i^2} < 0$, which gives

$$\frac{2A_m D}{s^3} + \frac{2(A_i + nF)D_i}{q_i^3} + n(h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{D_i^2 l_i^2}{q_i^3} f_L(l_i) dl_i > 0.$$

Proposition 8.8 The average expected profit function of the supply chain system is concave in p_i for given n , θ , and q_i if $2\beta_i + (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i > 0$.

Proof. Differentiating equation (8.19) with respect to p_i , we get

$$\begin{aligned} \frac{\partial AEP_S}{\partial p_i} &= D_i - \beta_i p_i + \frac{(A_i + nF)\beta_i}{Q_i} + h_i \int_0^{\frac{q_i}{R}} \frac{\beta_i (q_i - Rl_i)}{R} f_L(l_i) dl_i + c_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{(q_i - Rl_i)^2 \beta_i D_i}{q_i R^2} f_L(l_i) dl_i \\ &\quad + h_i \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{[q_i R + D_i (q_i - Rl_i)] \beta_i (q_i - Rl_i)}{q_i R^2} f_L(l_i) dl_i - c_i \int_{\frac{q_i(R+D_i)}{RD_i}}^{\infty} \frac{\beta_i (q_i - Rl_i)}{R} f_L(l_i) dl_i \\ &\quad + \frac{A_m \beta_i}{Q} + \frac{h_m \beta_i s}{R} - \frac{h_m Q \beta_i}{2R} + \frac{h_m \beta_i \sigma_i}{\sqrt{2\pi}} \end{aligned} \quad (8.28)$$

$$\frac{\partial^2 AEP_S}{\partial p_i^2} = -2\beta_i - (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i \quad (8.29)$$

The average expected profit function of the entire supply chain is concave in p_i , for given n , θ , and q_i , if $\frac{\partial^2 AEP_S}{\partial p_i^2} < 0$ i.e., if $2\beta_i + (h_i + c_i) \int_{\frac{q_i}{R}}^{\frac{q_i(R+D_i)}{RD_i}} \frac{\beta_i^2 (q_i - Rl_i)^2}{q_i R^2} f_L(l_i) dl_i > 0$.

8.4.3 Coordinated Model

Supply chain players agree to accept the joint-decision making policy only if it provides a better profit than the decentralized model scenario. To motivate supply chain members to make integrated decisions, an incentive strategy is required. In this section, we propose a coordination mechanism between the manufacturer and the retailers, which motivates the members to accept the integrated decision making policy. In this coordination mechanism, the manufacturer requests the retailers to decide their optimal batch sizes (q_i) and retail prices (p_i) according to the centralized policy and, in return, the manufacturer decreases his wholesale price (w).

Suppose that the manufacturer offers the price discount scheme to the i -th retailer as follows:

$$w = \begin{cases} w, & \text{if } q_i < q_i^c \\ w(1 - \phi_i), & \text{if } q_i \geq q_i^c \end{cases} \quad (8.30)$$

For this price discount scheme, the average expected profit of the i -th retailer becomes

$$\begin{aligned} AEP_i^{co}(q_i^c, p_i^c, \phi_i) &= p_i^c D_i^c - (1 - \phi_i)wD_i^c - \frac{(A_i + n^c F)D_i^c}{Q_i^c} \\ &- \left[\frac{h_i}{2} \int_0^{\frac{r_i}{D_i^c}} [q_i^c + 2(r_i - D_i^c l_i)] f_L(l_i) dl_i + h_i \int_{\frac{r_i}{D_i^c}}^{\frac{r_i + q_i^c}{D_i^c}} \frac{(q_i^c - D_i^c l_i + r_i)^2}{2q_i^c} f_L(l_i) dl_i \right] \\ &- \left[\frac{c_i}{2} \int_{\frac{r_i}{D_i^c}}^{\frac{r_i + q_i^c}{D_i^c}} \frac{(D_i^c l_i - r_i)^2}{q_i^c} f_L(l_i) dl_i + \frac{c_i}{2} \int_{\frac{r_i + q_i^c}{D_i^c}}^{\infty} [q_i^c + 2(D_i^c l_i - q_i^c - r_i)] f_L(l_i) dl_i \right] \end{aligned} \quad (8.31)$$

and the average expected profit of the manufacturer becomes

$$\begin{aligned} AEP_m^{co}(n^c, \theta^c, \phi_i) &= \sum_{i=1}^N (1 - \phi_i)wD_i^c - \frac{A_m D^c}{Q^c} - h_m \left[\frac{D^c \sum_{i=1}^N q_i^c}{R} + \frac{Q^c}{2} \left(1 - \frac{D^c}{R} \right) - \sum_{i=1}^N \frac{q_i^c}{2n^c} \right] \\ &- \sum_{i=1}^N \frac{h_m D_i^c \sigma_i}{\sqrt{2\pi}} - I\theta^{c2} \end{aligned} \quad (8.32)$$

Proposition 8.9 The minimum value of ϕ_i for which the i -th retailer accepts the coordination mechanism is

$$\phi_i^{min} = \frac{(p_i^d D_i^d - p_i^c D_i^c) - w(D_i^d - D_i^c) - \Delta^d + \Delta^c}{wD_i^c}, \text{ where,}$$

$$\begin{aligned} \Delta = & \frac{(A_i + nF)D_i}{Q_i} + \frac{h_i}{2} \left[\int_0^{\frac{r_i}{D_i}} [q_i + 2(r_i - D_i l_i)] f_L(l_i) dl_i + \int_{\frac{r_i}{D_i}}^{\frac{r_i + q_i}{D_i}} \frac{(q_i - D_i l_i + r_i)^2}{q_i} f_L(l_i) dl_i \right] \\ & + \left[\frac{c_i}{2} \int_{\frac{r_i}{D_i}}^{\frac{r_i + q_i}{D_i}} \frac{(D_i l_i - r_i)^2}{q_i} f_L(l_i) dl_i + \frac{c_i}{2} \int_{\frac{r_i + q_i}{D_i}}^{\infty} [q_i + 2(D_i l_i - q_i - r_i)] f_L(l_i) dl_i \right] \end{aligned}$$

