Critical Studies of Non-Minimum Phase Systems in Different Control Paradigm

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

Master of Electrical Engineering

in

Control System

By

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I hereby declare that this thesis entitled "**Critical Studies of Non-Minimum Phase Systems in Different Control Paradigm**" contains literature survey and original research work by the undersigned candidate, as part of his degree of **Masters of Electrical Engineering in Control System** in the department of **Electrical engineering, Jadavpur University**.

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ABSTRACT

Numerous real-world engineering systems exhibit non-minimum phase behaviour. The coupled-tank system, the cart-inverted pendulum, the mass-spring damper system, and many more are examples of well-known non-minimum phase systems. For such systems, controlling output tracking is a very difficult problem due to presence of zero(s) in right-half s-plane. There are several control strategies available to deal with problems caused by the system's non-minimum phase feature, but each has advantages and drawbacks of its own.

Practical non-minimum phase (NMP) systems are studied in this thesis article solely utilising their respective system transfer functions. Using simply the system transfer function model, system is analysed to determine the presence of anomalies in time-domain response such as zero-crossings, overshoot (owing to zeros), and initial undershoot in step response of NMP systems. Theorems of detection are presented for these anomalies, explained with some transfer function models, and confirmed using an actual system like a coupled-tank system. Some practical NMP systems are explained with their mathematical expression, showing non-minimum phase characteristics.

The major problem associated with the non-minimum phase system is the undershoot behaviour of the system caused by zeros of the right half plane and it becomes trivial as delay increases. In the control techniques, the ability of PI and PID controllers is widely accepted. Internal Model Controller (IMC)-based controller design is one of the ways that uses IMC, and its counterpart IMC-based PID is also one of the control approaches used in industries. As a result, using the IMC-PID tuning method, a clear trade-off between closed-loop performance and robustness to model inaccuracies is achieved with a single tuning parameter. This is because the PID controller algorithm is simple and robust to handle the model inaccuracies for practical applications or an actual process in industries.

The following suggested control strategies, which have been well documented in literature, provide good elimination of initial undershoot, set-point tracking and disturbance rejection. Conventional PID controller is not well efficient to handle the challenges of non-minimum phase systems. A two degree of freedom (2-DoF) PID controller has obvious advantages over a one degree-of-freedom controller because of it is able to control both the set-point tracking and disturbance rejections. Among various equivalent transformed form of 2-DoF controller, feed-forward equivalent form is used here for NMP system. Initial undershoot of NMP system and disturbance is considered here to verify the effectiveness of 2-DoF PID control strategy.

Fast disturbance rejection without significantly increasing overshoot in setpoint tracking is possible using a 2-DOF PID controller. Internal Model Control (IMC) based controller for NMP systems has some advantages over classical feedback control techniques. IMC is able to well analyse the effect on system response due to right hand plane zeros present in system transfer system. A single tuning parameter makes IMC-based controllers simple to adjust as well. Similar to IMC, the filter used in the IMC-based-PID controller is tuned with the different value of tuning parameter to achieve a desired response. IMC-based PID controller is well efficient to manage the model parameter uncertainty and disturbance occurring in the plant process. In order to attain the best PID performance, this study suggests a method to IMC and IMC-based PID controllers for process control applications. This technique claims that an optimal filter configuration exists for each distinct process model. To get the response of NMP system similar to a reference model, Model Reference Controller (MRC) is used. MRC can't be directly applied for the systems having right half plane zeros. Model Reference control can be implemented by using feed-forward compensation technique. This method of control results a good desired response.

MATLAB-Simulink is used to implement the real-time implementation. Figures are used to display the outcomes of simulations. The comparison results reveal that 2-DoF PID, IMC-based PID and Model Reference Control have good settling time, tracking ability, and disturbance rejection responses.

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1 Non-Minimum Phase Systems and It's Control Challenges

1.1 Introduction to Non-Minimum Phase System

Non-minimum-phase (NMP) systems can be defined as the systems which are causal and stable, while their inverses are causal and unstable [1]. The continuous-time NMP system's zeros are located on the right-hand side of the complex plane. Non-minimum phase features can also be seen in time delay systems. The Pade' approximation is used to turn a time delay system into a non-minimum phase system (explained in eqn. (2.2)).

A minimum phase (MP) system has all its transfer function's poles and zeroes in the left half side of the s-plane representation, whereas a general transfer function system does not (in discrete time, respectively, inside the unit circle of the z-plane) [1]. Because poles change from poles to zeros and vice versa when a system function is inverted, only the class of non-minimal phase systems is closed under inversion, and poles on the right side (s-plane imaginary line) or outside (z-plane unit circle) of the complex plane lead to unstable systems.

For a unique magnitude response, a non-minimum phase system shows a greater phase contribution than a minimum-phase system. And for one magnitude response, there is only one minimum phase system, however there are an endless number of NMP systems. To factor the most general causal LTI transfer function, a series of an all-pass and a minimum phase system might be utilised. The system response is the convolution of the two-part responses in the time domain, and the system function is the product of the two parts.

Because of the Right-hand plane zero dynamics, designing controllers for a non-minimum phase is a difficult challenge. The unique qualities of this sort of system make it unsuitable for any system. Many industrial processes have non-minimum phase features, such as hydro-turbines [2], cart-position in cart-inverted-pendulum systems [3], steam generators [4], aeroplanes, and flexible link manipulators [5]. This is critical when dealing with this sort of system since undesirable phenomena might be seen in the system's dynamic response.

To analyse the concept of non-minimum phase system behaviour, consider a real-world example that elucidates the concept of dynamic changes in the system output response caused by changing certain input conditions as the initial changes of the response in the wrong direction, which may cause the controller to make the incorrect control effort, resulting in poor system performance. For example, turning the hot water knob in the shower to "ON" will cause the water to fluctuate between being cold and hot for a short period of time.

Another common example in Control System texts is the change in aircraft height in reaction to elevator deflection. When an aeroplane tries to increase its height by applying elevator, the altitude lowers briefly as the aircraft tails down (resulting in a downward aerodynamic force) before increasing its altitude. This example may be found in Chapter 6 of Franklin's book "Feedback Control of Dynamic Systems" (7th edition) [6], along with the mathematical model.

A system's input is influenced by its zeros, while its response and stability are influenced by its poles. This can be well explained by analysing a transfer function model having zero on right hand side of s-plane. Let's assume two transfer functions one for minimum phase system with all zeros on left half of s-plane (G1(s)) and another one is for non-minimum phase system having one zero on right half of s-plane say G2(s).

$$G1(s) = \frac{s+2}{s^2+3s+2} \tag{1.1}$$

$$G2(s) = \frac{-s+2}{s^2+3s+2} \tag{1.2}$$

Now, Let's separate the pole and zero part of the transfer function for both the system, the zero portion of the transfer function behaves as the input modifier and the response and stability of the process is identified by the remaining portion of the transfer function. Step response of the system can be analysed by applying step input to the system and the zero block gives the updated input to the 2nd part of the system as shown below in Figure 1.1.

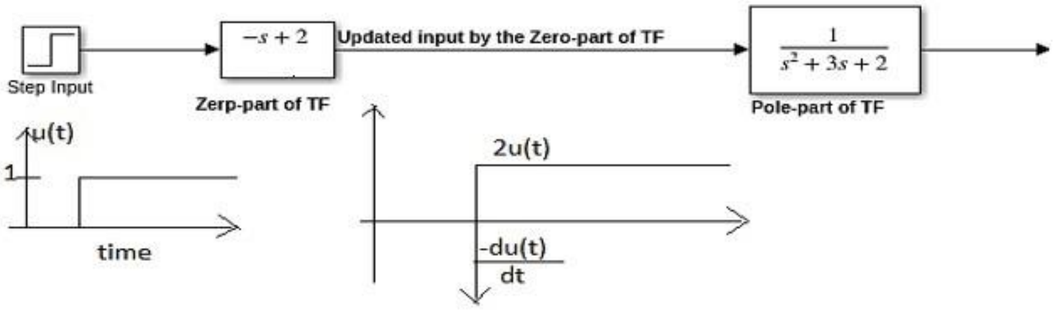

Figure 1.1: Separate Block Diagram for zero and pole TF with Input and Modified Input for System 2 [G2(s)]

The negative derivative of u(t) in System 2 (G2(s)) as shown in Figure 1.1, causes System 2's step response to move in the opposite direction of the intended response (steady-state value) before going in the expected way (the red curve). This contrasts with System 1's step response (blue curve), which does not have this undershoot at the start [Figure 1.2].

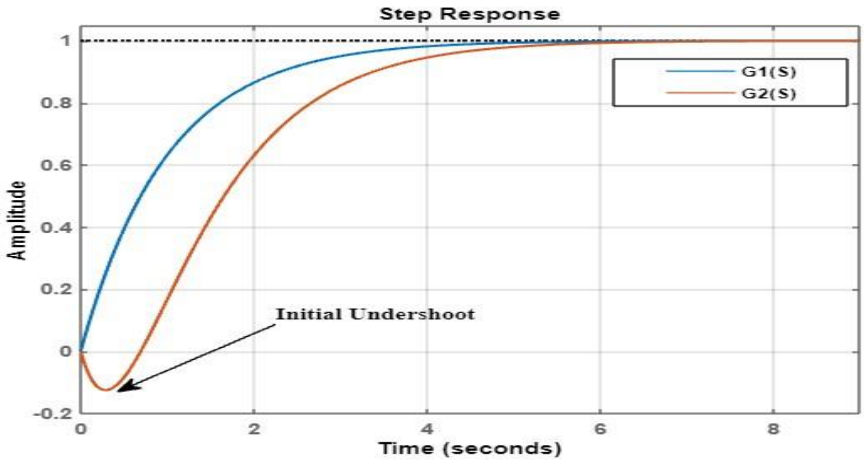

Figure 1.2: Unit Step Response for G1(s) and G2(s)

For such systems, we can also construct a controller/compensator. However, developing a controller for NMP systems is more difficult for a variety of reasons, including the possibility of the system being unstable or having a delayed reaction.

1.2 Anomalies Detection for NMP Systems

Non-minimum phase systems have unusual properties like:

- Initial Undershoot (due to odd no. of RHP zeros)
- Zero-Crossings
- Overshoot (due to even no. of RHP zeros)

1.2.1 Initial Undershoot (due to odd no. of RHP zeros):

Initial undershoot occurs when a stable system's response moves in the opposite direction of its convergence or steady state value but is subsequently rectified.

If the transfer function on the right half of the S plane has an odd number of zeros, the response of that system for a step input will shift direction [7] multiple times before achieving a steady state value. Initial error increase [8] is another term for this.

It can be stated that, if plant transfer function G(s) is strictly proper and having an add number of zeros then, the step response of the plant G(s), y(t) exhibits initial undershoot. If plant TF is proper, then the first undershoot occurs only when $G(s) - G(\infty)$ contains odd number of zeros in right half of s-plane [9].

Let's consider two TF, one with odd no. of zeros on RHP and another TF having even no. of zeros in right half of s-plane.

$$G1(s) = \frac{-6s + 6}{s^2 + 5s + 6} \tag{1.3}$$

And

$$G2(s) = \frac{24s^2 - 60s + 24}{s^3 + 11s^2 + 34s + 24} \tag{1.4}$$

Here, the above TF (1.3) has odd no. of zeros in right half of s-plane (at s=1) and 2nd TF (1.4) has even no. of zeros in right half s-plane (at s=0.5 and 2). Hence, from above statement, it is concluded that system G1(s) exhibits an initial undershoot whereas system G2(s) will not. This can be verified by finding step response of the aforesaid systems as given below Figure 1.3.

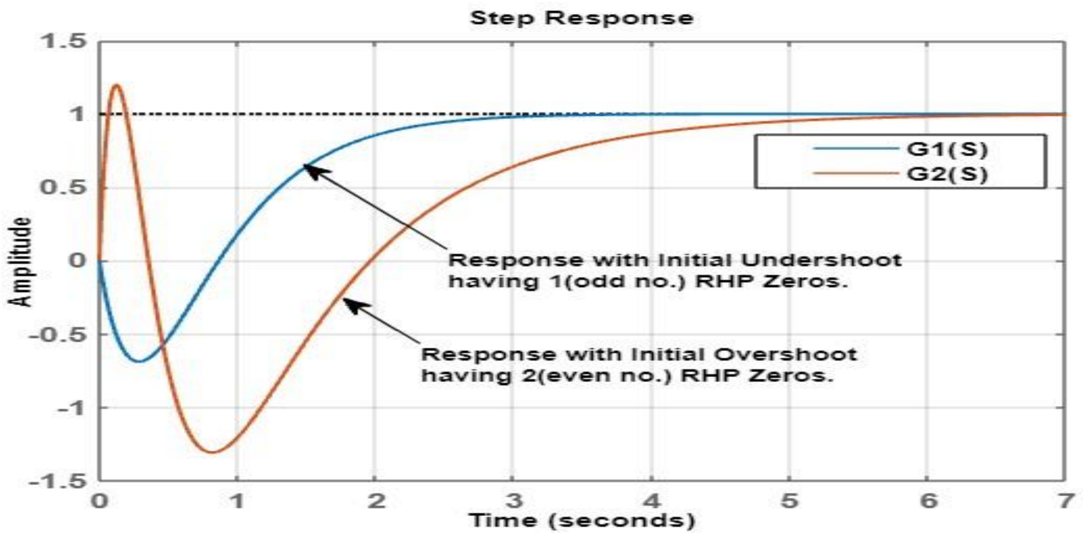

Figure 1.3: Unit Step Response with and without Initial Undershoot (for G1(s) and G2(s) process

1.2.2 Zero-Crossings

Like undershoot, the plant reaction changes its direction depending on the number of zeros in the right half of the S plane, resulting in zero crossing. A zero crossing occurs when a signal cross zero [9].

Theorem for finding the total no. of zero-crossing of a process G(s) can be summarised as [3]:

If system TF G(s) contains n_+ zeros in right half s-plane and a zero at s=0 of multiplicity n_0, then system step response, y(t) has atleast (n_+ + n_0 -1) no. of zero-crossing in case of $n_0 \geq 2$ else number of zero crossing will be n_+ for $n_0 < 2$ [9].

Consider a system G1(s) as given below:

$$G1(s) = -\frac{[(s-5)(s-6)]}{[(s+2)(s+5)(s+10)]} \tag{1.5}$$

Let, y1(t) is step response of G1(s).

