

**BACHELOR OF ENGINEERING (ELECTRICAL ENGINEERING)  
FOURTH YEAR  
SECOND SEMESTER EXAM 2024**

**INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHODS**

**Full Marks 100**

**Time: Three hours**

**(50 marks for each part)**

**Use a separate Answer-Script for each part**

**PART- I**

**Answer any *THREE* questions**

**Two marks reserved for neatness and well organized answers.**

1. (a) A random variable  $X$  is known to have a cumulative distribution function

$$F(x) = 1 - e^{-\frac{x^2}{b}} \text{ for } x \geq 0, \\ = 0 \text{ otherwise.}$$

6

where  $b > 0$  is a constant. Find its probability density function.

- (b) A mail-order computer business has six telephone lines. Let  $X$  denote the number of lines in use at a specified time. Suppose the probability mass function (pmf) of  $X$  is as given in the accompanying table.

$x$	0	1	2	3	4	5	6
$W(x)$	0.10	0.15	0.20	0.25	0.20	0.06	0.04

10

Establish that  $W(x)$  is a valid pmf.

Calculate the probability of each of the following events.

- a. {at most three lines are in use}
- b. {between two and five lines, inclusive, are in use}
- c. The expectation of  $X$ .
2. (a) What is 'Moment Generating Function' of a random variable?
- Derive the expression for the moment generating function of an exponential random variable, and hence obtain the expressions for the moments of different order and the variance. 2+8
- (b) Show that the expectation is the best predictor of a random variable. 6
- Write down the expression for the probability density function of a Gaussian random variable. 4
3. (a) Why does the computation of the probabilities related to Gaussian random variable requires the use of standard tables?
- (b) The life of a type of electricity meter is a normal distributed random variable. The power distribution company of a city installed 3000 such new

[ Turn over

meters having an average life of 10000 working hours with a standard deviation of 1500 working hours. 12

- (i) How many meters might be expected to fail in 1<sup>st</sup> 4000 working hours ?  
 (ii) In the above problem, how many meters are apprehended to fail between the first 1000 and 5000 working hours ?  
 (iii) After what period of working hours, would we expect 20% meters to have failed?  
 Use the attached table. Make linear interpolations wherever necessary.

4. (a) Enlist the properties of probability density function. 6  
 (b) The actual tracking weight of a stereo cartridge that has a prescribed weight of 3 gm on a particular changer, can be regarded as a continuous random variable  $Y$  with a probability density function (pdf) 10

$$f(y) = k[1 - (y - 3)]^2 \quad 2 \leq y \leq 4$$

$$= 0 \quad \text{otherwise}$$

What is the value of  $k$  ? Derive the expression for the cumulative distribution function (cdf) of  $Y$ , and the expectation of  $Y$ .

Also determine the following probabilities.

- (i)  $P(Y < 2.5)$   
 (ii)  $P(Y > 3)$   
 (iii)  $P(2 < Y < 3.5)$

5. Write short notes on any two of the following. 8+8  
 (a) Joint probability and conditional probability  
 (b) Uniform distribution  
 (c) Markov's inequality and Chebyshev's inequality

$F(z)$  = Probability of  $(Z \leq z)$

Table. Standard normal distribution  **$F(z)$  for  $z = 0.00 - 2.99$**

<b>Z</b>	0	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

**BACHELOR OF ENGINEERING (ELECTRICAL ENGINEERING) FOURTH  
YEAR SECOND SEMESTER - 2024**

**SUBJECT: INTRODUCTION TO STATISTICAL & PROBABILISTIC METHODS**

Time: Three Hours

Full Marks: 100  
(50 Marks for each part)

**Use a separate Answer-Script for each part**  
Two marks are reserved for neat and well-organized answers

Question No.	Part-II	Marks
--------------	---------	-------

Answer any three questions

To answer the following questions, you can use (with linear interpolation and extrapolation if necessary) the following standard normal distribution table where ever required:

z:	1.6	1.65	1.70	1.80	1.90	1.95	2.0	2.5
P (z):	0.9452	0.9505	0.9554	0.9641	0.9713	0.9744	0.9772	0.9938
z:	2.55	2.60	2.65	2.70				
P (z):	0.9948	0.9953	0.9960	0.9965				

- 
1. (a) Establish the relations between the expectation and the variance of the sample mean and the corresponding quantities of the population 6
- (b) Discuss about the significance of the central limit theorem. 4
- (c) An insurance company has 25,000 automobile policy holders. If the yearly claim of a policy holder is a random variable with mean 320 and standard deviation 540, approximate the probability that the total yearly claim exceeds 8.3 million. 6
2. (a) A Signal having value  $\mu$  is transmitted from station A and the value received at station B is normally distributed with mean  $\mu$  and variance 4. If the successive values received are 5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5 Find the 95% confidence interval for  $\mu$ . 8

- (b) If  $X_i$ ,  $i=1, 2, \dots, n$  constitute a random sample from a normal population with mean  $\mu$  and variance  $\sigma^2$  where both  $\mu$  and  $\sigma^2$  are unknown, then derive the expression for a  $100(1-\alpha)$  % confidence interval for  $\mu$ . 8
3. (a) Define the terms, (i) Simple Hypothesis, (ii) Composite Hypothesis, (iii) Type I error, (iv) Type II error, (v) Critical Region. 10
- (b) The ideal size of a first-year class at a particular college is 150 students. The college, knowing from past experience that, on the average, only 30 percent of those accepted for admission will actually attend, uses a policy of approving the applications of 450 students. Compute the probability that more than 150 first-year students attend this college. 6
4. (a) Derive the expressions for the Least Square Estimators of the Regression Parameters of a simple linear regression model. 8
- (b) Determine the expected values of Regression Parameters of a simple linear regression model. 8
5. Write short notes on the followings: 6+6+4
- (i) Test of the following hypothesis:  
 $H_0: \mu = \mu_0$  Vs.  $H_1: \mu \neq \mu_0$  for a normal population with mean  $\mu$  and variance  $\sigma^2$ , when both the  $\mu$  and  $\sigma^2$  are unknown.
- (ii) Chi-square distribution
- (iii) Continuity Correction