Proof. The retailer's goal in engaging in the coordination is to find the minimum discount level so that his profit becomes more or equal to the profit in the decentralized case. So,

$$AEP_i^{co}(q_i^c, p_i^c, \phi_i) \geq AEP_i^d(q_i^d, p_i^d) \quad (8.33)$$

Solving the inequality (8.33), we get

$$\phi_i \geq \frac{(p_i^d D_i^d - p_i^c D_i^c) - w(D_i^d - D_i^c) - \Delta^d + \Delta^c}{wD_i^c} \quad (8.34)$$

Therefore, if the wholesale price discount offered by the manufacturer does not satisfy the above condition, the i -th retailer will not accept the contract. So, to motivate the i -th retailer, the manufacturer should give at least ϕ_i discount level given by

$$\phi_i^{min} = \frac{(p_i^d D_i^d - p_i^c D_i^c) - w(D_i^d - D_i^c) - \Delta^d + \Delta^c}{wD_i^c}. \quad (8.35)$$

Proposition 8.10 The maximum discount level offered by the manufacturer to the i -th retailer is given by

$$\phi_i^{max} = \frac{w(D_i^c - D^d) - \nabla^c + \nabla^d + \sum_{j=1, j \neq i}^N (1 - \phi_j) w D_j^c}{w D_i^c} \quad (8.36)$$

$$\text{where, } \nabla = \frac{A_m D}{Q} + h_m \left[\frac{D \sum_{i=1}^N q_i}{R} + \frac{Q}{2} \left(1 - \frac{D}{R} \right) - \sum_{i=1}^N \frac{q_i}{2n} \right] + \sum_{i=1}^N \frac{h_m D_i \sigma_i}{\sqrt{2\pi}} + I\theta^2$$

Proof. The manufacturer will offer the price discount scheme if he/she gains more profit after giving price discount to all the retailers in this coordination than the decentralized scenario. So, if the manufacturer provides a ϕ_i discount level to the i -th retailer, then

$$AEP_m^{co}(n^c, \theta^c, \phi_i) \geq AEP_m^d(n^d, \theta^d) \quad (8.37)$$

Simplifying (8.37), we get,

$$\phi_i \leq \frac{w(D_i^c - D^d) - \nabla^c + \nabla^d + \sum_{j=1, j \neq i}^N (1 - \phi_j)wD_j^c}{wD_i^c} \quad (8.38)$$

So, if the manufacturer gives $\phi_i\%$ price discount to the i -th retailer, then the maximum allowable discount level for the manufacturer will be

$$\phi_i^{max} = \frac{w(D_i^c - D^d) - \nabla^c + \nabla^d + \sum_{j=1, j \neq i}^N (1 - \phi_j)wD_j^c}{wD_i^c}.$$

From Propositions (8.9) and (8.10), it can be observed that, the i -th retailer will accept the discount offer for all $\phi \geq \phi_i^{min}$. Therefore, all the N retailers will accept the discount scheme (8.30) if $\phi \geq \phi_{min}$, where $\phi_{min} = \max\{\phi_1^{min}, \phi_2^{min}, \phi_3^{min}, \dots, \phi_N^{min}\}$ and the manufacturer will provide this price discount only if

$\phi \leq \phi_{max} = \min\{\phi_1^{max}, \phi_2^{max}, \phi_3^{max}, \dots, \phi_N^{max}\}$. Hence, for all ϕ in $[\phi_{min}, \phi_{max}]$ the coordination through the price discount scheme (8.30) will result better profit level for both the manufacturer and the retailers than the decentralized scenario. Since the manufacturer sells the product to all the retailers at the same wholesale price, therefore, we assume that he/she offers the same price discount ratio ϕ to each retailer.

8.5 Numerical Analysis

In this section, we consider three numerical examples to analyze the behaviour of our proposed model and its applicability. Here we focus on the scenario where one manufacturer is trading with two retailers.

Example 1: The following set of parameter-values are considered to demonstrate the proposed model numerically. As it is difficult to get access to the actual industrial data, some of the parameter-values are taken from Hoque, 2013 and the rest are hypothetical.

$R = 3000$; $A_m = 400$; $\sigma_1 = 0.12$; $w = 100$; $a_1 = 1500$; $a_2 = 1500$; $\sigma_2 = 0.13$; $F = 10$;
 $h_1 = 5.8$; $h_2 = 5$; $\beta_1 = 4$; $\alpha_1 = 2$; $\alpha_2 = 1.5$; $h_m = 3.5$; $c_1 = 7$; $\beta_2 = 4.5$; $c_2 = 7$;
 $A_1 = 40$; $A_2 = 45$; $N = 2$; $I = 40$. (with appropriate units)

It is also observed from Table 8.1 that the channel coordination can be achieved through the price discount mechanism between the manufacturer and the retailers. The optimal results of the models reflect that embracing the price discount coordination mechanism boosts not only the total system profit but also the profits of individual supply chain members. For the first retailer, the minimum discount ratio to undertake the coordination mechanism is obtained as 0.12 and for the second retailer, it is 0.14. It is clear that, if the manufacturer offers 12% discount, the first retailer will accept the offer but the second retailer will not, as it will cause a loss to him. Therefore, to motivate both the retailers for participating in the coordination, the manufacturer has to give at least 14% price discount. Again, the maximum discount ratio for which the manufacturer does not face any loss is obtained as 0.27. So, the manufacturer can offer each retailer a maximum discount of 27%.