G1(s) has zero at s=0 of multiplicity n_0=0 (n_0<2) and two positive zeros in right half s-plane at s=5 and s=6, so n_+=2. So, y1(t) has n_+ (=2) number of zero-crossings. It can be verified by finding step response of the system G1(s) as given below (Figure 1.4):

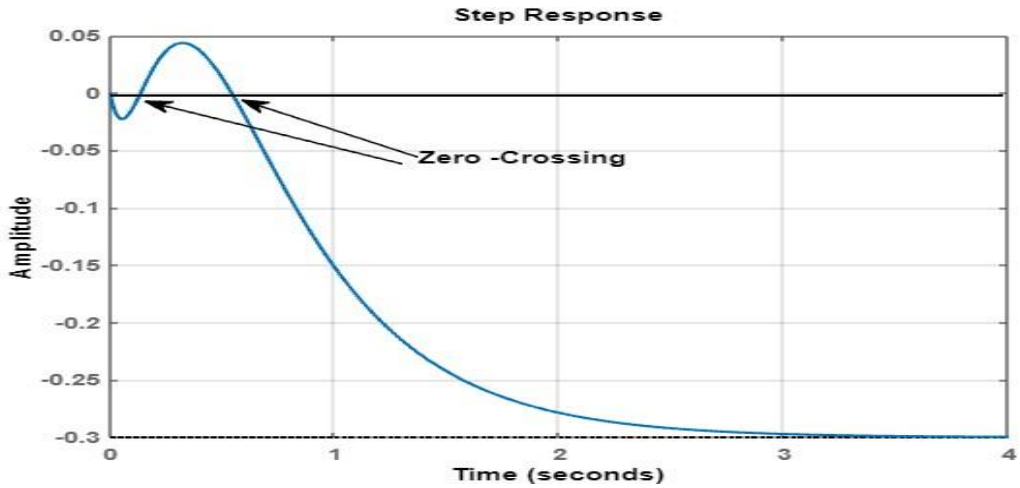

Figure 1.4: Illustration of Zero-Crossing

The above system transfer function (TF) has two zeros in the right half of the S-plane, and its step response crosses zero twice, which is same as the number of zeros in the right half of the S-plane.

1.2.3 Overshoot (due to even no. of RHP zeros):

The maximum value (largest one) obtained by an asymptotically stable system's response in relation to its steady-state value is known as overshoot. Overshoot is most caused by complex poles. However, even if the system is over-damped, some systems exhibit overshoot owing to RHP zeros, causing the responses to overshoot.

Presence of overshoot in system with transfer function G(s) can be confirmed by using following rule. Let $G(s) - G(0)$ has m no. of zeros in right half s-plane and zero at s=0 of multiplicity m_0. Then, if $m_0 < 2$, y(t) has at least m crossings of $y(\infty)$. In case of $m_0 \geq 2$, y(t) has at least $(m + m_0 - 1)$ crossings of $y(\infty)$.

Let's consider two transfer functions G1(s) and G2(s) given as,

$$G1(s) = \frac{-s^2 + 7s + 8}{(s+2)(s+3)} \tag{1.6}$$

and,

$$G2(s) = \frac{-s^2(s+40)}{(s+2)(s+4)(s+5)} \tag{1.7}$$

The step responses for G1 and G2 is given as:

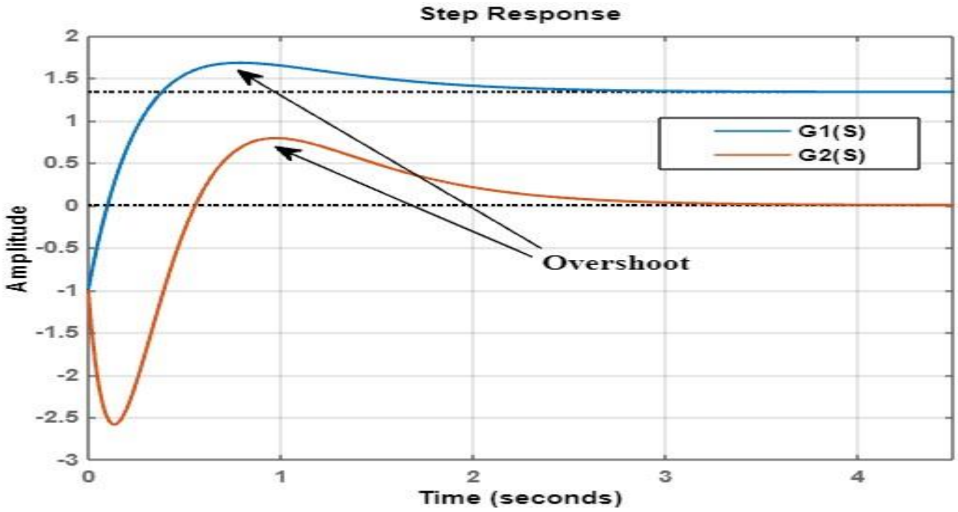

Figure 1.5: Analysing Overshoot for NMP Systems G1(s) & G2(s)

1.3 Inverse Response process

NMP systems can be analysed as the inverse response process. This means that the step response of the controlled process is initially responds in the opposite direction to the steady-state value. This analyses the system by considering the system as two parallel dynamic components. It occurs when the output is produced by two effects, and they are opposing. For example, it's found in industrial operations like distillation columns and chemical reactors [10].

Let's consider a system with transfer function G(s) as given below (1.8):

$$G(s) = \frac{3-s}{[(s+1)(s+5)]} \tag{1.8}$$

So, this NMP transfer function can be modelled as the combination of two parallel TF, such that the step-response of the TF are opposing each-other.

$$G(s) = \frac{1}{s+1} - \frac{2}{s+5} \tag{1.9}$$

Here, it is clear that the two components of the TF G(s) have opposing nature for their step responses, which arises the undershoot for the overall system and finally settled to its overall steady-state value.

1.4 Control Challenges of NMP Systems

As the response of the NMP systems have several anomalies, so an advanced controller is needed for accurate controlling of these systems. To design an effective controller for NMP systems is critical task due to the RHP zero-dynamics of the system, which causes the initial-undershoot, zero-crossing in the response. Various strategies for controlling processes with non-minimum phase characteristics have been developed. NMP systems have certain limitations compared to the MP systems. As the MP systems have a stable inversion, result in a good regulation, but NMP systems can't. NMP systems have constraint over simple feedback control because of they may give unstable closed-loop transfer function, which lead a limitation for root-locus controller designing techniques [11]. As explained below, by increasing the forward gain, after a certain value, system becomes unstable. So, we can say that the RHP zero adds an extra limitation to the controller parameters to maintain stability. By restricting the region of stability (by RHP zeros), NMP systems arise difficulties to achieve the desired characteristics. Sometimes, it may become uncontrollable too [12]. Some controller techniques, which needs the inverted model of the NMP system also uses the approximate modelling of the system to eliminate the RHP zero (as in case of IMC Controller of power system load-frequency control) [13] [14], which leads to the decrease in efficiency of system response.

Minimum phase systems have long been known as having some advantages over non-minimum phase systems; specifically, minimum phase systems have the desirable property of being able to respond arbitrarily rapidly with no "peaking" in the output. This is not the case for non-minimum phase systems.

Let us consider a NMP system transfer function model to analyse the limitation on availability of variation in gain:

$$G(s) = \frac{(s^2 - 1)}{s^3 + 7s^2 + 12s + 10} \tag{1.10}$$

The closed loop feedback diagram for above transfer function model is given below in Figure 1.6.

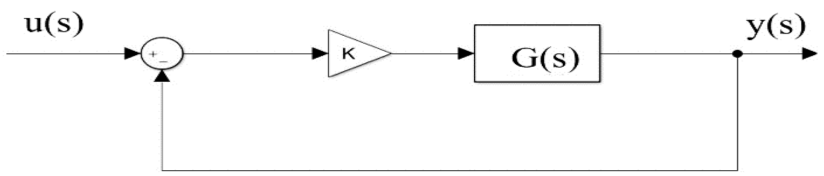

Figure 1.6: Block Diagram of G(s) with Feedback Gain K

Root-Locus, Bode and Step Response Plot for K=1 & 20 is given below:

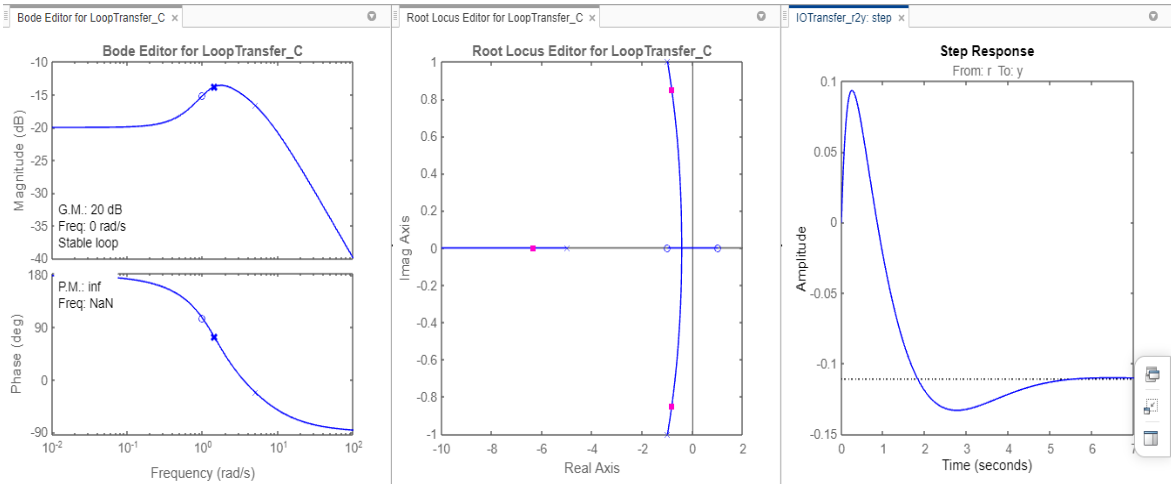

Figure 1.7: Bode-Plot, Root-Locus, and Unit Step Response plot of G(s) for gain K=1.

Now, increasing the gain to 20.

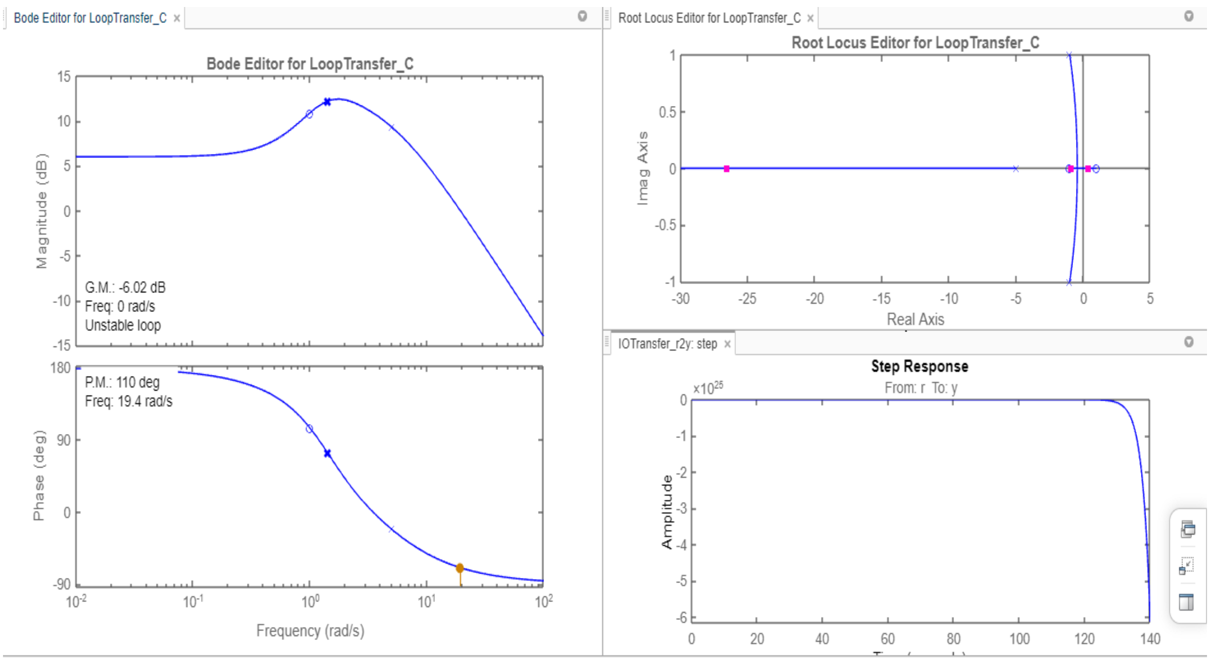

Figure 1.8: Bode-Plot, Root-Locus, and Unit Step Response plot of G(s) for gain K=20.

Figure 1.7 shows the root-locus plot, bode-plot and step response plot of the system G(s) with gain K=1. For gain K=1, the system is stable, but the step response does not give the desired output as it has a large steady-state error. In Figure 1.8, the system becomes unstable when the gain rises to K=20 because one pole of the previously stable open-loop transfer function moves to the right side of the s-plane. This leads to the conclusion that the highest possible feedback gain for NMP systems is constrained, which may prohibit us from obtaining the requisite transient responsiveness and steady-state accuracy.

1.4.1 Issues of NMP Systems

- The system initially responds in the opposite direction of the steady state.
- Limitation on choice of arbitrary high feedback gain.
- Internal stability (States of the system not bounded may result the system towards instability).
- Problems in phase response (phase response is greater than 90 degree implies reduces phase margin).
- Limitation of control bandwidth, which result into disturbance rejection.
- The non-minimum phase system has a slower response (a positive zero system is somehow like a time delay system).

There are several ways in the literature for dealing with the limits provided by non-minimum phase zeros. One among them is pole-zero cancellation, in which an unstable pole cancels the right half plane zero [15]. The problem of this strategy is that it results in an unstable controller design [16] [17] to cancel out the transfer function's non-minimum phase zeros. Outside of the left half of the S plane, pole-zero cancellation occurs, resulting in unbounded internal states associated with non-zero starting conditions. In addition, if the plant's characteristics change over time, inexact pole-zero cancellations occur.

However, all NMP systems do not act the same way; for example, some non-minimum phase systems provide results that are "almost as great" as minimum phase systems, while others are "absolutely impossible" to manage.

1.5 Thesis Objective

The objective of this thesis is to present different types of controllers for non-minimum phase system as PID, 2-DoF PID, IMC, IMC based PID, Feed-Forward Compensator and Model Reference Controller to obtain the desired response. The purpose of these controllers is to minimise or eliminate the initial undershoot and overshoot and other anomalies and to get the effective steady state response. The advantages and limitations of each type of controller is analysed for three NMP systems. The PID tuning is done by using Ziegler-Nichols tuning method and then doing fine tuning and compare its results. 2-DoF PID controller is used to get the better results than PID controller. After that the model-based controller (IMC & IMC-PID) is used for more effective response of NMP systems which are also capable to handle model mismatch and noise in the system. The model reference control is implemented to get the response as the reference model response for different reference signals as step, and square-wave signal. The model-reference control-based system uses feed-forward compensators to compensate the effect of non-minimum phase part. The responses of these controllers are also viewed in case of the stable and unstable NMP system.

