Ref. No.: Ex/EE/5/T/512A/2024(S)

BACHELOR OF ENGINEERING (ELECTRICAL ENGINEERING)

5TH YEAR 1ST SEMESTER SUPPLEMENTARY EXAMINATION, 2024

Subject: DIGITAL CONTROL TECHNIQUES   Time: Three Hours   Full Marks: 100

Use a separate Answer-script for each Part

Part I (50 marks)

Question No.	**Question 1** is compulsory Answer ***Any Two*** questions from the rest (2×20)	Marks

Q1 Answer ***any Two*** of the following:

(a) Draw a schematic diagram of a discrete-time control system and show the various types of signals associated with it. — 5

(b) What is an *Impulse Sampler*? Why is it also referred to as an *Impulse Modulator*? — 5

(c) What is a Data Hold Circuit? Show how Zero Order Hold can be used to reconstruct analog signals from their sampled versions. — 5

(d) Obtain the Z-Transform of the function $x(t)$ defined as $x(t) = e^{-t}$ $(for\ t \geq 0)$. — 5

Q2 (a) Derive the expressions for the *Static Position*, *Velocity* and *Acceleration Error Constants* for the discrete-time control system shown in Figure Q2(a). — 12

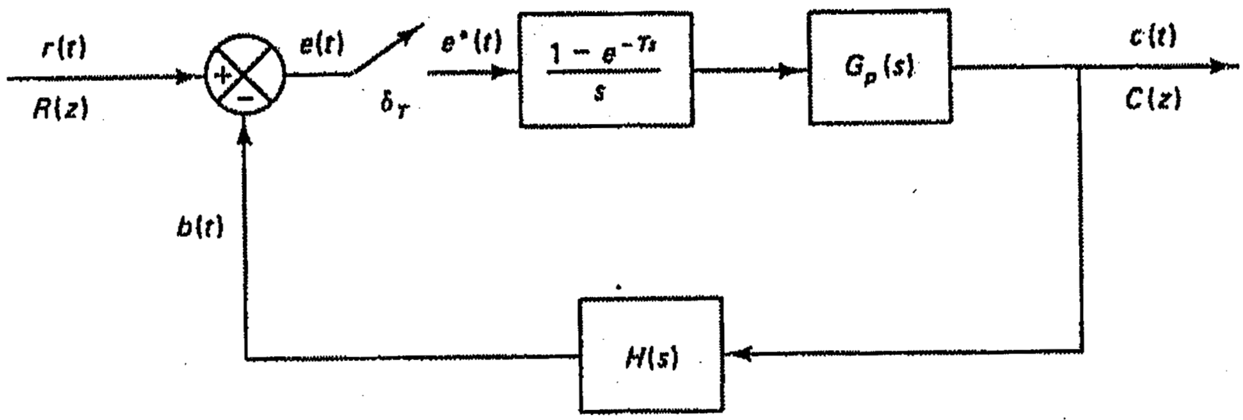

Figure Q2(a)

(b) Show how the followings will be mapped from the left half of the s-plane to the z-plane: — 3+5

(i) constant frequency (ω) loci, (ii) constant damping ratio (ζ) loci.

Q3 (a) What is Pulse Transfer Function? — 2

Show how the Pulse Transfer Function can be derived from the Convolution Summation for discrete-time systems. — 4

(b) Draw the schematic diagram of sampler and Zero Order Hold circuit used in a discrete-time system. — 2

Derive the transfer function of the Zero Order Hold circuit. — 6

[Turn over

Ref. No.: Ex/EE/5/T/512A/2024(S)

Q3 (c) Given the system shown in Figure Q3(c), with input $e(t)$ being a unit step function, determine the output function $C(z)$. 6

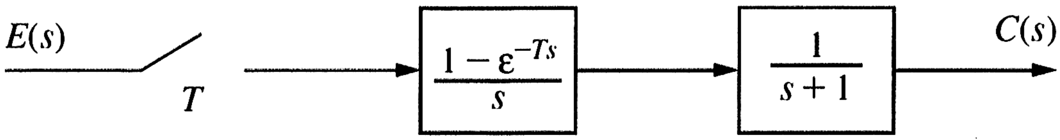

Figure Q3(c)

Q4 (a) State and prove the "*Initial Value Theorem*" and "*Final Value Theorem*" in respect of Discrete-time Systems. 4+4

(b) Determine, using the Final Value theorem, the value of $x(\infty)$ for $X(z)$ given as 4

$$X(z) = \frac{1}{1 - z^{-1}} - \frac{1}{1 - e^{-aT} z^{-1}}, \quad a > 0$$

(c) Obtain, with the help of starred Laplace Transform, the transfer function $C(z)/R(z)$ for the closed loop configuration shown in Figure Q4(c). 8