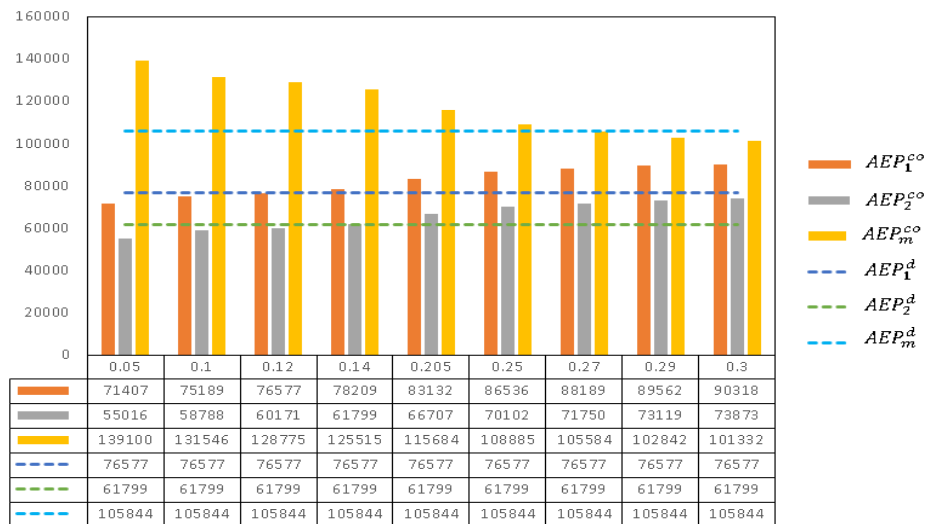


Figure 8.5: Price discount rate vs. average expected profit

Therefore, the win-win situation which occurs in the interval $[\phi_{min}, \phi_{max}]$ is appeared as $[0.14, 0.27]$. For any value of ϕ in this interval, the price discount mechanism becomes profitable for the manufacturer and both the retailers than the decentralized scenario. For ϕ lying in the interval $[0.14, 0.27]$, the average expected profits of the first and second retailers vary within the intervals $[\$78215, \$88049]$ and $[\$61799, \$71610]$, respectively and the average expected profit of the manufacturer lies within the interval $[\$105584, \$125503]$. In all cases, the average expected system profit remains $\$265523$ i.e., our suggested coordination method effectively achieves channel coordination and results in the supply chain members sharing extra profit

that occurs in the centralized scenario. Naturally, whenever the value of ϕ increases from ϕ_{min} , the retailers profitability increases gradually and attains their maximum profits at ϕ_{max} while the manufacturer's profit decreases, and at ϕ_{max} , the manufacturer attains the same profitability as that of the decentralized case. This fact is plotted in Figure 8.5. The supply chain members can fix the value of ϕ through bargaining. Here we take the value of ϕ as the mean of the feasible interval $[0.14, 0.27]$ i.e., 0.205.

Example 2: We consider the following set of parameter values to demonstrate the model, and the optimal results thus obtained are provided in Table 8.2.

$R = 4000; A_m = 500; \sigma_1 = 0.12; w = 90; a_1 = 2000; a_2 = 1800; \sigma_2 = 0.13; F = 15; h_1 = 6; h_2 = 5.5; \beta_1 = 4.2; \alpha_1 = 3; \alpha_2 = 2.5; h_m = 3; c_1 = 7.4; \beta_2 = 5; c_2 = 7.4; A_1 = 50; A_2 = 50; N = 2; I = 30.$

Table 8.2 shows that, when compared to a decentralized system, integrated decision making provides higher supply chain profit. Both the order quantity of each retailer as well as the product's greening improvement level increase in the centralized scenario compared to the decentralized scenario. In addition, the product's retail price falls at both the retailers. As a consequence, customers are enticed by a greener product at a lesser cost, which significantly increases market demand. In the coordinated model, the minimum wholesale price discount ratios for the two retailers are obtained as 2% and 7%, respectively, while the maximum allowable price discount ratio for the manufacturer is 17%. Therefore, for any price discount lying in the interval $[7\%, 17\%]$, a win-win situation arises, i.e., the wholesale price contract benefits every member of the supply chain. The value of ϕ is taken as the mean of this feasible interval $[7\%, 17\%]$ i.e., 12%.

Table 8.2: Optimal results for different models

Models	n^*	θ^*	q_1^* (unit)	q_2^* (unit)	p_1^* (\$/unit)	p_2^* (\$/unit)	ϕ_{min}	ϕ_{max}	ϕ	AEP_1^* (\$/unit time)	AEP_2^* (\$/unit time)	AEP_m^* (\$/unit time)	AEP_s^* (\$/unit time)
Decentralized	4	8.22	132.42	161.82	286.18	227.19	-	-	-	161024	93638	131443	386105
Coordinated	5	19.94	112.12	124.16	245.58	185.35	0.07	0.17	0.12	170783	97731	140097	408611
Centralized	5	19.94	112.12	124.16	245.58	185.35	-	-	-	-	-	-	408611

From Table 8.2, it can be noticed that, by accepting the wholesale price discount contract, the profits of the two retailers are increased by 6% and 4%, respectively.

Furthermore, the manufacturer earns about 7% more profit from this contract.

8.5.1 A comparative study

In this section, we attempt to compare the findings of our study to some previous research. Sarkar et al., 2020 developed a single-vendor single-buyer model with equal-sized batch shipment policy and price-dependent demand in this direction. They did, however, take into account variable backorder and the inspection process, that are not considered in this study. Furthermore, their model didn't take into account for stochastic lead time and greening investment. To compare the proposed model to Sarkar et al., 2020, common parameter values from Sarkar et al., 2020 are used, while the remaining parameter values are chosen at random. The proposed model is compared to Sarkar et al., 2020 model in two different situations: without greening investment and with greening investment. The parameter values considered are given as follows:

$$a_1 = 11000; A_m = 200; A_1 = 20; w = 10; F = 5; h_m = 2; h_1 = 5; c_1 = 7.5; \beta_1 = 320; N = 1; \alpha_1 = 3; \sigma_1 = 0.02; I = 80; \frac{D}{R} = 0.4$$

For the case of without green sensitivity of the customer demand, we set, $\alpha_1 = 0, I = 0$. Figure 8.6 shows a comparative graphical representation of the average expected supply chain profit. Their centralized model obtains optimal batch size as 279 units, optimal number of shipments as 5, optimal retail price as \$17 and optimal profit of the entire supply chain as \$92021. Whereas our proposed model without green investment results the optimal batch size as 155.2 units, optimal number of shipments as 9, optimal retail price as \$17.30, and the average expected supply chain profit as \$92287. Furthermore, the proposed model with stochastic lead time and greening investment provides the optimal batch size as 223.26 units, optimal number of shipments as 8, optimal retail price as \$28.24, optimal green level as 35.07 and the average expected supply chain profit as \$152186. As a conclusion of the above numerical results, it is apparent that adding the stochastic lead time and greening investment strategy makes the supply chain significantly more profitable.