Let's choose three transfer function models of NMP systems, which are used in this whole thesis to analyse the effectiveness of the different controller techniques (as mentioned above) are:

1) At first, let's consider a simple 2nd order NMP system with one zero in right half of s-plane which arises initial undershoot in the step-response. System-1 is defined as:

$$G1(s) = \frac{(-1.4s + 1)}{(4.2s^2 + 4.4s + 1)} \tag{1.11}$$

2) Secondly, considering a TF model of NMP system with two zeros in the right half of s-plane, which induces initial overshoot and undershoot both [18]. System-2 is:

$$G2(s) = \frac{(24s^2 - 60s + 24)}{(s^3 + 11s^2 + 34s + 24)} \tag{1.12}$$

3) In last, Let's consider a 2nd order unstable NMP system and analyse its stability and desired response characteristics. System-3 is defined as:

$$G3(s) = \frac{-5(s - 5)}{(s - 2)(s + 7)} \tag{1.13}$$

1.6 Thesis Organisation:

Chapter 1 gives a brief idea about the general characteristics of the NMP systems and its control challenges. Chapter 2 includes some benchmark examples with their mathematical models, which shows NMP characteristics. Effectiveness of PID and 2-DoF Controller are shown in chapter 3 and 4 respectively. Chapter 5 and 6 explains the IMC and IMC based controller for stable and unstable NMP systems. A feed-forward compensator-based controller is implemented in chapter 7 to enhance the controller abilities. Chapter 8 explains the model reference controller to get the plant response as the reference model output. Finally concluding remarks are provided in chapter 9.

2 Few Benchmark NMP Systems and Their Mathematical Models

There are many systems in industries which shows the behaviour of non-minimum phase. Few among those as benchmark NMP systems with their mathematical models and responses are discussed below to understand their characteristics. Also, the above-mentioned theorems (as discussed in 1 can be applied here to theoretically examine the NMP-anomalies present in systems using transfer function analysis. Some benchmark NMP systems are:

- DC-DC Boost Converter
- Cart-Inverted Pendulum System
- Coupled-tank system
- Hydro Turbine of Hydro-Electric Power Plant
- Longitudinal Aircraft Dynamics
- Drum-Boiler Dynamics

  The Drum-boiler dynamics is one of the practical examples of non-minimum phase system. Actually, the complicated shrink and swell dynamics are present which creates a non-minimum phase behaviour which changes significantly with the operating conditions.

2.1 Cart Inverted Pendulum (CIP) System:

For many years, the cart-inverted pendulum (CIP) system has been the one among the most popular benchmark systems, that has been widely used in research in non-linear control theory [19]. This is considered as a well-defined benchmark problem as it provides many challenges to the control design. This is a simple mechanical system that can be easily set-up at laboratory which has highly unstable and highly non-linear dynamics [20]. This system is also famous due to its simple structure and rich non-linear model. The fundamental focus is to analyse this system characteristics using its mathematical model and identify its behaviour as non-minimum phase system. Further in later sections, this model is used to validate the effectiveness of different control techniques and verify its implementation using MATLAB Simulink.

2.1.1 Mathematical Modelling of CIP:

The mathematical model of the dynamics of the CIP system can be easily obtained by doing the free body analysis of the system as shown in Figure 2.1: Cart Inverted Pendulum. This is a simple model with two DoF: one DoF is the rotational movement of pendulum and another one DoF is the linear horizontal movement of the cart [22].

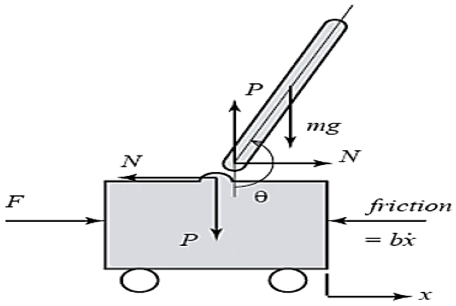

Figure 2.1: Cart Inverted Pendulum [21]

In the Figure 2.1, the horizontal force "F" is applied to the cart to push the cart horizontally forward and backward using DC motor (generally) and hence it changes the dynamics of inverted pendulum and analyse the relationship between the cart and pendulum to maintain the pendulum in upright position while the cart is moved with desired position or velocity.

The state-space model of CIP system shows non-minimum phase behaviour, and the state-space model is obtained in [23].

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0\\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0\end{bmatrix} . \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{I+ml^2}{I(M+m)+Mml^2}\\ 0\\ \frac{ml}{I(M+m)+Mml^2}\end{bmatrix} .u \tag{2.1}$$

Where, x_1=x, x_2=ẋ, x_3=Φ, and x_4=Φ(dot).

The transfer function of this system for the cart position with respect to the applied force will be given, as obtained in [21]:

$$P_{pend}(s) = \frac{\Phi(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}} \tag{2.2}$$

$$P_{cart}(s) = \frac{X(s)}{U(s)} = \frac{\frac{(I+ml^2)s^2 - gml}{q}}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \tag{2.3}$$

where, $\Theta = \pi + \Phi$ and

$$q = [(M+m)(I+ml^2) - (ml)^2 \tag{2.4}$$

The parameter values are specified in Table 2.1, as given in [3].

Table 2.1: Parameters of CIP System

Symbol	Parameter	Value
M	Mass of the cart	2.4 kg
m	Mass of the pendulum	0.23 kg
l	Length of pendulum	0.4 m
I	Inertia of pendulum	0.099 kg m^2
b	Friction of the cart	9.8 m/s^2
F	Force applied	$\pm 24\ N$
X	Cart position w.r.to reference	$\pm 0.3\ m$
Φ	Angle of pendulum	$< 0.1\ rad\ (from\ vertical)$

By putting the value of the parameters as provided in Table 2.1, the cart transfer function (eqn. (2.3)), has zeros at 2.5767 and -2.5767 and the poles are located at 0, 2.6075, -2.6080 and -0.0209. Due to the right-hand side pole located in s-plane, the open-loop step response will be unstable.

By using two-degree of freedom (2-DoF) PID control for stabilizing the process, the close loop transfer function is given in [3].

$$G_{close-loop}(s) = \frac{0.73854(s+0.3571)(s+1.22)}{(s+1.854)(s^2+0.6705s+0.2598)} \cdot \frac{(s+1.485)(s-1.485)}{(s^2+0.7365s+1.473)} \qquad (2.5)$$

Here, from transfer function analysis, it is well clear that the cart-position response will exhibit non-minimum phase characteristics dur to the presence of "right hand side zero" in s-plane.

2.2 Coupled-Tank System:

Coupled tank (CT) system is considered here as the 2nd example to explain the non-minimum phase behaviour of practical system. The coupled-tank system can be considered as a benchmark problem because it has a wide application in the process industries such as biochemical, pharmaceutical industries, spray-coating, and Petro-chemical industries. The control of liquid-level in CT is a typical control process.

Let's examine the process reaction curve for tank-2 using empirical modelling [24], it shows as a time-delay process. The linear model of CT is written as:

$$G(s) = \frac{K_p e^{-\theta s}}{\tau s + 1} \qquad (2.1)$$

where, K_p is plant dc-gain, and τ is plant-time constant and θ is dead-time.

From [24] , value of parameters is obtained as: $K_P = 1$, $\tau = 22.8\ sec$, and $\theta = 2.2\ sec$.

Using Pade' 1st order approximation, $e^{-\theta s}$ can be written as:

$$e^{-\theta s} = \frac{1 - \frac{s\theta}{2}}{1 + \frac{s\theta}{2}} \tag{2.2}$$

By using the above approximation used in equation (2.2), G(s) in equation (2.1) is obtained as:

$$G(s) = \frac{1}{(22.8s + 1)} \cdot \frac{(1 - 1.1s)}{(1 + 1.1s)} \tag{2.3}$$

From above transfer function, it is clear that all the poles of the process are real and those are in left half s-plane, so the system is stable, and it has one right-hand side zero, which introduces the non-minimum phase characteristics. The farthest pole from origin is at s = -0.91. Thus, the ROC of G(s) is given by $\{s \in \mathbb{C} \mid \mathbb{R}(s) > -0.91\}$.

The step response of CT liquid-level process is shown below, Figure 2.2.

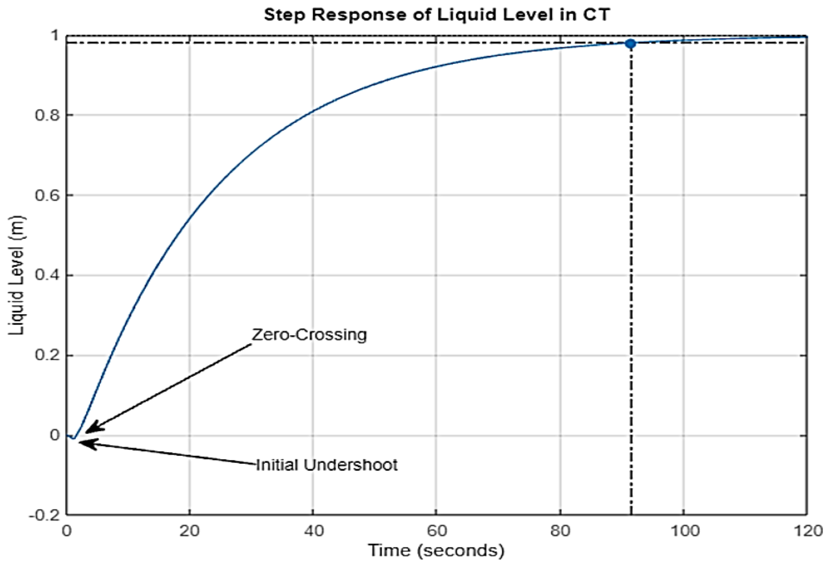

Figure 2.2 Process Reaction Curve of Liquid Level for CT

From Figure 2.2, it can be understood that the response of liquid-level has initial undershoot and one zero-crossing, which confirms the analysis of the process as NMP system.

This process is taken as the plant for rest of the thesis paper for controlling purpose. This thesis paper deals with the anomalies properties shown by this system, which behaves like non-minimum phase system and the final controlled system provide specific desired response by eliminating the anomalies.

2.3 Hydro Turbine of Hydro-Electric Power Plant

In 1973, IEEE committee releases a report on "Dynamic Models for the Hydro and Steam Turbines " [25]. Since then, several mew techniques are used to advanced control of the Hydraulic turbine. Here, I will analyse the NMP characteristics shown by the hydro-turbine using its transfer function model.

The linearised model of "Turbine Conduit Analysis" [26] [2], using the small perturbation operation to the steady-state point, the block-diagram is developed [2]. The transfer function for the change in mechanical power output w.r.to the change in the gate position [27] is written as:

$$G(s) = \frac{(1 - T_w s)}{(1 + 0.5 \cdot T_w s)} \tag{2.4}$$

here,

$$T_W = \frac{LU_0}{gH_{d_0}} \tag{2.5}$$

The values of the above parameters are given in below table from [3] :

Table 2.2: Values of Parameters for Hydro Turbine

Symbol	**Parameter**	**Unit**
L	Length of the Conduit	m
U_0	The initial velocity of water	m/sec
g	Acc. due to gravity	m/sec^2
H_{d_0}	Initial Height	m

Using time constant of the system is $T_W = 4$ sec from [3].

The transfer function becomes:

$$G(s) = \frac{(1 - 4s)}{(1 + 2s)} \tag{2.6}$$

Here, from equation (2.6), the system transfer function has one real pole in the left-half s-plane and one real-zero on the right-half s-plane. So, due to positive zero, system will exhibit the non-minimum phase characteristics.

The response of mechanical power output due to the unit step change done in gate position is given in below Figure 2.3.

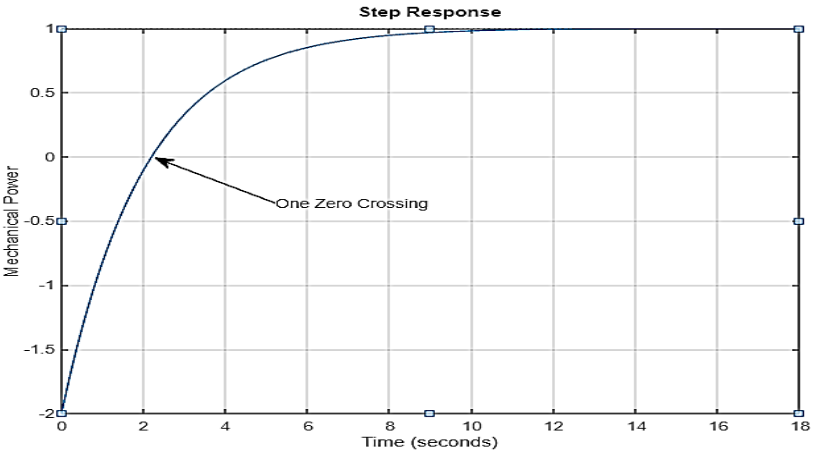

Figure 2.3: Mechanical Power output with step change in the gate position of Hydro-Turbine

2.4 Mass-Spring Damper Mechanical System [28]

Freeman et al. [29] constructed a mechanical experimental setup in which the nonminimum phase component was positioned in the top left corner of the test bed and two additional mass-spring-damper systems were linked to the nonminimum phase system. The electrical equivalent of the mechanical system above exhibits non-minimum phase characteristics. Inertias, a damper, a torsional spring, a timing belt, pulleys, and gears make up this NMP system. The plant has been built to be linear time invariant (LTI) in order to expand the variety of algorithms that may be used. To raise the relative degree of the overall system, two spring mass damper systems have been installed before the non-minimum phase component. The following transfer function model has been used to design the mechanical experimental setup.

$$G(s) = \frac{123.853 \times 10^4 (3.5 - s)}{(s^2 + 6.5s + 42.25)(s + 45)(s + 190)} \tag{2.7}$$

Step response of system is given below shows the characteristics as NMP system:

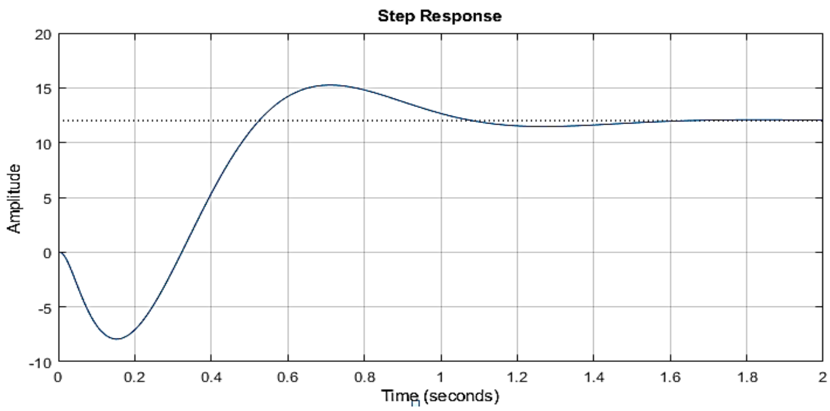

Figure 2.4: Unit Step Response of Mass-Spring Damper System as NMP System

3 Performance of NMP Systems Using PID Controller with Filter

3.1 Introduction to Conventional PID Controller

PID controllers are most widely used across industries. PID control is used as a main approach or with slight modifications in most practical feedback systems to operate well under a variety of operational circumstances. These types of controllers are the most efficient in the absence of a perfect understanding of the process. The three control laws, proportional, integral, and derivative, are arranged in such a way that the needed control signal is generated, driving the closed loop system to fulfil the desired characteristics [30].

The integral and derivative parts are in responsibility of the accumulation of previous errors and the rate of change of errors in the process respectively, while the proportional element is responsible to maintain the intended set-point. The detailed effect of the variations in these parameters is explained in Table 3.1: The Characteristics of P, I and D controllers.

