

Master of Nuclear Engineering 1st Semester Examination, 2024

Subject: Analytical & Computational Tools in Nuclear Engineering

Full marks: 100

Answer all Questions

1. a) Show that the system of equations
 $2x + 6y = -11, 6x + 20y - 6z = -3, 6y - 18z = -1$ is not consistent. 5

or

- b) Solve the following system of equations using Cramer's rule.

$$\begin{aligned} 5x - 7y + z &= 11 \\ 6x - 8y - z &= 15 \\ 3x + 2y - 6z &= 7 \end{aligned} \quad 5$$

- c) If $\begin{vmatrix} a & a^2 & a^3 - 1 \\ b & b^2 & b^3 - 1 \\ c & c^2 & c^3 - 1 \end{vmatrix} = 0$ prove that $abc = 1$. 5

- d) Show that any square matrix can be expressed as the sum of two matrices, one symmetric and the other anti-symmetric. 5

or

- e) The matrix A is defined as $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$. Find the Eigen values of

$$3A^3 + 5A^2 - 6A + 2I. \quad 5$$

- f) Use Gauss-Jordan elimination to solve 5

$$\begin{aligned} x_1 + 3x_2 - 2x_3 &= -7 \\ 4x_1 + x_2 + 3x_3 &= 5 \\ 2x_1 - 5x_2 + 7x_3 &= 19 \end{aligned}$$

2. a) Prove that $\Gamma(n + 1) = n!$ 5

or

- b) Proved that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ 5

- c) Find the value of $L(t^n)$ using Laplace transforms where n is positive. 5

- d) Using Laplace transforms, find the solution of the initial value problem

$$D^2 + 9y = 9u(t - 3), \quad y(0) = D(0) = 0$$

Where $u(t - 3)$ is the unit step function. 6

- e) Find the inverse Laplace transform of $\frac{1}{s(s^2+1)}$. 4

3. a) Find the Fourier sine and cosine transforms of

$$f(x) = \begin{cases} 1, & 0 < x < a \\ 0, & x > a \end{cases} \quad 5$$

or

b) Show that the Fourier cosine transform of $\frac{1}{1+x^2}$ is $\sqrt{\frac{\pi}{2}} e^{-s}$ 5

c) Established relation between Laplace and Fourier transform. 5

d) Use the method of separation of variables to solve the equation:

$$\frac{\partial u}{\partial x} + u = \frac{\partial u}{\partial t} \quad \text{if } u = 4e^{-3x} \text{ when } t = 0. \quad 10$$

or

e) A rod of length l with insulated sides is initially at a uniform temperature u . Its ends are suddenly cooled to 0°C and are kept at that temperature. Prove that the temperature function $u(x, t)$ is given by

$$u(x, t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \cdot e^{-\frac{c^2 \pi^2 n^2 t}{l^2}} \quad 10$$

where b_n is determined from the equation. $U_0 = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$

4. a) Find the volume of the parallelepiped with adjacent sides.

$$\overline{OA} = 3\hat{i} - \hat{j}, \overline{OB} = \hat{j} + 2\hat{k}, \text{ and } \overline{OC} = \hat{i} + 5\hat{j} + 4\hat{k} \quad 5$$

extending from the origin of co-ordinates O.

or

b) If four points whose position vectors are $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar, show that

$$[\vec{a} \vec{b} \vec{c}] = [\vec{a} \vec{d} \vec{b}] + [\vec{a} \vec{d} \vec{c}] + [\vec{d} \vec{b} \vec{c}] \quad 5$$

c) Find the directional derivative of $f(x, y) = 2x^2y^3 + 6xy$ at $(1, 1)$ in the direction of a unit vector whose angle with the positive x-axis is $\pi/6$. 5

d) Prove that $(y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational. 5

e) A vector field is given by $\vec{F} = (\sin y)\hat{i} + x(1 + \cos y)\hat{j}$. Evaluate the line integral over a circular path $x^2 + y^2 = a^2, z = 0$. 5

or

f) Use Divergence Theorem to evaluate $\iint_S \vec{A} \cdot d\vec{S}$, 5

where $\vec{A} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

5. a) Using Euler's modified formula, find an approximate value of y when $x = 0.06$, given that $dy/dx = x^2 + y$ and $y(0) = 1$, taking the interval 0.01. 10

or

- b) Apply Runge-Kutta method (4th order), to find an approximate value of y when $x = 0.2$, given that $\frac{dy}{dx} = x + y^2$ and $y = 1$ when $x = 0$. 10

- c) Apply Runge's formula of order 2 approximate value of y when $x = 1.1$ given $dy/dx = 3x + y^2$ and $y = 1.2$ when $x = 1$. 5

- d) Explain Euler's Method for solving the first order differential equations. 5