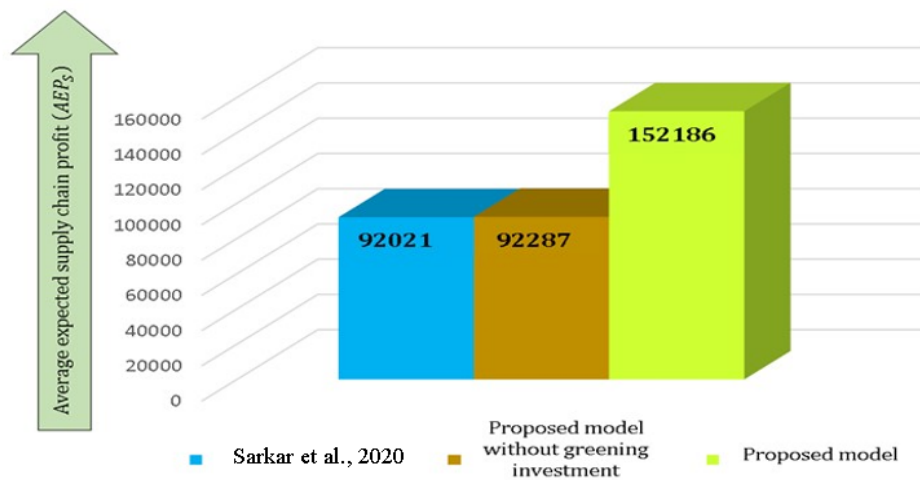


Figure 8.6: Comparison with existing literature

8.6 Sensitivity Analysis

In order to explore the impact of model parameters on the optimal decisions as well as the average expected system profit, in this section we vary one parameter-value at a time while keeping other parameter-values unchanged in Example 1. The results are shown in Table 8.3 from which the following conclusions can be drawn:

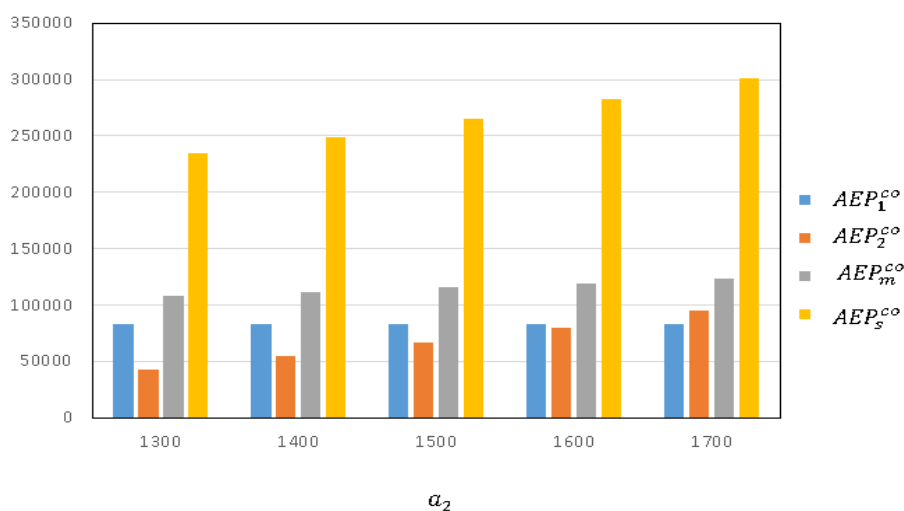


Figure 8.7: Average expected profit vs. a_2

From Table 8.3 and Figure 8.7, a significant change in overall profit of the system under the price discount coordination mechanism is observed for higher basic market

demand. The first retailer can charge a higher price for the product whenever the customer demand increases at his side. This is because the first retailer compensates the effect of higher price by the higher market demand. He places order for more quantity from the manufacturer. Consequently, the profit of the first retailer as well as the manufacturer increases significantly. The changes in the order quantity and retail price of the product for the second retailer are almost negligible. As a result, the overall profit of the system increases. Similar scenario occurs whenever the market demand increases at the second retailer's side.

Table 8.3: Sensitivity analysis of the parameters a_1 , β_1 , α_1 and F

Parameters	Values	n^*	θ^*	q_1^* (unit)	q_2^* (unit)	p_1^* (\$/unit)	p_2^* (\$/unit)	AEP_1^{co} (\$/unit time)	AEP_2^{co} (\$/unit time)	AEP_m^{co} (\$/unit time)	AEP_S^{co} (\$/unit time)
a_1	1300	6	7.24	65.20	78.88	164.687	168.26	55569	66592	108057	230219
	1400	6	7.55	68.50	79.41	177.253	168.301	68719	66649	111879	247242
	1500	6	7.87	71.77	79.96	189.82	168.34	83132	66707	115684	265523
	1600	6	8.19	75.10	80.55	202.387	168.383	98812	66765	119486	285065
	1700	6	8.50	78.20	81.17	214.955	168.424	115761	66823	123285	305865
β_1	3	6	9.47	71.90	80.07	253.508	168.607	131649	67014	114825	313488
	3.5	6	8.55	71.83	80.01	217.082	168.455	103873	66838	115343	286055
	4	6	7.87	71.77	79.96	189.82	168.34	83132	66707	115684	265523
	4.5	6	7.34	71.58	79.57	168.652	168.253	67049	66605	115917	249571
	5	6	6.92	71.48	79.90	151.736	168.183	54229	66524	116088	236841
α_1	1.5	6	6.67	71.58	79.87	189.104	168.143	82276	66478	116083	264837
	2	6	7.87	71.77	79.96	189.82	168.34	83132	66707	115684	265523
	2.5	6	9.08	72.01	80.07	190.69	168.543	84177	66940	115210	266327
	3	6	10.31	72.27	80.18	191.718	168.747	85419	67177	114654	267250
	3.5	6	11.56	72.58	80.30	192.908	168.954	86868	67418	114012	268298
F	0	11	7.87	38.99	43.67	189.752	168.282	83199	66751	115816	265766
	10	6	7.87	71.77	79.96	189.82	168.34	83132	66707	115684	265523
	20	4	7.87	97.95	111.89	189.89	168.41	83084	66680	115588	265352
	30	4	7.87	103.46	116.70	189.911	168.421	83022	66626	115563	265211
	40	3	7.87	124.43	144.46	189.98	168.48	82983	66605	115496	265084

Figure 8.8 depicts how the price sensitivity of the consumer demand affects the decision variables and the system profit. The figure shows that when the price sensitivity of consumer demand increases, the greater price of the product influences the customers' choice of alternatives. As a result, if customer demand becomes more price sensitive, the corresponding retailers lower their product prices to meet market demand, reducing the product's greenness. Figure 8.8 and Table 8.3 show that, under the coordination scheme, the average expected profit of both retailers and the total system profit decrease at a decreasing rate as price elasticity increases.