Output of the PID controller can be mathematically represented as:

$$u(t) = Kp \cdot e(t) + K_i \int_0^t e(x) \cdot dx + Kd \cdot \frac{de(t)}{dt} \tag{3.1}$$

Where, Error, e(t) = (Setpoint – Plant output), K_p = proportional gain, K_i = integral gain, K_d = derivative gain, and u(t) is control signal.

3.2 Tuning of PID parameters

The three parameters P, I, and D of a PID controller is adjusted to deal with the required performance specification of individual process.

A table below (Table 3.1: The Characteristics of P, I and D controllers) is described to understand the change in control parameters with the variation in the controller gain (K_p, K_i, K_d).

Table 3.1: The Characteristics of P, I and D controllers

(Effects of increasing PID controller gain independently)

Controller Gain	**Rise Time**	**Overshoot**	**Settling Time**	**S-S Error (Ess)**
↑ K_p	Decrease	Increase	Small Change	Decrease
↑ K_i	Decrease	Increase	Increase	Eliminate
↑ K_d	Small Change	Decrease	Decrease	Small Change

The effectiveness of the controller depends upon the terms that how quickly it responds to an error, how much it provides the steady-state error and the maximum overshoot and oscillation present in the transient response.

The adoption of the PID algorithm does not ensure that the system will be well regulated or stable. The tunning of the adjustable gain is done either by trial-and-error method or by a particular tuning method. Here, PID controller is used for controlling NMP systems So, Ziegler-Nichol's tuning has been used here. Here, the effectiveness of PID controller is analysed for some NMP systems.

3.3 Simulation for PID Control

3.3.1 PID Control for System-1 (Stable NMP System)

From 1.5 Thesis Objective, the transfer function of system-1 is:

$$G1(s) = \frac{(-1.4s + 1)}{(4.2s^2 + 4.4s + 1)} \tag{3.2}$$

System-1 contains one zero on right half side of s-plane, behaves as a NMP system. Here the effectiveness of PID controller is analysed for this system.

The block diagram of simulation is:

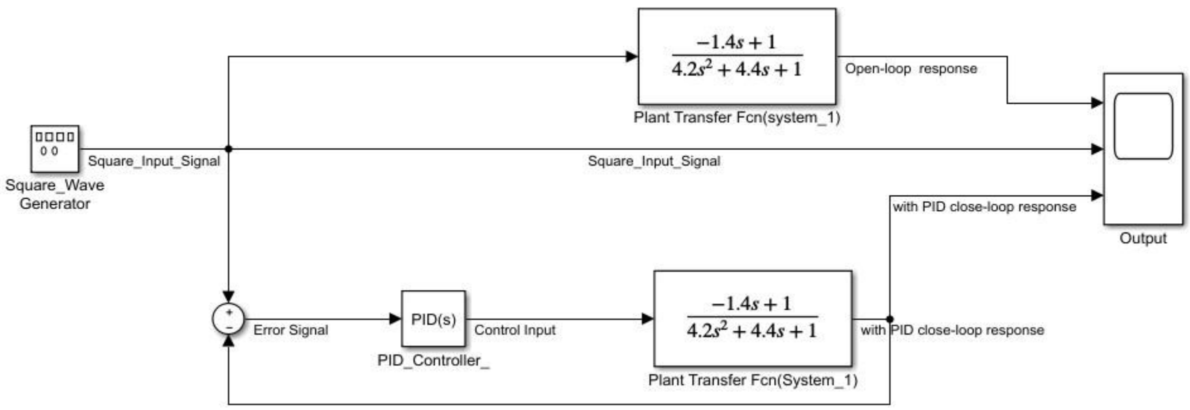

Figure 3.1: Simulink Block Diagram for System-1 with PID Controller

Here, a square wave (with amplitude =1) is applied as the input signal to the plant, which can be considered as a series combination of step-signals, and PID controller is implemented to control the plant (system-1) performance and robustness. PID controller tries to minimize the undershoot introduced due to non-minimum phase component and try to keep balance between the settling time, maximum overshoot and the overall stability of the closed-loop system.

The final characteristics (output) is shown in comparison to the input-signal and open-loop response.

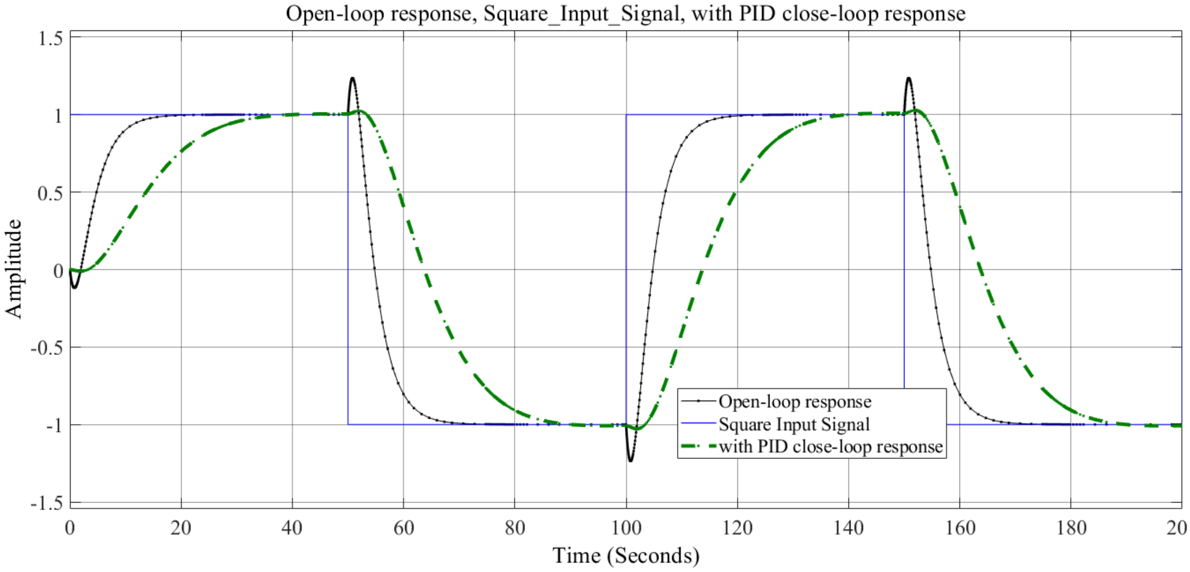

Figure 3.2: Square Wave Input Response of System-1 with PID+f Controller

3.3.2 PID Control for System-2

The transfer function of system-2 is given as:

$$G2(s) = \frac{(24s^2 - 60s + 24)}{(s^3 + 11s^2 + 34s + 24)} \tag{3.3}$$

Syatem-2 contains two zeros in right half of s-plane (at s=2 and s=0.5), behaves as a NMP system and it has 3 (three) poles, all are in left-half s-plane, so system is stable. Here PID controller is used to decrease the initial overshoot and undershoot of the system open-loop response and try to keep the system stable. The simulation result is given as:

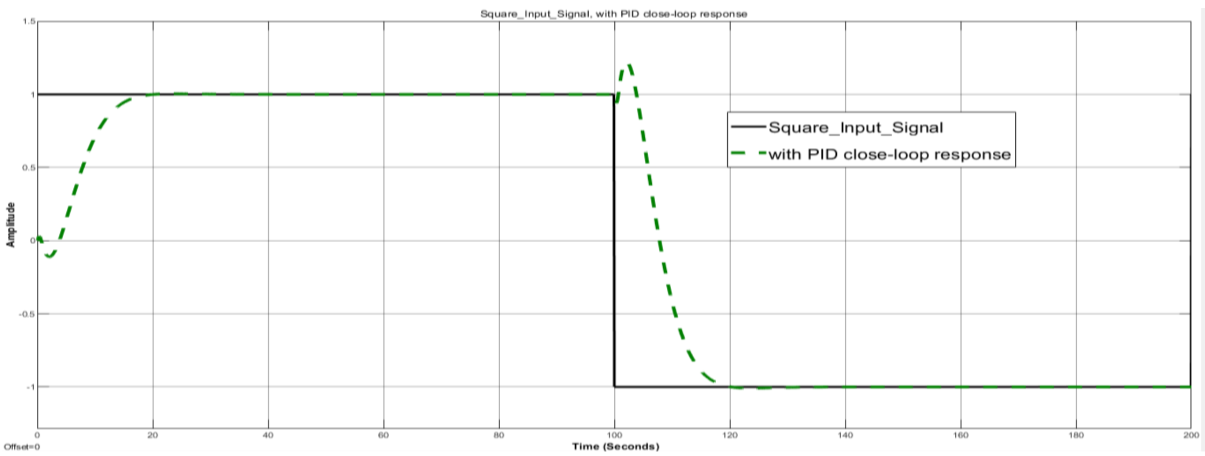

Figure 3.3: Square Wave Input Response of System-2 with PID+f Controller

3.3.3 PID Control for System-3 (Unstable System)

The transfer function of system-3 is given as:

$$G3(s) = \frac{-5(s-5)}{(s-2)(s+7)} \quad (3.4)$$

System-3 contains one zero in right half of s-plane (at s=5), behaves as a NMP system and it has 2 (two) poles, one in the left-half s-plane and one in the right-half s-plane, so system is unstable. Here PID controller is used to decrease anomalies present due to NMP characteristics and try to keep the system stable. The simulation result is given as:

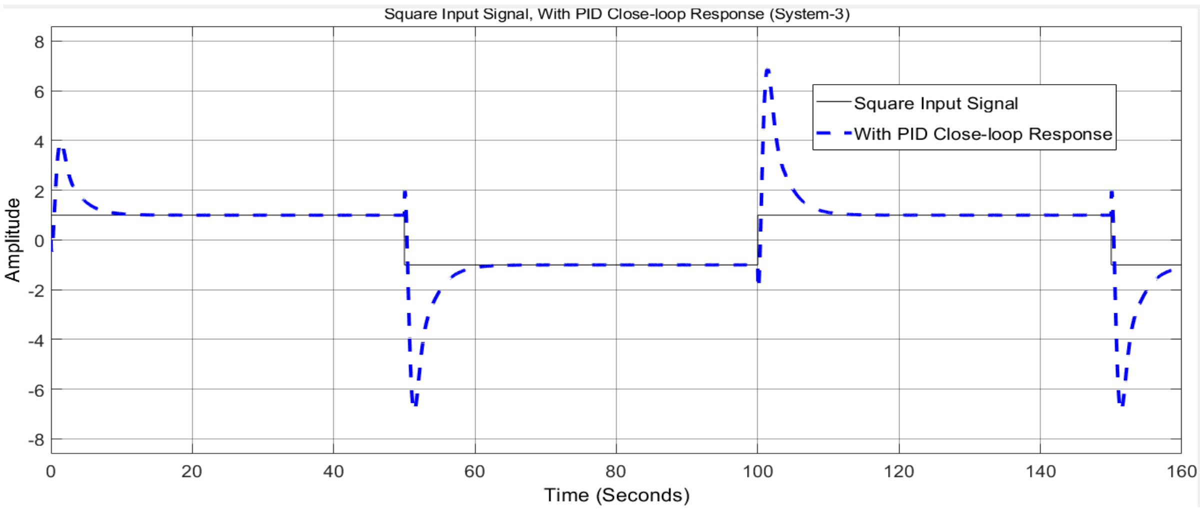

Figure 3.4: Square Wave Input Response of System-3 with PID+f Controller

Simulations are carried out using PID controller for three systems as shown above. For different types of NMP systems, PID controller tries to eliminate the anomalies and ensure their stability. The parameter of PID controller after fine tuning and performance of each system can be summarised and provided in below table:

Table 3.2: Controller Parameters of PID Simultaion (System-1,2,3)

Systems / PID Parameters	**System-1** (Coupled-Tank System) $G1(s) = \frac{(-1.4s+1)}{(4.2s^2+4.4s+1)}$	**System-2** (System with overshoot) $G2(s) = \frac{(24s^2-60s+24)}{(s^3+11s^2+34s+24)}$	**System-3** (Unstable System) $G3(s) = \frac{-5(s-5)}{(s-2)(s+7)}$
K_p	0	0	0.78
K_i	0.066865	0.11482	0.074368
K_d	0	0	0.055
N	100	100	100

Table 3.3: Performance and Robustness of PID Simultaion (System-1,2,3)

Systems / **Performance Robustness**	**System-1** (Coupled-Tank System) $G1(s) = \frac{(-1.4s+1)}{(4.2s^2+4.4s+1)}$	**System-2** (System with overshoot) $G2(s) = \frac{(24s^2-60s+24)}{(s^3+11s^2+34s+24)}$	**System-3** (Unstable System) $G3(s) = \frac{-5(s-5)}{(s-2)(s+7)}$
Rise Time (sec)	19.5	8.75	0.154
Settling Time (sec)	33.5	16.9	9.86
Overshoot (%)	0.428	0.409	294
Minimum amplitude (Undershoot)	-0.0102	-0.111 0.03 (Initial Overshoot)	-0.483
Gain Margin (dB)	16.1	9.99	-2.5
Phase Margin (deg)	68.4	64	12.8
Open-loop Stability	Stable	Stable	Unstable
Close-loop Stability	Stable	Stable	Stable

3.4 Conclusion:

The PID control approaches for NMP stable and unstable systems are explained in this chapter. The traditional Ziegler-Nichols (1942) technique, which requires very minimal knowledge about the process, is one way. However, there are a few drawbacks. To begin, the system must be brought to its limit of instability, which may require a number of tries. Another drawback is that the Ziegler-Nichols (1942) tunings do not function with all processes. It is well acknowledged that the suggested parameters for delay-dominant processes are rather slow (Skogestad, 2003). For comparison purposes, Table 3.3 shows that fine tuning provides good steady-state accuracy and overshoot, as well as minimising the initial undershoot present in NMP systems. It also works well in the case of an unstable NMP system, but it is very slow to track the reference signal, resulting in a long settling time.

4 Performance of NMP Systems Using 2-DoF PID Controller

4.1 Introduction

2-DoF stands for two degrees of freedom, can be defined as the number of closed-loop transfer functions that may be modified independently [31]. A two-degree-of-freedom control system offers obvious benefits over a one-degree-of-freedom (abbreviated as 1DOF) control system when designing control systems [28]. Only two decades after Horowitz's invention, in 1984, was a study published to take use of the 2DOF structure's benefits in PID control systems [32]. Various 2DOF PID controllers for industrial usage were proposed in [33] [34], and comprehensive studies were performed, including appropriate transformations and further research towards appropriate tuning has been done [35] [36]. The 2DOF arrangement not only ensures stability, but also attempts to accomplish the desired objectives. The diagram below shows a typical control design for a 2-DOF PID controller.