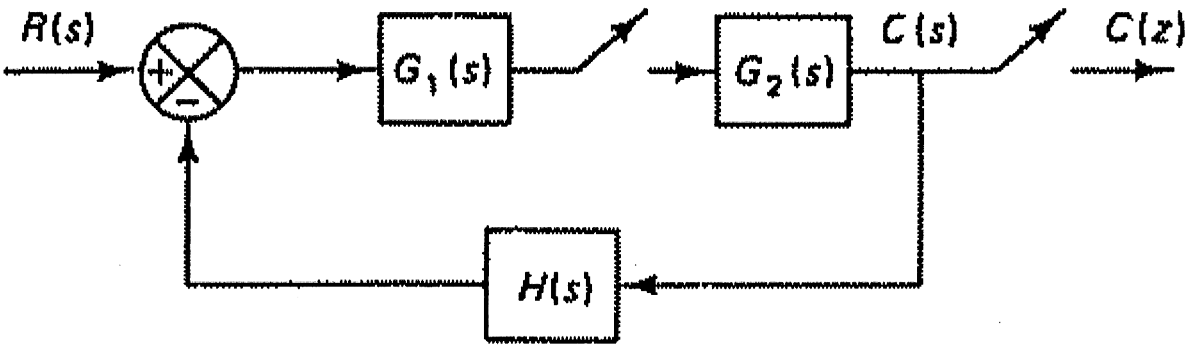

Figure Q4(c)

Q5 (a) State Jury's Stability Test for a closed-loop discrete-time system. 4

Determine the stability of a discrete-time system having the characteristic equation 6

$$Q(z) = z^3 - 1.1z^2 - 0.1z + 0.20 = 0$$

(b) Show how the left half of the s-plane will be mapped into the z-plane. 2

Briefly discuss the concepts of "*Primary Strip*" and "*Complementary Strip*" in respect of the mapping from s-plane to z-plane. 4

(c) Draw the output response of a real sampler with First Order Hold circuit for a unit step input. 4

Ref. No.: Ex/EE/5/T/512A/2024(S)

B.E. ELECTRICAL ENGINEERING 5TH YEAR 1ST SEMESTER
SUPPLEMENTARY EXAMINATION-2024

Subject: **DIGITAL CONTROL TECHNIQUES** **Part: II** **Full Marks:50**

Question No.	Question No 1 is compulsory Answer any **two question** from the rest (2x20)	Marks
Q1.	Answer any two question **(2X5=10)**	
(a)	Illustrate non-uniqueness of state space representations of discrete-time system.	5
(b)	Find the controllable canonical realization of the following difference equation $y(k+2)+0.4y(k+1)-0.8y(k)=u(k)$	5
(c)	Obtain the discrete-time state space representation of the following pulse transfer function in diagonal canonical form. $G(z)=\frac{1+6z^{-1}+8Z^{-2}}{1+4z^{-1}+3z^{-2}}$	5
(d)	Using the nested programming method, obtain the state equation and output equation for the system defined by $\frac{y(z)}{u(z)}=\frac{z^{-1}+5z^{-2}}{1+4z^{-1}+3z^{-2}}$	5
Q2. (a)	Derive the State Transition matrix (G^k) of discrete-time system using Z transform method.	7
(b)	Write down the characteristics equation and find the eigen values of the following discrete-time system $\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$	3
(c)	Obtain the state transition matrix (G^k)of the discrete-time system defined in 2 (b)	10
Q3. (a)	Define complete state controllability and complete output controllability for a discrete-time system. How do you find the rank of a matrix?	2+2+2
(b)	Check the state controllability and observability of the following discrete-time system. $\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ -0.4 & -1.3 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}+\begin{bmatrix} 0 \\ 1 \end{bmatrix}u(k)$, $y(k)=\begin{bmatrix} 0.8 & 1 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$.	8

[Turn over

Ref. No.: Ex/EE/5/T/512A/2024(S)

(c) Consider the discrete-time system defined by 6

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} m & o \\ n & p \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k), \quad y(k) = [1 \ 0] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Determine the conditions for complete state controllability and observability.

Q4. (a) Derive the pulse transfer function matrix of discrete-time system. 5

(b) Show that the pulse transfer function matrix is invariant under similarity transformation. 5

(c) Draw the simulation block diagram of the following discrete-time state model. 5

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k), \quad y(k) = [5 \ -13] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + 2u(k)$$

(d) Obtain the pulse transfer function matrix of the above discrete-time state model. 5

Q5. (a) Draw and explain the block diagram of closed loop discrete-time control system with input vector, $u(k) = -Kx(k)$. 2+3

(b) Consider the complete state controllable n^{th} order discrete-time system, $x(k+1) = Gx(k) + Hu(k)$. Define the controllability matrix as M, where, $M = \left[H : GH : \ldots : G^{n-1}\right]$, show that the, 7

$$M^{-1}GM = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_n \\ 1 & 0 & \cdots & 0 & -a_{n-1} \\ 0 & 1 & \cdots & 0 & -a_{n-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & & \vdots & \vdots \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -a_1 \end{bmatrix}$$

, where, $a_1, a_2, \ldots a_n$ are the co-efficient of the characteristics polynomial $[zI - G] = z^n + a_1 z^{n-1} + \ldots + a_{n-1}z + a_n$. Take $n = 3$.

(c) Consider the discrete-time double integrator system defined by the following 8

$x(k+1) = Gx(k) + Hu(k)$

$y(k) = Cx(k)$

Where, $G = \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$, $H = \begin{bmatrix} 0.02 \\ 0.2 \end{bmatrix}$ $C = [1 \ 0]$.

By use of the pole placement design technique, determine the state feedback gain matrix K such that the closed loop poles of the system are located at $z_1 = 0.6 + j0.4$, and $z_1 = 0.6 - j0.4$