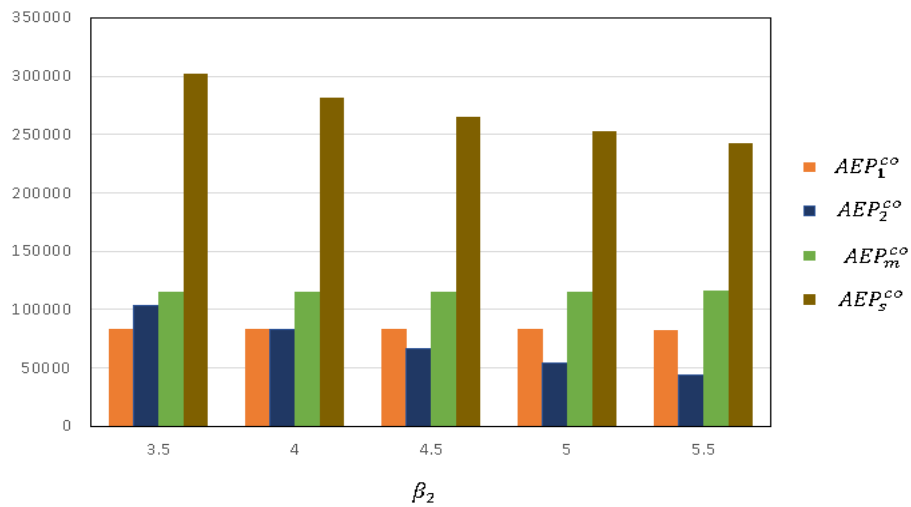


Figure 8.8: Average expected profit vs. β_2

Table 8.3 shows the effect of customers' environmental awareness on optimal decisions and supply chain members' profitability. When the values of α_1 and α_2 increase, customers are more concerned about the environmental performance of the product and they are willing to spend more for environmentally friendly products. In such a scenario, to satisfy the customers' requirement, the manufacturer increases the greening level of the product. This fact is presented in Figure 8.9.

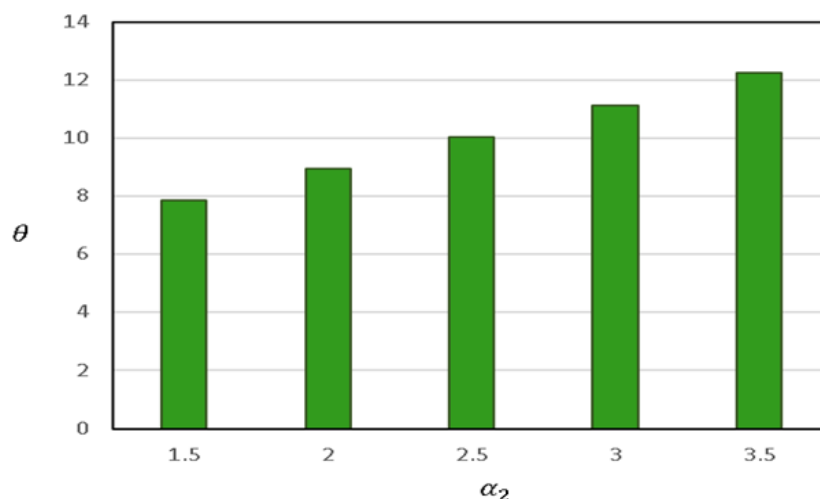


Figure 8.9: Product's greening level vs. α_2

However, the higher greening level increases the expense of the manufacturer. So, the average expected profit of the manufacturer gradually decreases. On the other hand, the retailers can enhance the retail price of the product and achieve higher

profitability with higher greener product. It is further observed that the average expected system profit increases for greater values of α_1 and α_2 .

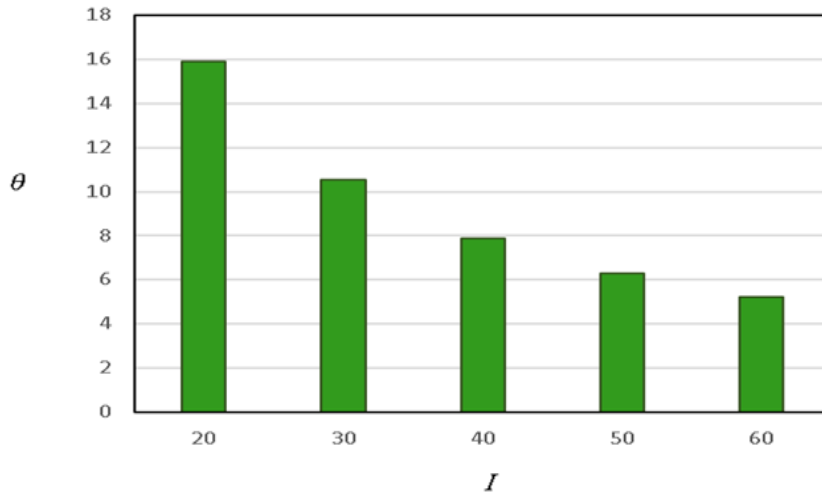


Figure 8.10: Product's greening level vs. I

Table 8.3 illustrates the effect of the greening investment on the coordinated profit of the supply chain members and the profit of the entire supply chain. Figure 8.10 shows that the product's greening level falls rapidly while I increases. When I increases, the manufacturer produces lower greener product in order to curb his expenditure but it makes a negative impact on customer demand. So, for higher I , the profitability of the retailers decreases and the average expected profit of the entire supply chain also decreases gradually.