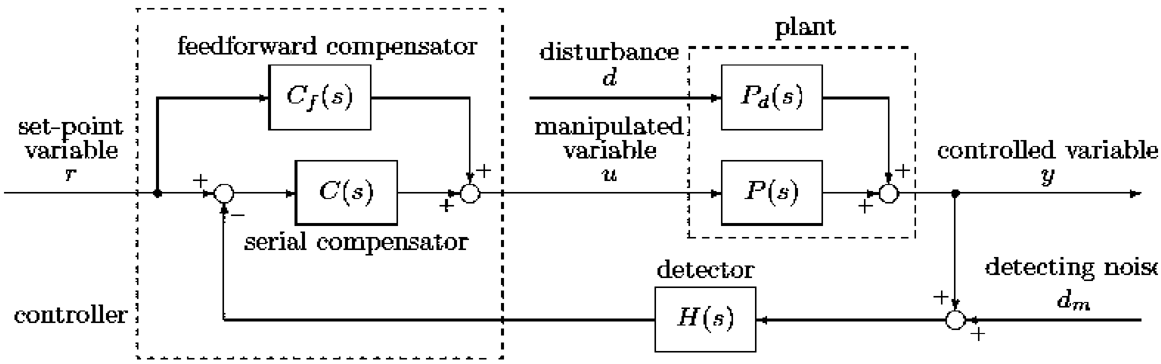

Figure 4.1: 2-DoF Control Sturucture Block Diagram [37]

The feed-forward and feedback controllers of the 2DOF control structure are coupled in numerous ways. The two controllers are independent of one another. Feed-forward controllers assist to fulfil the desired dynamic features of the provided system, whilst feed-back controllers serve to stabilise the system [28]. The purpose of this chapter is to analyse the effectiveness of 2-DoF PID controller to achieve the desired performance and robustness of NMP systems (stable and unstable systems).

4.2 Assumptions on Control System

In Figure 4.1, $C(s)$ is main compensator and $C_f(s)$ is the feed-forward compensator. The closed-loop TF of system for reference input "r" and disturbance "d" can be written as:

$$G_{yr}(s) = \frac{P(s)\{C(s) + C_f(s)\}}{1 + P(s)C(s)H(s)} \tag{4.1}$$

And,

$$G_{yd}(s) = \frac{P_d(s)}{1 + P(s)C(s)H(s)} \tag{4.2}$$

Certain requirements must be met in order to achieve zero steady-state error for the unit step change in the reference input as well as owing to unit step disturbance. They can be summarised as follows:

$$\lim_{s\to 0} C(s) = \infty, \qquad and \ \lim_{s\to 0} \frac{C_f(s)}{C(s)} = \infty \tag{4.3}$$

$$\lim_{s\to 0} H(s) = 1 \tag{4.4}$$

$$\lim_{s\to 0} P(s) \neq 0, \qquad and \ \lim_{s\to 0} \left|\frac{P_d(s)}{P(s)}\right| < \infty \tag{4.5}$$

To satisfy the aforementioned conditions, the controller TF $C(s)$ must contain an integrator, whereas $C_f(s)$ does not, and the sensor must be accurate. Eq. 4.5 indicates that the plant is not of a differentiating type, and that the disturbance is not integrated more than the controlled variable. These are the necessary requirements for the steady-state errors to be reliably zero.

Assumption 1: Sensor is ideal and accurate with enough speed of response, i.e.,

$$H(s) = 1, and \ d_m = 0 \tag{4.6}$$

Assumption 2: The disturbance is analysed from manipulating point, i.e.,

$$P_d(s) = P(s) \tag{4.7}$$

4.3 Equivalent Form of 2-DoF PID Controller

The controller in Figure 4.1 is a 2-DOF PID controller, with $c(s)$ being a standard PID element and $C_f(s)$ being any acceptable element matching the second requirement of Eq. (4.3). Because the PID controller's main benefit is its simplicity, it was suggested that just the proportional and/or derivative components be included in $C_f(s)$ [32]. The 2-DoF controller is a two-input, one-output system with set-point and controlled variables as inputs and the manipulated variable as output. When assumptions 1 and 2 are taken into account, the 2-DoF controller may be stated in a variety of ways, including feed-forward (FF-type), feed-back (FB-type), filter type, and so on. Here, for the simulation purpose a feed-forward type (FF-type) configuration is used.

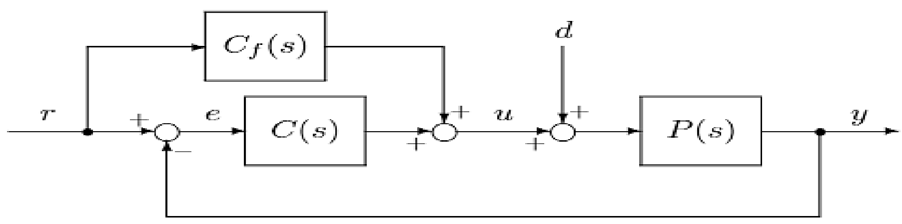

Figure 4.2: Feed-Forward type 2-DoF PID Controller [37]

4.4 Simulation

Let's consider two types of NMP systems one is stable and the another is unstable one to analyse the effectiveness of 2-DoF controller for NMP Systems. Simulation is performed using MATLAB to simulate the overall performance and robustness of the system with controller.

4.4.1 Simulation for System 1: Stable NMP System

Assume the stable NMP system's transfer function is:

$$G(s) = \frac{(-1.4s + 1)}{(4.2s^2 + 4.4s + 1)} \quad (4.8)$$

From Figure 4.2: Feed-Forward type 2-DoF PID Controller Figure 4.2, the controller of 2-DoF will have the form as:

$$C(s) = K_p\{1 + \frac{1}{T_I s} + T_D D(s)\} \quad (4.9)$$

$$C_f(s) = -K_p\{b + cT_D D(s)\} \quad (4.10)$$

Simulation in MATLAB is used to select appropriate values for the controller parameters. Simulation diagram for 2-DoF PID is given below.

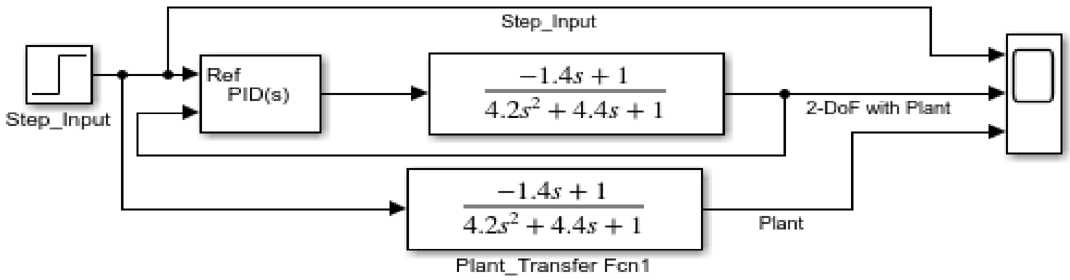

Figure 4.3: Simulation Block Diagram of 2-DoF PID Controller

2-DoF controller is fine-tunned to produce the optimal response for the given plant by modifying its parameter values and several responses are shown below to examine the controller efficacy.

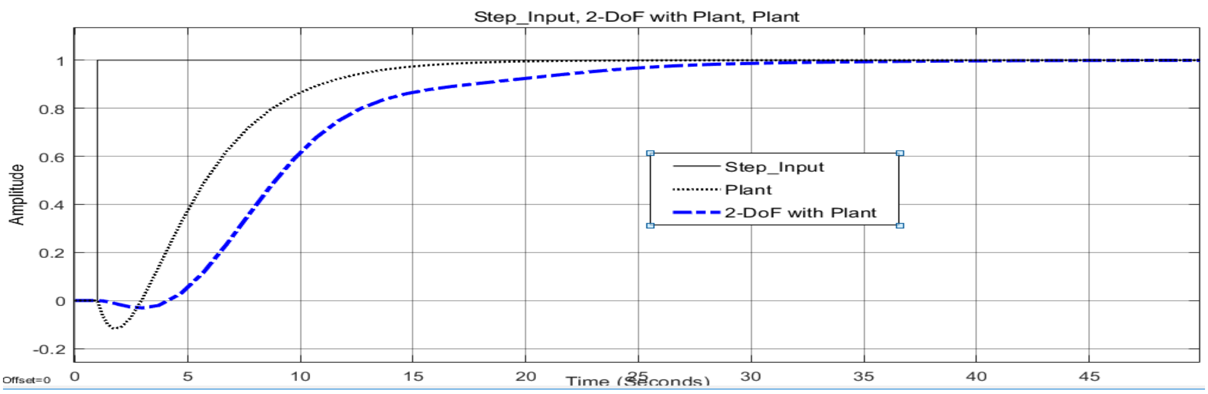

Figure 4.4: Unit Step Response of System-1 with 2-DoF Controller

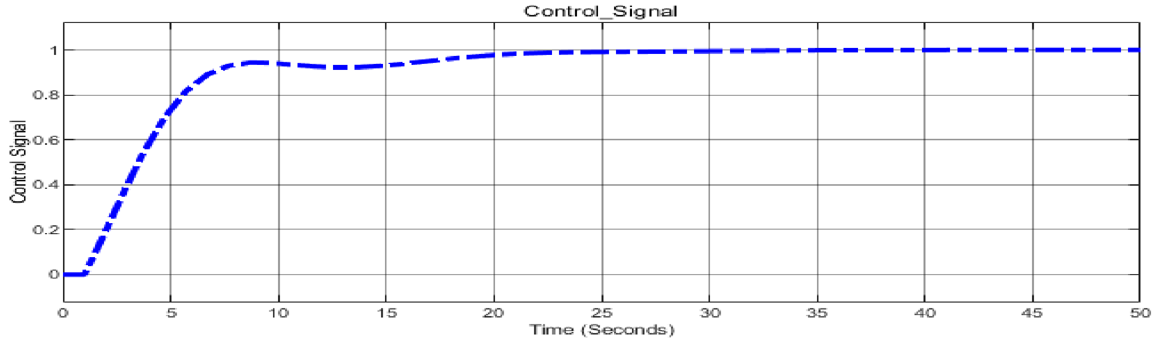

Figure 4.5: Control Signal of System-1 for Unit Step Input with 2-DoF Controller

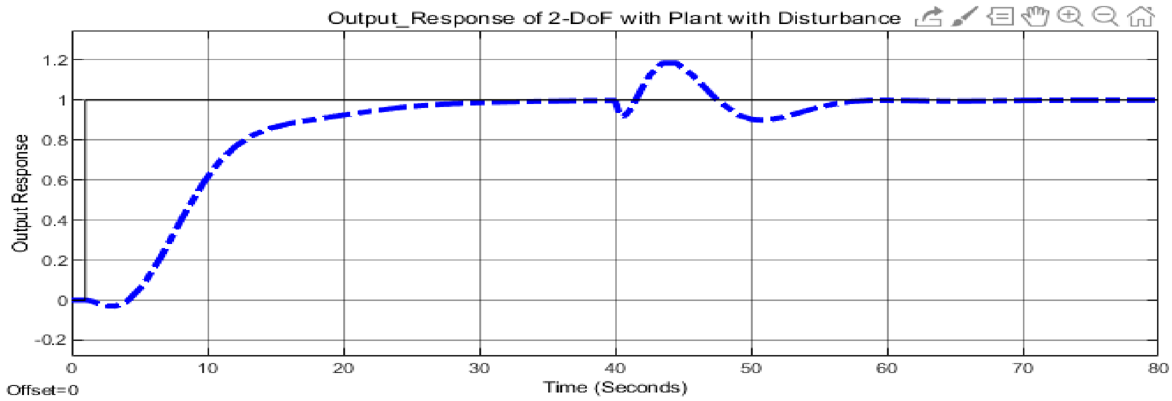

Figure 4.6: Unit Step Response of System-1 for 2-DoF PID Controller with Disturbance

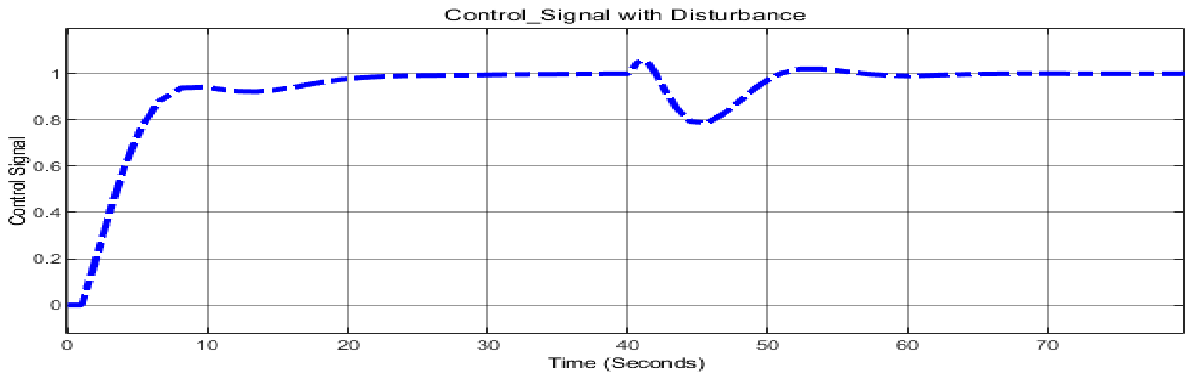

Figure 4.7: Control signal of System-1 for 2-DoF PID Controller with Disturbance

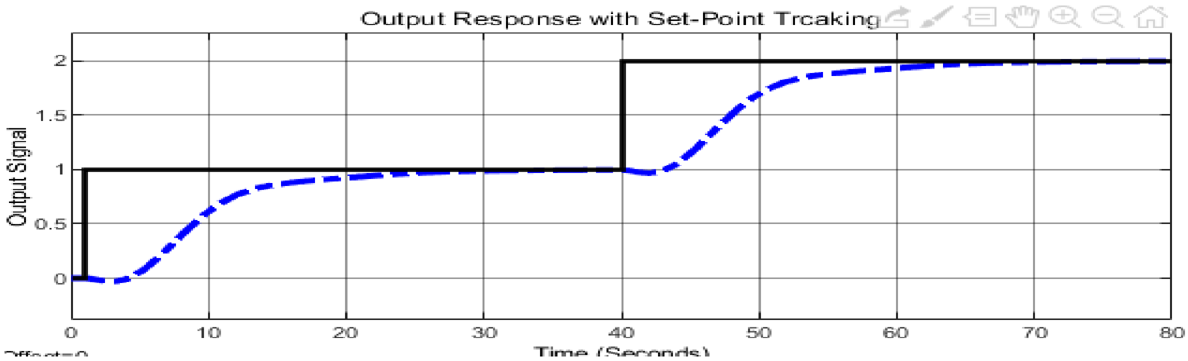

Figure 4.8: Output Response of System-1 for 2-DoF PID Controller with Set-Point Tracking

4.4.2 Simulation for System 2: Unstable NMP System

Assume the transfer function of unstable NMP system as:

$$G(s) = \frac{-5(s-5)}{(s-2)(s+7)} \tag{4.11}$$

"Set-point response" and "disturbance response" have generally been used to evaluate the performance of PID controllers during tuning. We'll include these responses in our analysis as well, to evaluate how the implementation of the 2DOF structure improves them all. "Set-point response" and "disturbance response" are the responses of the controlled variable y to the unit change of the set-point variable r and the unit step disturbance d, respectively. They've generally been used to evaluate the performance of PID controllers during tuning. System-2 is an unstable NMP system with RHP zeros and poles, a 2-DoF controller is implemented here to improve the performance of the overall systems. Different response curves are given below to illustrates the effectiveness of controller:

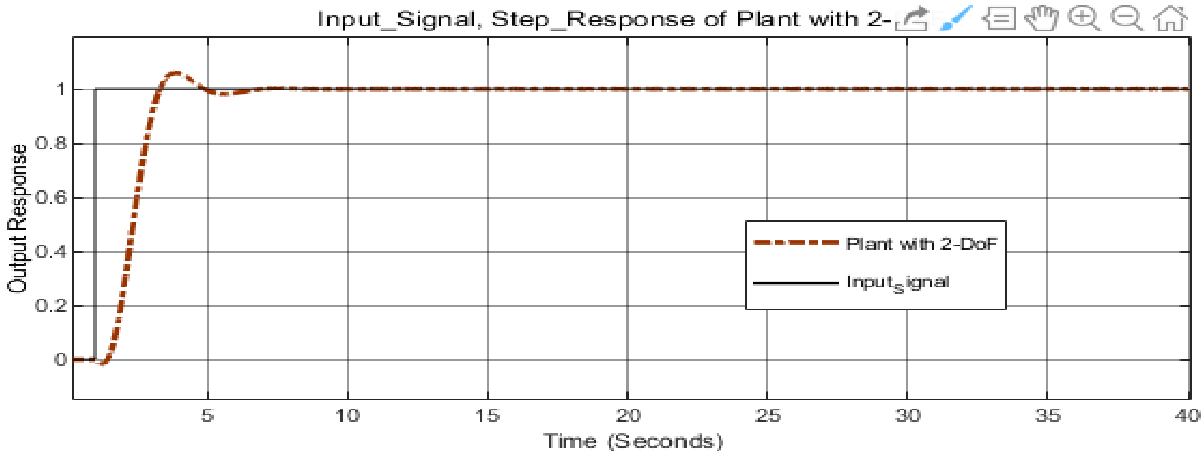

Figure 4.9: Output Step Response of System-2 (Unstable) with 2-DoF Controller

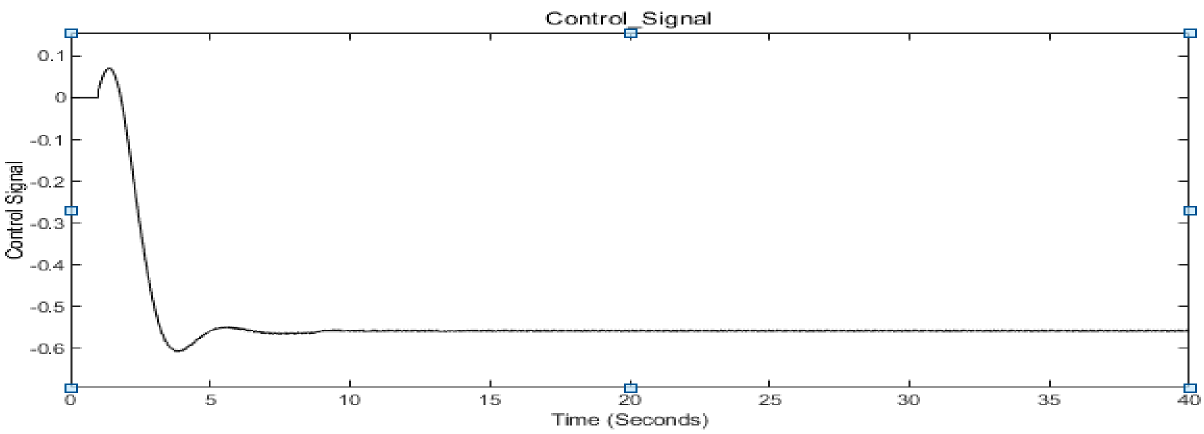

Figure 4.10: Control Signal of System-2 for 2-DoF Controller

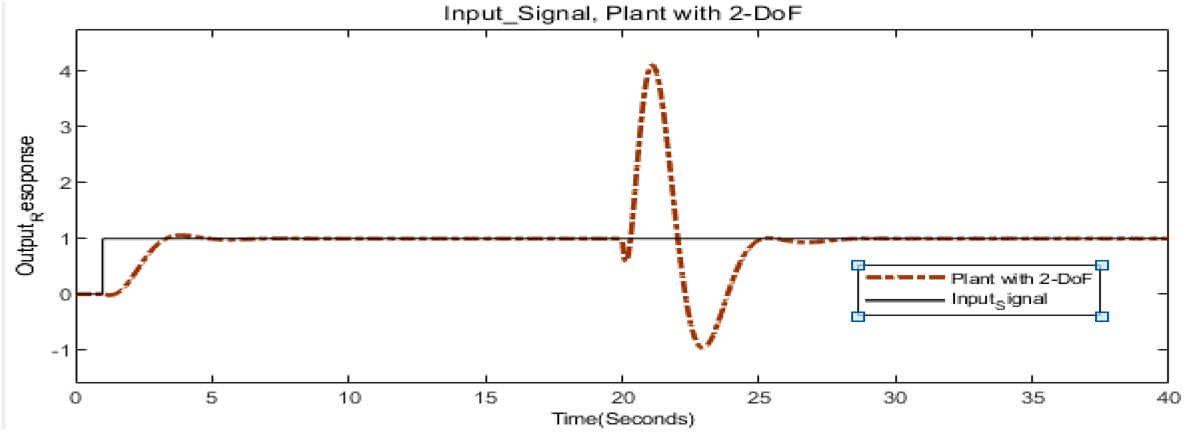

Figure 4.11: Output Response of System-2 for 2-DoF PID Controller with Disturbance

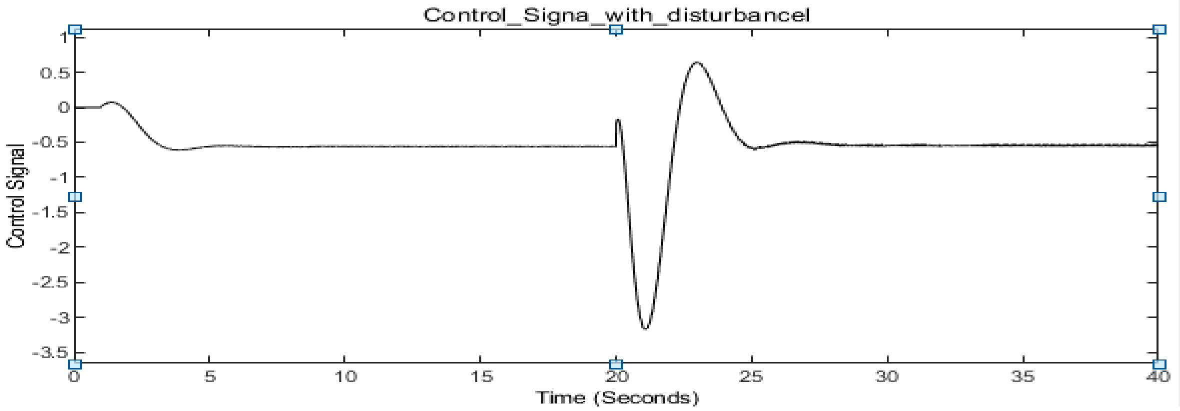

Figure 4.12: Control signal of System-2 for 2-DoF PID Controller with Disturbance

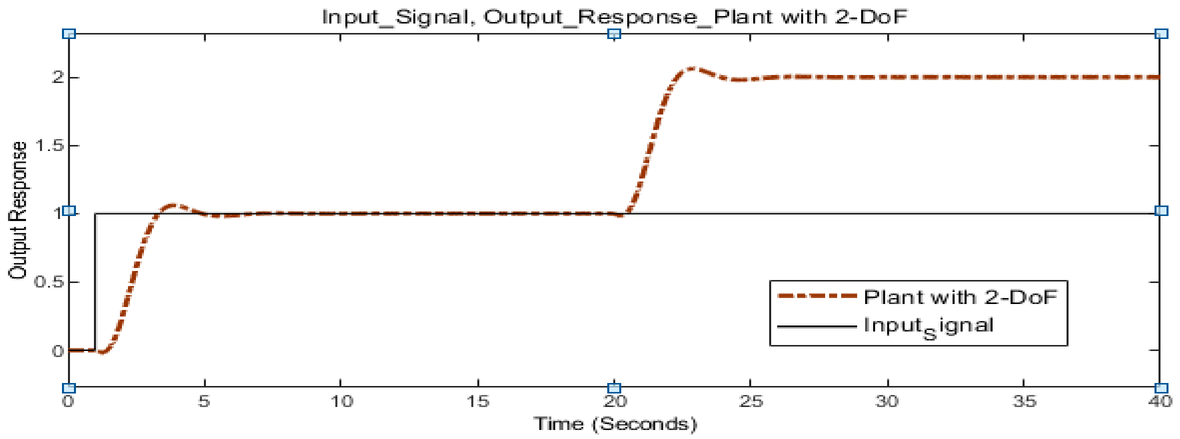

Figure 4.13: Output Response of System-2 for 2-DoF PID Controller with Set-Point Tracking

Simulations are carried out for different types of NMP systems (stable and unstable) as shown above. 2-DoF PID controller tries to eliminate the anomalies and ensure their stability. The parameters of 2-DoF PID controller after fine tuning and performance of each system can be summarised and provided in below table:

Table 4.1: Controller Parameters of 2-DoF PID Simultaion (System-1,2)

Systems / 2-DoF PID Parameters	**System-1** (Coupled-Tank System) $G1(s) = \frac{(-1.4s+1)}{(4.2s^2+4.4s+1)}$	**System-2** (System with overshoot) $G2(s) = \frac{-5(s-5)}{(s-2)(s+7)}$
K_p	0.7691	0.7661
K_i	0.1864	0.1454
K_d	-0.0334	0.0477
N	2.2999	230.32
b	0.00362	0.0174
c	0.00362	0.0003

Table 4.2: Controller Performance and Robustness of 2-DoF PID Simultaion (System-1,2)

Systems / Performance Robustness	**System-1** (Coupled-Tank System) $G1(s) = \frac{(-1.4s+1)}{(4.2s^2+4.4s+1)}$	**System-2** (System with overshoot) $G2(s) = \frac{-5(s-5)}{(s-2)(s+7)}$
Rise Time (sec)	12	1.27
Settling Time (sec)	26.3	3.59
Overshoot (%)	0	6.04
Minimum amplitude (Undershoot)	-0.03	-0.0163
Gain Margin (dB)	7.88	-1.89
Phase Margin (deg)	62.1	8.99
Steady-state Error	0	0
Open-loop Stability	Stable	Unstable
Close-loop Stability	Stable	Stable
IAE (Integral Absolute Error)	9.472	1.449
ISE (Integral Square Error)	6.817	1.106

4.5 Summary

Two-degree-of-freedom (2-DOF) PID controllers feature setpoint weighting on the proportional and derivative terms. With a 2-DOF PID controller, fast disturbance rejection is achievable without a significant increase in set - point tracking overshoot. Two-degree-of-freedom PID controllers are also good in reducing the impact of changes in the reference and control signals. With a high level of performance, 2-DoF PID works well for both stable and unstable NMP systems.

5 Performance of NMP Stable Systems with Internal Model Controller (IMC)

5.1 Introduction to Internal Model Control

Internal Model Controller (IMC) is a model-based control strategy which utilises the internal model of the process to implement effective control. IMC is one among the frequently used controller in the process industry to keep tracking the set-point trajectory and for low disturbance rejection. Even, theoretically the perfect control can also be achieved, if the system process has the exact model. But due to inaccurate information of the process it needs feedback control to accomplish the performance. The structure of IMC is well sketched below.

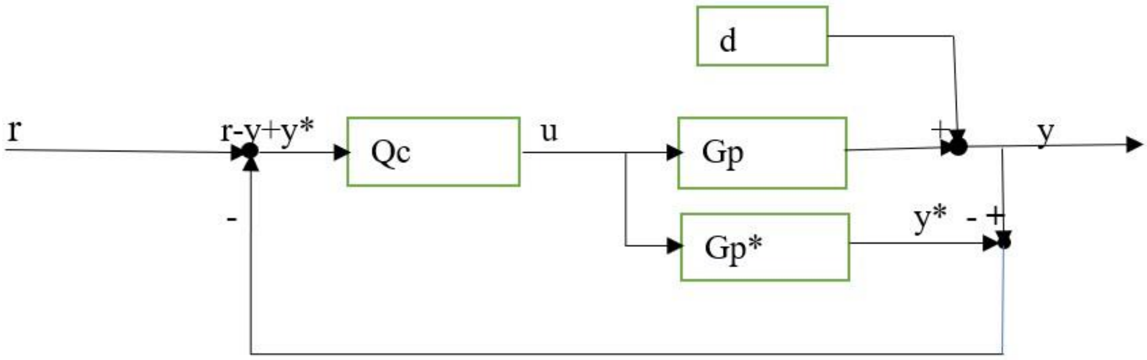

Figure 5.1: IMC Basic Structure

Here,

Q_c = IMC Controller, G_p = Actual plant or process TF, G_p^* = Plant model,

r = SetPoint, R′ = Updated SetPoint, u = manipulated input, d = disturbance,

y = Measured process output,

y^* = process model output, $d^* = \text{feedback} = (G_p - G_p^*)u + d$.

In case of perfect model with no disturbance, if the controller Q_c is stable and the process G_p is also stable then the closed loop system becomes stable. Here, Q_c is the inverse of the plant TF model (Minimum phase systems). But, in case of NMP System, the controller Q_c will become unstable due to the right hand-side zero (s-plane) present in the plant TF. So, controller Q_c needs some modification on basis of plant model G_p^*.

5.2 IMC Design Procedure for NMP Systems

In IMC controller, the controller is designed based on the inverse of the plant model TF. For NMP systems due to the presence of dead-time or RHP zeros, during the development of controller, the inverse of the plant model makes the controller unrealizable. As a result, this is a non-invertible component that must be removed from the system. So, the 'perfect control' can't be achieved due to the limitation of NMP systems. To deal with this difficulty, the plant model is considered as the combination of two components consisting invertible and non-invertible part [38].

Mathematically, it can be described as:

$$\mathrm{G_p^*} = G_p^{*(+)} \cdot G_p^{*(-)} \tag{5.1}$$

Here, $G_p^{*(+)}$ *is non − invertible part and* $G_p^{*(-)}$ *is invertible part.*

Further, the controller uses the inverse of the invertible component of the plant model and may also use a filter to make the controller TF proper.

So, the proper controller TF can be represented as:

$$Q_c(s) = Q_c^*(s) \cdot f(s) = \frac{1}{G_p^{*(-)}(s)} \cdot f(s) \tag{5.2}$$

Where, *f(s)* is a low pass filter.

$$f(s) = \frac{1}{(\lambda s + 1)^n} \tag{5.3}$$

Here, "λ" is the filter tuning parameter to vary the speed of the response of closed loop system and "n" is the relative order of the invertible component (Minimum phase) of the plant model.

