Jadavpur University
Master of Nuclear Engineering
1^{st} year 2^{nd} semester - 2024
Nonlinear and Adaptive Control
Ex/PG/NuE/T/128A/2024

Time: 3 hours

Full marks: 100

Instructions: Q1 to Q5 are compulsory. Answer either Q6 or Q7

1. (a)How a chaotic system differs from and a noisy system? (2)

   (b)What do you mean by the "filtering hypothesis" in reference to the describing function analysis of a nonlinear system? (2)

   (c)Consider the following system equation of a Van der Pol oscillator

   $$\ddot{x} + \alpha(x^2 - 1)\dot{x} + x = 0.$$

   Show that it can represented as a system with a feedback. (4)

   (d) First assume the existence of a limit cycle with undetermined amplitude and frequency, and then determine whether the system equation of 1(c) can indeed support such a solution. (8)

2. Consider an electrical system (Fig. 1) with a hypothetical nonlinear device with the I-V characteristics shown in Fig. 2.

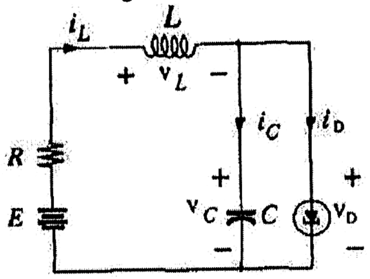


Fig. 1

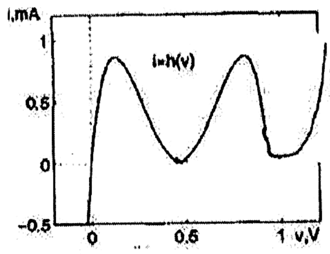


Fig. 2

   (a)Select the appropriate state variables and form the state equations for the system. (8)

   (b)Determine the equilibrium point or points of the system. (4)

   (c)Show whether it is possible to change the number of equilibrium points. (4)

3. (a)Consider the following system equation

   $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

   Determine the eigen values and draw a conclusion about the nature of stability of the equilibrium point. (4)

   (b)Consider the following system equation

   $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ -4 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

   Determine the eigen values and draw a conclusion about the nature of stability of the equilibrium point. (4)

   (c)Consider the following nonlinear system

   $$\dot{x}_1 = -x_1 + 2x_1^3 + x_2$$
   $$\dot{x}_2 = -2x_1 - x_2.$$

   (i)Determine the equilibrium points of the system. (3)

   (iii)Determine the nature of stability of each of the equilibrium point. (6)

[Turn over

4. (a) Why it is essential to impose the condition of Lyapunov stability in the definition of Lyapunov asymptotic stability? (3)

(b) Consider the following system

$$\dot{x} = \alpha(y - x)$$
$$\dot{y} = sx - y - xz$$
$$\dot{z} = xy - cz$$

where α, s and c are positive constants.

Define a suitable Lyapunov function and determine the stability of the origin under the condition $0 < s < 1$. (7)

(c) Consider the following system

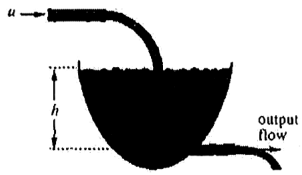
$$\dot{x}_1 = -x_1 + \alpha x_1 x_2$$
$$\dot{x}_2 = -4x_2$$

where $\alpha > 0$ is constant.

(i)Determine the equilibrium point of the system. (2)

(ii)Determine whether the system is globally or locally asymptotically stable and how its stability depends on the value of α. (5)

5. (a) Consider the problem of fluid level control in a tapered tank.



It is intended to control the fluid level h in the tank to a desired level h_d. Consider the following

h_0: initial level,

u : input volume flow rate,

$A(h)$: cross section of the tank as a function of h.

(i)What is the dynamic system equation and under what condition the problem becomes a nonlinear one? (2+1)

(ii)Determine the control law $u(t)$ such that the dynamic system equation becomes linear when considered

(a)h_d is fixed (stabilization problem). (4)

(b)h_d is changing with time (tracking problem). (4)

(b)Consider a system described by the following general nonlinear dynamic system equation

$$\dot{x} = Ax + B\gamma(x)[u - \alpha(x)]$$

where

$\gamma(x)_{n\times1}$: nonlinear vector function of the state vector $x_{n\times1}$ and $|\gamma(x)| \neq 0$,

$\alpha(x)_{n\times1}$: nonlinear vector function of the state vector $x_{n\times1}$.

Determine the general form of the control law $u(t)$ that can linearize the general nonlinear dynamic system equation. (3)

(c)What is the relative degree of a nonlinear system? If the order of a nonlinear system is n, what is the maximum value of its relative degree? (2+1)

6. (a)Consider the following for a Model Reference Adaptive Control (MRAC) scheme:

The model of a SISO plant is

$\dot{x} = ax + bu;, \ a, b$: scalars and unknown; $x, a, b \in \Re, b \neq 0$.

The model of the reference is

$\dot{x}_m = a_m x_m + b_m r;\ r$: reference input; a_m, b_m: parameters of the ref. model. $a_m < 0$.

The controller model is

$u(t) = \hat{K}_x(t)x + \hat{K}_r(t)r$; $\hat{K}_x(t)$: time varying estimation of K_x; $\hat{K}_r(t)$: time varying estimation of K_r.

(b)Why it is not possible to achieve the perfect tracking of the reference output x_m in MRAC? (4)
(c)Draw a block diagram of the MRAC scheme. (3)
(CO6)

7. Consider the following 2-dimensional single input system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = h(x) + g(x)u, \; g(x) \geq g_0 > 0 \text{ for all } x,$$

where the functions $h(x)$ and $g(x)$ are unknown.
(a)Determine a suitable control law $u(t)$ which can drive the system to a manifold $s = 0$, where $s = ax_1 + x_2, a > 0$ and hold it there. (12)
(b)Show that the equilibrium point (0,0) of the system is asymptotically stable when its state trajectory lies in the manifold $s = 0$. (5)