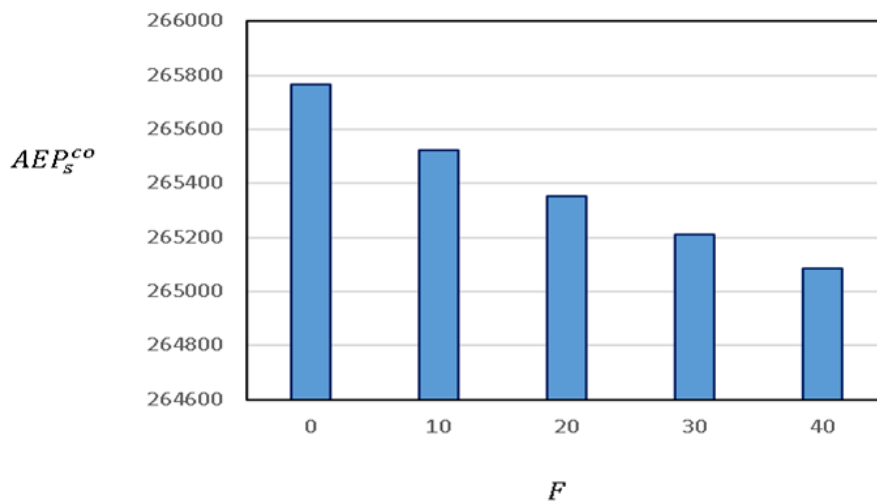


Figure 8.11: Average expected profit vs. F

The effect of the transportation cost is found to be negligible on the supply chain's profitability. If we ignore the transportation cost then the optimal number of shipments is obtained as 11. As the transportation cost increases, the optimal number of shipments declines from 11. Figure 8.11 reflects that the system profit decreases at a diminishing rate as F increases.

8.7 Concluding Remarks

In this chapter, we have designed a two-level supply chain model consisting of a single manufacturer and multiple retailers. To develop a realistic model, the lead time between placing an order and receiving its delivery is taken to be stochastic in nature. The retailers face a price sensitive demand from the end customers. The customer demand is also affected by the greening improvement level of the product as determined by the manufacturer. We have studied the decentralized model where supply chain members optimize their own profits without worrying about the profit of others. Stackelberg gaming approach is used where the retailers are assumed to act as the leader and the manufacturer as the follower. A solution algorithm is suggested to find the optimal solution of the proposed model. The performance of the whole supply chain is also investigated under integrated decision-making model. Though the entire supply chain experiences a better economical and environmental performances in the centralized scenario but it may not be beneficial for all the members individually. Since the retail price of the product is decided by the retailers and the manufacturer determines the greening level of the product, and both these factors influence the customers' demand, it is therefore essential to make these decisions in an efficient and coordinated manner which enriches the profit levels of each members. A price discount mechanism has been proposed to convince the supply chain members to make decisions in a coordinated manner. The maximum and minimum satisfactory discount rates are found so that all the members become interested for participating in this price discount coordination. This coordination mechanism is effective in both cases whether the market demand is high or low. From the numerical study and sensitivity analysis of our proposed model, the following key managerial insights are derived:

(1) Business managers can improve the sales volume and economic efficiency by

adopting green manufacturing technologies and suitable coordination scheme. Though, from the point of view of social welfare, it is always desirable to produce green products, the firms should estimate the profitable growth before adopting green manufacturing. From the outcomes of sensitivity analysis, it is evident that environmental awareness of the consumers and greening investment play a crucial role in the profitability of the supply chain members. By participating in the proposed price discount coordination mechanism, the business managers can improve the greening quality of the product to a remarkable higher level. It not only increases their profits but also maintains their social responsibility and increases their reputation in the business market for adopting such green initiative.

(2) The proposed price discount scheme is capable of coordinating the supply chain. Under this mechanism, the manufacturer reduces his wholesale price and increases greening level of the product and encourages the retailers to set their prices and ordering quantities according to the centralized model. This improves the economic level of all members. Moreover, by participating in such coordination, the end customers get more eco-friendly product at a cheaper price than if they used an individual optimization strategy. Retailers should also remember that when consumers' price sensitivity is too high, they should lower their sales price to retain profitability.

(3) The business managers may not always agree to adopt joint decision-making process even though it yields higher profit for the entire supply chain but it may not be profitable for all the chain members. To convince the members to make coordination, such price discount scheme is very effective as in this scheme increment of each member's profitability is guaranteed. All the members could enjoy the coordination agreement as it is beneficial both socially and economically.

(4) The delivery of the order quantity may not reach to the retailer's end in time due to various reasons such as variation in transportation time, inspection time, loading and unloading times, etc. Therefore, the business managers should understand the stochastic nature of the lead time and account for all possibilities of early arrival, on time arrival, and late arrival to conduct the business efficiently.

There are some limitations and the present model can be extended in many directions to further enhance the scope of study. It is widely adopted in the literature but the policy of equal sized batch shipment is very limited in nature, and it may not

be always possible to supply the order quantities of all the retailers in some integer number of equal sized batches. So, it would be more realistic to consider a combined equal and unequal sized batch shipment policy (Hoque, 2013). Another limitation of this model is that it is based on a single product being traded between the manufacturer and the retailers. To simulate a real-world scenario, it can be expanded to include many items (Barman et al., 2021) and multiple manufacturers. Another shortcoming of our study is the consideration of complete backlogging strategy. It is desirable to consider partially backlogging of shortages for a more realistic approach (Duary et al., 2022). In our model, we have considered constant production rate and perfect production system at the manufacturer. One can enrich the study by taking into account variable production rate (Sarkar et al., 2018) and/or imperfect production system (Sepahri and Gholamian, 2023). The competition between the retailers will be another interesting research idea (Mondal and Giri, 2022). Our developed model can be modified by considering bargaining between manufacturer and retailers to share the profits among all the members (Nouri et al., 2018). In this chapter, we have proposed a price discount coordination scheme. It would be interesting to employ other contracts such as greening cost sharing contract between the manufacturer and the retailers. Consideration of set up cost reduction investment (Sarkar et al., 2017), and promotional effort (Ebrahimi et al., 2019) would also be fruitful extensions of this model.

CHAPTER 9

Conclusion and future research prospects

Expeditious globalization, population expansion, public health and safety concerns, labour disputes, and most importantly, a fiercely competitive corporate environment, have compelled business managers to re-evaluate their prevailing traditional strategies and take environmental thinking into account. In this direction, this thesis has addressed several pivotal issues faced by organizations such as optimal pricing and ordering strategy, batch shipment policy, imperfect production, lead time, back-order quantity, coordination, greening investment, and so on.