Now the low pass filter f*(s)* can be of following three types based upon requirement:

a) To track setpoint changes, the form of filter used is

$$f(s) = \frac{1}{(\lambda s + 1)^n} \tag{5.4}$$

b) For good tracking of ramp set point changes the filter of the form used is

$$f(s) = \frac{(n\lambda s + 1)}{(\lambda s + 1)^n} \tag{5.5}$$

c) For good rejection of step input load disturbances, the filter of the form use is

$$f(s) = \frac{(\gamma s + 1)}{(\lambda s + 1)^n} \tag{5.6}$$

here, gamma, γ is any constant.

5.3 Simulation

5.3.1 Simulation for Stable NMP System:

As the stable NMP system taken into consideration, the TF for system-1 is:

$$G1(s) = \frac{(-1.4s + 1)}{(4.2s^2 + 4.4s + 1)} \quad (5.7)$$

Here the TF of system-1 can be factorised into invertible and non-invertible part as:

$$G1^{+}(s) = \frac{(-1.4s + 1)}{1} \quad (5.8)$$

$$G1^{-}(s) = \frac{1}{(4.2s^2 + 4.4s + 1)} = \frac{1}{(3s + 1)(1.4s + 1)} \quad (5.9)$$

and, the proper controller TF can be represented as:

$$Q_c(s) = \frac{1}{G_1^{*(-)}(s)} \cdot f(s) = \frac{(3s + 1)(1.4s + 1)}{1} f(s) \quad (5.10)$$

Where, *f(s)* is a low pass filter and can be expressed as:

$$f(s) = \frac{1}{(\lambda s + 1)(\gamma s + 1)} \quad (5.11)$$

Which will give the following manipulated signal:

$$U(s) = \frac{(3s + 1)(1.4s + 1)}{(\lambda s + 1)(\gamma s + 1)} R(s) \quad (5.12)$$

And the output variable response will be given by:

$$Y(s) = \frac{(-1.4s + 1)}{(\lambda s + 1)(\gamma s + 1)} R(s) \quad (5.13)$$

From equation (4), it can be seen that there are two adjustable parameters employed here: lambda and gamma. The process model is regarded excellent in this case, and there is no disruption. In simulation, the parameters are fine-tuned and analysed under various scenarios. The criteria are applied to two entities: one is the filter time constant, and the other is the filter order. The filter time constant range is chosen to achieve rapid responsiveness and durability. To make the IMC controller TF perfect, the filter order is taken.

5.3.2 Simulation

The enhanced response of plant by using the IMC controller is analyse by using MATLAB SISOTOOL for stable NMP System.

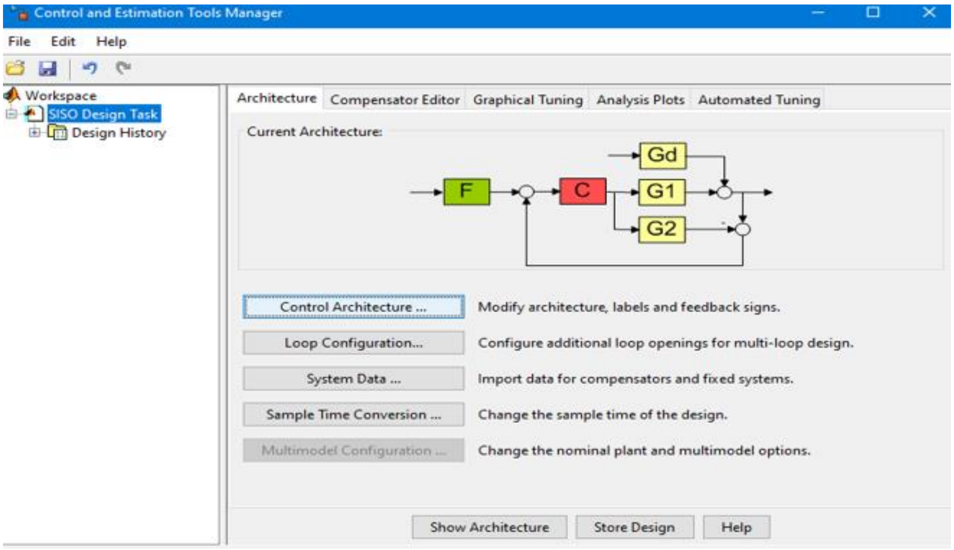

Figure 5.2: IMC Designing using SISOTOOL (Control Architecture)

Now to analyse the closed loop response of the process, we need to go to the Controls and Estimation Tools Manager, click Analysis Plots, choose Step as the plot type for Plot 1, and set Closed Loop 'r' to 'y' as the content of Plot.

Let, the transfer function of the process plant is given by:

$$G1(s) = \frac{2(-1.4s + 1)}{(4.2s^2 + 4.4s + 1)} \tag{5.14}$$

Here,

Step response of close-loop plant with unity feedback is:

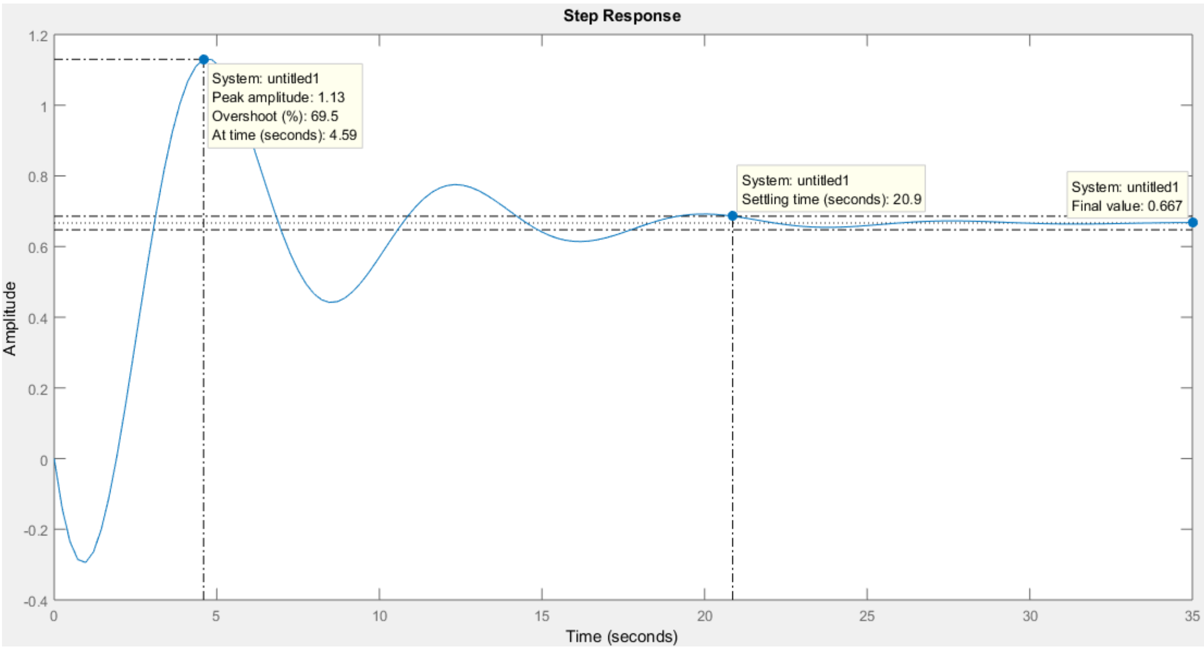

Figure 5.3: Unit Step Response of close-loop plant with unity feedback

Here, it is clear that the above system response has:

- Settling time = 20.9 sec
- Steady-state value = 0.667
- Overshoot = 69.5%

so, an IMC controller is design with specific parameters values [lambda] to improve the performance such that the final controlled system response will be desired response.

Now, tuning this non-minimum phase process (2nd order) using IMC Structure with the help of SISOTOOL. Taking the different value of tuning parameter, Lambda, λ as 0.5,0.75 and 1.0.and, analysing the response of output and manipulated variable.

For Lambda=0.5

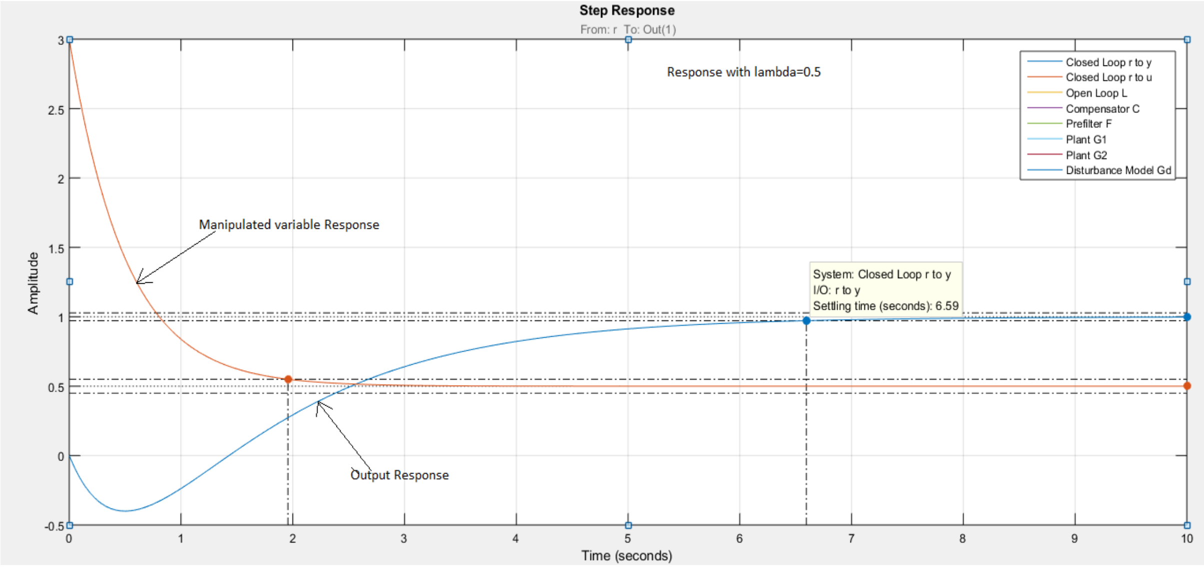

Figure 5.4: Unit Step Response of IMC for 2nd Order NMP System [lambda = 0.5]

For Lambda = 0.72

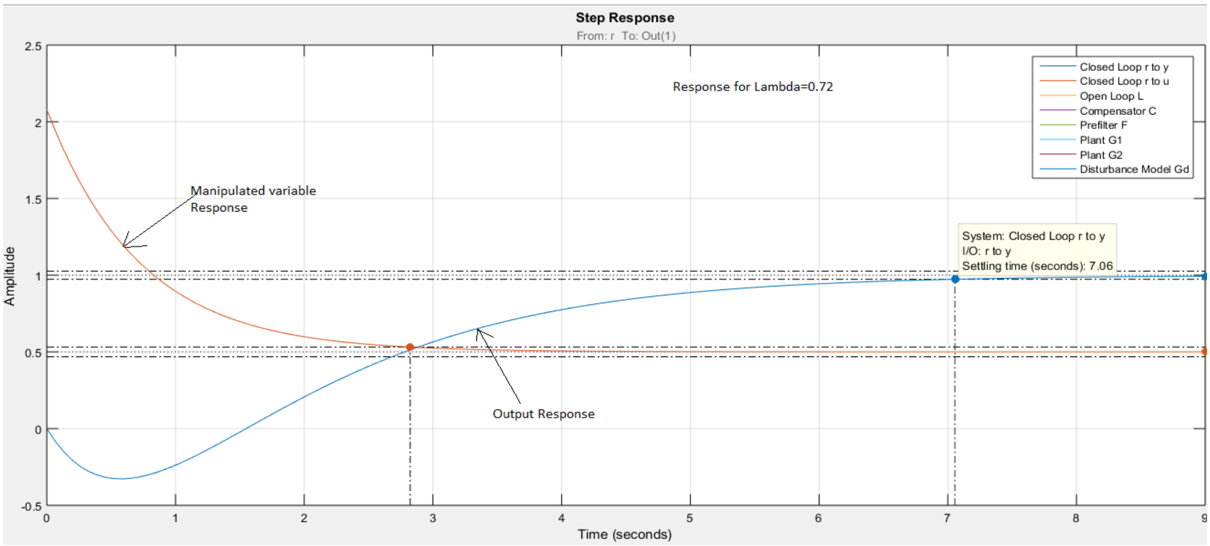

Figure 5.5: Unit Step Response of IMC for 2nd Order NMP System [lambda = 0.72]

For Lambda=1.0

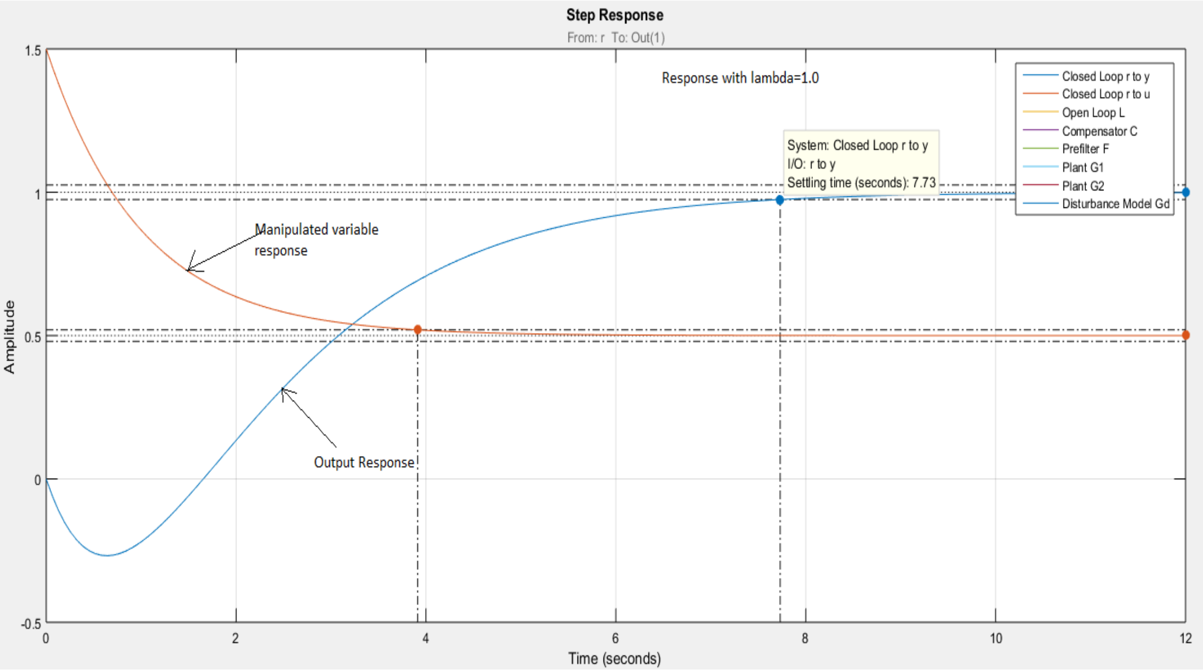

Figure 5.6: Simulation of IMC for 2nd Order NMP System [lambda = 1.0]

Table 5.1: Settling time for NMP System for different lambda value

Lambda, λ	**Settling time (Sec)**
$\boldsymbol{\lambda = 0.5}$	6.59
$\boldsymbol{\lambda = 0.72}$	7.06
$\boldsymbol{\lambda = 1.0}$	7.73

5.4 Summary

IMC is used to minimize the effect of load disturbances and parameter change. The tuning of controller is also easily done due to single tuning parameter. The response of plant by using IMC is well enhanced. From the above Table 5.1, it is clear that as the value of Lambda increases from 0.5 to 1.0, the settling time also increases. It means that the lower value of lambda results the faster response. But, with the lower lambda value the manipulated variable should have large effort at starting, which physically may not possible after a certain value of lambda. According to table 4.2, lambda=0.72 gives the optimal response.