In order to achieve environmental sustainability and reduce carbon footprint, a strategic defect management policy has been introduced in the Chapter 3. Moreover, a trade credit mechanism and a price discount contract have been implemented to coordinate the channel. Chapter 4 considers a combined equal and unequal batch shipment policy in a two-level supply chain model with a price and green sensitive demand. In this chapter, we have proposed a cost sharing contract which effectively enhances both economic and environmental performance of supply chain members. However, these models comprise several constraints from sundry practical aspects. Firstly, we have considered the defective rate to be constant in the first model, but it would be more realistic and an immediate extension of this study to account the random defective rate in the manufacturer's production process. Secondly, apart from

a product's retail price and greening improvement level, advertising is also among the most significant aspects influencing modern market demand. We have dealt with these limitations in the next chapter. We have discussed an imperfect manufacturing system with an unequal batch shipment policy in Chapter 5. The impact of the product's retail price, advertising, and greening level on customer demand has been studied and the defective rate considered to follow a uniform distribution. Consideration of an advertisement cost sharing contract and bargaining between the manufacturer and the retailer on the sharing proportion would be an intriguing extension of this model.

Chapter 6 has investigated the effectiveness of a proactive continuous investment to reduce set-up costs and the acceptability of sample inspection over the full inspection policy. For further investigation, the proposed sample inspection policy could be modified by introducing an acceptance criterion of samples. In addition, in today's cutthroat marketplace, retaining customer satisfaction and loyalty is of utmost importance. It is heavily impacted by timely delivery of the products. Vanquishing limitations like early or late arrival of shipment and the modeling of multi-retailer business situations are therefore essential. In this regard, chapter 7 has focused on these two crucial factors: stochastic lead time and multi-retailer business scenario. Henceforth, we have extended this model by incorporating greening sensitivity of customers and implemented a price discount contract to coordinate the channel in chapter 8. Our findings demonstrate that, the inclusion of environmental awareness and the proposed coordination mechanism become advantageous for each supply chain member.

The most traditional approach to developing a supply chain model is to adopt a single manufacturer and single retailer scenario. However, in practice, this is rarely acceptable. Consequently, to be closer to reality, a multi-manufacturer and/or multi-retailer supply chain model dealing with multiple items can be established for future research purposes. Another significant flaw is the assumption of entirely backlogged shortages because there is an abundance of marketable alternatives in the market. Therefore, to be more plausible, it should be considered a partially backlogged shortage. Again, in contrast to forward logistics, reverse logistics can also be taken up for future studies. Furthermore, taking into account several realistic issues such as deteriorating items, time-dependent demand, carbon cap-and-trade policy, dual channels, etc., leads to some useful future research directions.

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List of publications

The thesis is based on the following papers:

1. **Anamika Dash**, Bibhas C. Giri, & Ashis Kumar Sarkar (2022). Coordination and defect management strategy for a two-level supply chain under price and sales effort-sensitive demand. *International Journal of Systems Science: Operations & Logistics*, 10(1), 2103198.
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Coordination and defect management strategy for a two-level supply chain under price and sales effort-sensitive demand

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ABSTRACT

This paper presents a single-manufacturer single-retailer supply chain model with an imperfect production system at the manufacturer. The manufacturer meets the order quantity of the retailer in unequal-sized shipments. After receiving each delivery, the retailer performs an inspection process that is erroneous and results in misclassification errors (Type-I and Type-II errors). Defective items are returned to the manufacturer, who conducts a rigorous screening to classify the defective items into repairable, design-flawed and non-repairable ones. The manufacturer incurs carbon emissions cost due to transportation and disposal of scrap items to get one step closer to sustainability. Customers at the retailer are influenced by retail price and sales effort. Optimal shipment policy, pricing and inventory decisions are determined in both the decentralised (based on the Stackelberg leader-follower gaming approach) and the centralised scenarios. A trade credit mechanism and wholesale price discount contract are employed and a fair profit-sharing strategy is implemented based on the supply chain member's bargaining power. The conditions for participating in these coordination mechanisms are examined from the point of view of each supply chain member. Finally, a numerical experiment along with sensitivity analysis is carried out to illustrate the applicability of the proposed coordination mechanisms and defect management policy.

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KEYWORDS

Supply chain management; imperfect production; unequal shipments; inspection error; defect management; trade credit mechanism; wholesale price discount contract

1. Introduction

The world is on the verge of catastrophe due to pollution and other regrettable anti-environmental activities. So, this is high time to take the initiative in reducing environmental degradation. In this regard, supply chain members have been enterprising in reducing their pernicious impact on the environment along with achieving a high economic level and maintaining customer satisfaction. Carbon emissions from transportation contribute significantly to global warming. Moreover, waste disposal is one of the major reasons behind environmental degradation as it releases toxic substances and greenhouse gases. Therefore, implementing sustainable strategies for transportation and waste disposal has become an inevitable task for the supply chain players in mitigating environmental impacts. Various companies all over the world have adopted eco-friendly business strategies for reducing their carbon footprint. Honda, Toyota, Ford and Valeo are some of the leading automotive industries which have embraced innovative practices to achieve sustainable goals by reducing carbon emissions. Public environmental awareness and rising raw material prices have driven companies to reduce waste. Disney has adopted a

zero-waste policy. Starbucks redesigned their cold drink cups aimed at reducing the use of plastic straws. Samsung has offered e-waste take-back programs to renovate old electronic items and reuse old parts in manufacturing new products. Adidas has run a recycling program by using the shreds of returned shoes as raw materials and alternative fuels. In the real business scenario, the production of defective items is an undeniable phenomenon. So, efficient management of these defective items will be tactful for waste reduction. Companies can reap the financial benefits of extracting reusable products from these defective products.

Demand forecasting is a crucial issue for the smooth running of a supply chain. There are several factors that significantly affect the market demand. Selling price of the product is one of them. Higher selling price can drop the demand or force customers to buy less. In addition to the selling price, the retailer's sales effort plays an important role in influencing customer demand. Retailers can enhance customer demand in many different ways, viz. hiring more skilful salespeople, increasing advertisement frequency, attractive and large shelf space, etc. Even though this sales effort attracts more

COORDINATION OF A SINGLE-MANUFACTURER MULTI-RETAILER SUPPLY CHAIN WITH PRICE AND GREEN SENSITIVE DEMAND UNDER STOCHASTIC LEAD TIME

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Abstract: *When dealing with uncertainties in supply chain and ensuring customer satisfaction, efficient management of lead time plays a significant role. Likewise, besides managing inventory and pricing strategies adeptly in multi-retailer supply chain, it has become inevitable to the firms to embrace green and sustainable business practices. In this context, this paper considers a two-level supply chain consisting of a single manufacturer and multiple retailers in which the manufacturer produces a single product and delivers it to the retailers in some equal-sized batches. Each retailer faces a price and green sensitive market demand. The lead time is assumed to be a random variable which follows a normal distribution. Shortages for retailer inventory are allowed to occur and are completely backlogged. The centralized model and a decentralized model based on leader-follower Stackelberg gaming approach are developed. A price discount mechanism between the manufacturer and retailers is proposed. For the acceptance of this contract, the upper and lower limits of the price discount rate are established. Numerical outcomes exhibit that the price discount mechanism effectively coordinates the supply chain and enhances both environmental and economical performances. A sensitivity analysis with respect to some key parameters is performed, and certain managerial insights are emphasized.*

Key words: *Two-level supply chain, multiple retailers, stochastic lead time, price and green sensitive demand, price discount mechanism.*

1. Introduction

The growing importance of environmental protection and pollution reduction has been felt all over the world in recent years. Green supply chain management aims to prevent pollution while also producing environmentally friendly products. It

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A single-vendor single-buyer supply chain model with price and green sensitive demand under batch shipment policy and planned backorder

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Abstract: This paper considers a two-level supply chain which consists of a single-vendor and a single-buyer. The vendor produces a single product in lot and transfers the whole lot to the buyer in a combination of equal and unequal sized batches. All unequal sized batches are in a geometric progression with a fixed common ratio. Consumer's demand is sensitive to the retail price and greening improvement level of the product. At the buyer, planned shortages are allowed and are completely backlogged. Assuming that the buyer has higher decision power than the vendor, we develop non-coordinated decentralised model and coordinated model with cost sharing contract, and the integrated centralised model. The numerical results show that the cost sharing contract gives an impressive improvement in the profits of both the vendor and the buyer over their profits in the non-coordinated system. A sensitivity analysis is performed to explore how the changes in the key model-parameters effect the optimal profits of the vendor and the buyer and the whole supply chain.

Keywords: two-level supply chain; greening improvement level; planned shortage; cost sharing contract.

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A Single-Manufacturer Multi-Retailer Integrated Inventory Model with Price Dependent Demand and Stochastic Lead Time

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Abstract

This paper considers a two-level supply chain which is composed of a single manufacturer and multiple retailers. The ordered quantities of the retailers are delivered in some equal sized batches from the manufacturer. Customer demand is linearly dependent on the retail price of the product. Lead time is random and it follows a normal distribution. The proposed models are developed for both the centralized and the decentralized scenarios. In the decentralized model, a Stackelberg gaming approach is used to find the optimal solution. The developed models are illustrated by a numerical example. It is found that integration of the supply chain players gives an impressive increment in profit of the whole supply chain. Sensitivity analysis is also carried out to explore the impacts of key parameters on the expected average profit of the supply chain system.

Keywords: Two-level supply chain; single manufacturer; multiple retailers; price dependent demand; stochastic lead time.


1. Introduction

The supply chain management involves with the activities for coordinating raw materials, information, production and financial flow to fulfil the customer demand with the aim of maximizing customer value and gaining competitive advantage in the market. In addition to efficient business management, supply chain spans various fields such as livestock management, waste management. Nowadays, supply chain plays an important role even in disaster management also (Khalilpourazari et al., 2020). In a supply chain, raw materials are delivered to the manufacturer from the supplier. Then produced items are transferred from the manufacturer to the retailer and finally delivered to the end customers to meet their demands. The most important factor that affects the consumer demand is the retail price of the product. The customer demand falls with the increase in retail price. So, many companies are now focusing on achieving the best pricing strategy to increase the sales volume. Many researchers (Ray and Jewkes, 2004; Banerjee, 2005; Yang et al., 2009) have also developed their models considering price-sensitive demand.

Nowadays, due to the globalization of the marketplace and rapidly increasing competition between various organisations, companies are facing many obstacles to compete exclusively. So, collaboration between different business units leads to a significant way to obtain spirited advantage. For better efficiency, the supply chain players are showing great interest in making their decisions jointly. Goyal (1976) was the first author who introduced integration between supply chain members in inventory model with finite production rate. Later, his work has encouraged many researchers (Banerjee, 1986; Goyal and Nebebe, 2000; Ben-Daya and Hariga, 2004; Pandey et al., 2007; Glock, 2012) to develop various integrated inventory models.

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Optimal batch shipment policy for an imperfect production system under price-, advertisement- and green-sensitive demand

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In this paper, we consider an imperfect production-inventory system which consists of a single manufacturer and a single retailer. The manufacturer delivers the order quantity to the retailer in some unequal-sized batches. To separate the defective items, the retailer performs an error-free screening process after receiving each delivery from the manufacturer. Shortage in retailer's inventory is allowed and completely backlogged. The customer demand is influenced by the retail price, advertisement frequency and greening level of the product. The centralized model and the decentralized model based on a Stackelberg gaming approach are developed to determine optimal pricing, advertising and inventory decisions. A cost-sharing contract between the manufacturer and the retailer is implemented, which enhances the environmental performance, advertisement frequency and profitability of the supply chain significantly. The proposed model is illustrated with a numerical example. Sensitivity analysis for some key parameters is carried out and several managerial insights are also highlighted.

Keywords: Imperfect production; unequal batch shipment; price-; advertisement- and green-sensitive demand; cost-sharing contract

1. Introduction

Supply chain management deals with all the activities associated with the transformation of raw materials into finished product such as raw material collection, information gathering, manufacturing, packaging, delivery of the goods and services to the right people at the right time. In addition to proficient business management, researchers and practitioners are focusing on various streams of the supply chain. For instance, global supply chain (Li, 2013; Zhang & Wang, 2019a), agile supply chain (Bicocchi, Cabri, Mandreoli, & Mecella, 2019), service supply chain (Zhang & Wang, 2019b), order distribution problem (Zandieh & Aslani, 2019), augmentation of cryptocurrency and smart contracts (Viriyasitavat, Hoonsopon, & Bi, 2020), data integration (Vieira, Dias, Santos, Pereira, & Oliveira, 2020), food supply chain (Haleem & Sufiyan, 2020), etc. In recent times, environmental protection and pollution reduction have drawn global attention. Most developed countries are focusing on reduction of carbon footprint and harmful impacts of various supply chain activities on the environment. In this context, green supply chain management balances environmental issues and economic performance quite efficiently (Dey &

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