6 Performance of NMP Stable Systems with IMC Based PID Controller

6.1 Introduction

As with the IMC procedure, there is just only one tuning parameter lambda (λ) which makes the controller design simpler. For minimum phase system it is equivalent to the time-constant of the process or the speed of response of the closed-loop system. IMC performs well for non-minimum phase systems. Even IMC is easy to implement and simpler to tune, the PID controller is the most widely used all across industries. In this chapter, IMC procedure is analysed as the feedback configuration by rearranging the control structure and implement it as a PID controller. The IMC-PID controller tuning technique not only benefits from internal model control, but also incorporates traditional PID controller characteristics [39]. Typical tuning approaches [40] [41] based on attaining a desirable closed-loop response include the IMC-PID tuning methods and the direct synthesis (DS) method [42]. For lag-dominant (including integrating) processes with τ/θ larger than around 10, both techniques provide excellent performance for setpoint changes but slow reactions to input (load) perturbations. Shamsuzzoha and Lee (2007) presented an IMC filter for a variety of processes in order to produce a PID controller with improved disturbance rejection.

To tune the controller, there is one simple classical method is Ziegler-Nichols (1942) tuning approach, which needs less amount of actual information about the process, it needs mainly the input and output process characteristics. But it has several disadvantages so, it is not well suited for all types of systems. Shamsuzzoha and Skogestad (2010) have proposed a novel approach for the closed-loop adjustment of the PI/PID method. Their solution is based on the SIMC tuning rule, and it produces excellent results in terms of both performance and resilience. To get the PI controller setting, Shamsuzzoha and Skogestad used a one-step closed-loop test. On the basis of the past knowledge obtained from the P controller test, they must redo the experiment with a PD controller for the PID tuning parameter. They suggested that the derivative action be included only for dominating second-order processes. This tuning method provides better performance and robustness.

6.2 IMC-PID Controller Design

The IMC structure may be reconfigured to build a typical feedback control system that can readily manage open loop unstable systems. The controller Qc(s) in the IMC technique is directly based on the acceptable part of the process transfer function. In addition, when using the IMC formulation, there is usually just one tuning parameter, the close loop time constant (filter tuning factor). The PID tuning settings based on IMC are thus a function of this time constant. The robustness is closely connected to the closed loop time constant selection (sensitivity to the modular of the closed loop system). The point of comparison between the process and the model output can be shifted in the IMC structure to produce a standard feedback structure, which is nothing more than another equivalent feedback form of the IMC structure known as IMC based PID structure, as illustrated in the picture below.

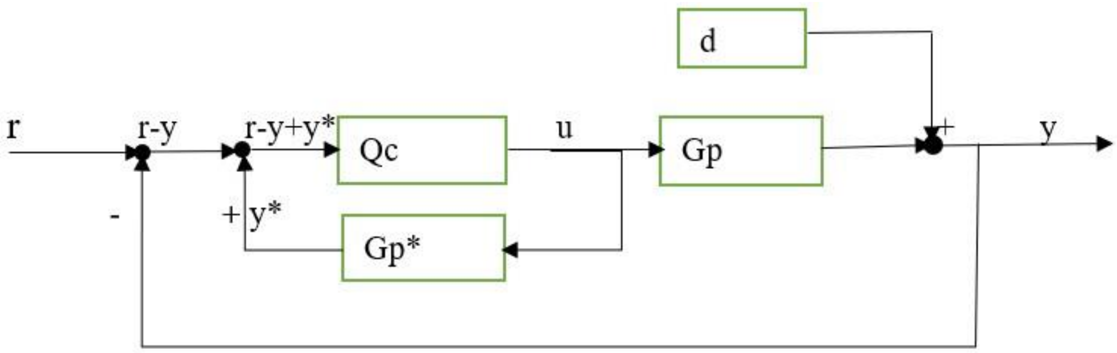

Figure 6.1: IMC-Based PID Control Architecture

Consider a $G_p^*(s)$ process model for a real-world process or plant $G_p(s)$. The disturbances d(s) enter the system, and the controller Qc(s) is employed to govern the process. The following are the steps involved in designing an Internal Model Control (IMC) system:

6.2.1 Factorization

It entails breaking down a transfer function into invertible (good) and non-invertible (bad) parts. Mathematically it is given as,

$$G_p^* = G_p^{*(+)} \cdot G_p^{*(-)} \qquad (6.1)$$

Here, $G_p^{*(+)}$ is non − invertible part and $G_p^{*(-)}$ is invertible part.

Factorization is same as explained in chapter-4 (5.2 IMC Design Procedure for NMP Systems).

6.2.2 Ideal IMC Controller with filter

The Ideal IMC controller deals with the inverse of the invertible portion of the process model. Then, it uses a filter to further make the controller TF proper to fully utilize the controlling techniques. It’s transfer function can be written as:

$$Q_c(s) = Q_c^*(s) \cdot f(s) = \frac{1}{G_p^{*(-)}(s)} \cdot f(s) \tag{6.2}$$

The filter designing techniques are same as discussed in Chapter-4 (5.2 IMC Design Procedure for NMP Systems).

6.2.3 Equivalent Feedback Structure

By re-configuring the IMC structure as given in Figure 5.1, we obtain as given in Figure 6.1, Now, by simplifying the circuit diagram of Figure 6.1 an equivalent standard feedback structure is formed, then the controller TF Gc(s) can be written as:

$$Gc(s) = \frac{Qc(s)}{(1 - Qc(s) \cdot \mathrm{G_p^*}(s))} \tag{6.3}$$

6.3 IMC-PID Tuning Rules for Typical NMP Stable Process Models

The aim of this section is to find the most suitable IMC based PID controller tuning for the NMP systems with different transfer function model (as specified in Chapter-1).

System-1 TF model is Pade' 1^{st} order approximated model for a time-delay system and System-2 are stable systems, which show non-minimum phase behaviour:

Consider a 2^{nd} order NMP system with RHP zero. So, let's assume the generalised form for process model is:

$$Gp^*(s) = \frac{Kp^*(-\beta s + 1)}{\tau^2 s^2 + 2\zeta\tau s + 1} \tag{6.4}$$

I need to control this plant behaviour when applying step input to it with equivalent PID controller and PID with filter control.

6.3.1 IMC-PID Control

Step-1: Factorization of Transfer-function model

Using the Integral-Absolute-Error (IAE) optimal factorization for step input, transfer-function may be written as a combination of invertible and non-invertible component as:

$$\mathrm{G_p^*} = G_p^{*(+)}(s) \cdot G_p^{*(-)}(s) = (-\beta s + 1) \cdot \frac{Kp^*}{\tau^2 s^2 + 2\zeta\tau s + 1} \tag{6.5}$$

$$Q_c^*(s) = \frac{1}{G_p^{*(-)}(s)} = \frac{\tau^2 s^2 + 2\zeta\tau s + 1}{Kp^*} \tag{6.6}$$

Step-2: Apply a filter

Let's apply a 1st order filter to the controller, even it presents an improper IMC controller model.

$$f(s) = \frac{1}{(\lambda\, s+1)} \tag{6.7}$$

$$Q_c(s) = Q_c^*(s)\cdot f(s) = \frac{1}{G_p^{*(-)}(s)}\cdot f(s) = \frac{\tau^2 s^2 + 2\zeta\tau s + 1}{Kp^* \cdot (\lambda\, s+1)} \tag{6.8}$$

Step-3: Classical feedback Controller

Equivalent feedback controller structure obtained via transformation from IMC structure results a form of PID controller, leads to a tuning rule with the help of tuning parameter (lambda, λ) for an ideal PID controller.

$$\mathrm{Gc}(s) = \frac{\mathrm{Qc}(s)}{(1-\mathrm{Qc}(s)\cdot \mathrm{G_p^*}(s))} = \frac{\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{\mathrm{Kp}^*}\cdot \frac{1}{(\lambda\cdot s+1)}}{1-\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{\mathrm{Kp}^*}\cdot\frac{1}{(\lambda\cdot s+1)}\cdot\frac{\mathrm{Kp}^*(-\beta s+1)}{\tau^2 s^2 + 2\zeta\tau s + 1}} \tag{6.9}$$

$$= \frac{2\zeta\tau}{(\beta+\lambda)\cdot \mathrm{Kp}^*}\cdot\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{2\zeta\tau\cdot s}$$

(This transfer function is regarded as the equivalent standard feedback controller).

Transfer Function for ideal PID controller for second order is written as:

$$Gc(s) = Kc\cdot\left(1+\frac{1}{Ti\cdot s}+Td\cdot s\right) = Kc\cdot\frac{(Ti\cdot Td\cdot s^2 + Ti\cdot s + 1)}{Ti\cdot s} \tag{6.10}$$

Now, by comparing equivalent transfer function of IMC based PID controller with transfer function for ideal PID controller, PID tuning parameters are expressed as:

$$Kc = \frac{2\zeta\tau}{(\beta+\lambda)\cdot Kp^*}$$
$$Ti = 2\zeta\tau \tag{6.11}$$
$$Td = \frac{\tau}{2\zeta}$$

6.3.2 IMC-PID with Filter Control

Step-1: Factorization of Transfer-function model

Using the Integral-square-Error (ISE) optimal factorization for step input, transfer-function may be written as a combination of invertible and non-invertible component as:

$$G_p^{*(+)}(s) = \frac{(-\beta s+1)}{(\beta s+1)} \text{ and } G_p^{*(-)}(s) = \frac{Kp^*(\beta s+1)}{\tau^2 s^2 + 2\zeta\tau s + 1} \tag{6.12}$$

$$Q_c^*(s) = \frac{1}{G_p^{*(-)}(s)} = \frac{\tau^2 s^2 + 2\zeta\tau s + 1}{Kp^*(\beta s + 1)} \quad (6.13)$$

Step-2: Apply a filter

Let's apply a 1st order filter to the controller, leads to semi-proper IMC model.

$$f(s) = \frac{1}{(\lambda s + 1)} \quad (6.14)$$

$$Q_c(s) = Q_c^*(s) \cdot f(s) = \frac{1}{G_p^{*(-)}(s)} \cdot f(s) = \frac{\tau^2 s^2 + 2\zeta\tau s + 1}{Kp^*(\beta s + 1) \cdot (\lambda s + 1)} \quad (6.15)$$

Step-3: Classical feedback Controller

Equivalent feedback controller structure obtained via transformation from IMC structure results a form of PID controller, leads to a tuning rule with the help of tuning parameter (lambda, λ) for an ideal PID controller.

$$Gc(s) = \frac{Qc(s)}{(1 - Qc(s) \cdot \mathrm{G}_\mathrm{p}^*(s))}$$

$$= \frac{\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{Kp^* \cdot (\beta s + 1)} \cdot \frac{1}{(\lambda \cdot s + 1)}}{1 - \frac{\tau^2 s^2 + 2\zeta\tau s + 1}{Kp^* \cdot (\beta s + 1)} \cdot \frac{1}{(\lambda s + 1)} \cdot \frac{Kp^*(-\beta s + 1)}{\tau^2 s^2 + 2\zeta\tau s + 1}} \quad (6.16)$$

$$= \frac{2\zeta\tau}{(2\beta + \lambda) \cdot Kp^*} \cdot \frac{\tau^2 s^2 + 2\zeta\tau s + 1}{2\zeta\tau \cdot s} \cdot \frac{1}{\frac{\beta \cdot \lambda}{2\beta + \lambda} \cdot s + 1}$$

(This transfer function is regarded as the equivalent standard feedback controller)

Transfer Function for ideal PID + filter controller for second order is written as:

$$Gc(s) = Kc \cdot (1 + \frac{1}{Ti \cdot s} + Td \cdot s) \cdot (\frac{1}{T_f s + 1})$$

$$= Kc \cdot \frac{(Ti \cdot Td \cdot s^2 + Ti \cdot s + 1)}{Ti \cdot s} \cdot (\frac{1}{T_f s + 1}) \quad (6.17)$$

Now, by comparing equivalent transfer function of IMC based PID with filter controller with transfer function for ideal PID controller, PID tuning parameters are expressed as:

$$\mathrm{Kc} = \frac{2\zeta\tau}{(2\beta + \lambda) \cdot \mathrm{Kp}^*}$$

$$\mathrm{Ti} = 2\zeta\tau, \mathrm{Td} = \frac{\tau}{2\zeta}, \text{and } \mathrm{T_f} = \frac{\beta \cdot \lambda}{2\beta + \lambda} \quad (6.18)$$

6.4 Performance Measure

To check the quality of performance of the closed-loop system, let's consider a unit-step set-point change and a unit step change in the input (load) disturbance for both input and output performance.

6.4.1 Input-Performance

It measures the smoothness of the manipulated variable. Here, total variation (TV) can be found as the total sum of the ups and downs in the input signal u(t). In continuous time-domain, it is difficult to find TV, so the discrete time-domain can be utilised here by discretizing the continuous time signal as given below:

$$TV\ (total\ variation) = \sum_{i=1}^{\infty} |u_{i+1} - u_i| \tag{6.19}$$

6.4.2 Output-Performance

The output-performance can be calculated by finding the total integral sum of deviations between the actual path followed and the desired path to be followed in terms of IAE (Integral Absolute Error) or ISE (Integral Square Error). It should be minimum as much as possible.

Here, the control error is:

$$e = y(t) - y_s(t) \tag{6.20}$$

y(t) is actual output and $y_s(t)$ is the set-output or desired output.

IAE (Integral Absolute Error) can be calculated as:

$$IAE = \int_0^{\infty} |e(t)| dt \tag{6.21}$$

And, ISE (Integral Square Error) can be calculated as:

$$ISE = \int_0^{\infty} (e(t))^2 dt \tag{6.22}$$

And, ITAE (Integral Time Absolute Error) can be calculated as:

$$ITAE = \int_0^{\infty} t|e(t)| dt \tag{6.23}$$

6.5 Simulation Results

The efficiency of an IMC-based PID Controller for a practical NMP system (Coupled-Tank System) is demonstrated through simulation results.

6.5.1 Simulation Results for Stable NMP System

Let's consider transfer function of Stable NMP System (System-1) is given as:

$$G1(s) = \frac{(-1.4s + 1)}{(4.2s^2 + 4.4s + 1)} \quad (6.24)$$

Let's apply IMC-PID with filter controller for simulation of system-1. Then the parameters for PID controller can be expressed by using equation (6.18) is:

$$Kc = \frac{4.387}{(2.8 + \lambda)} \text{ and } Ti = 4.387 \text{ and } Td = 0.96 \text{ and } T_f = \frac{1.4\lambda}{2.8 + \lambda} \quad (6.25)$$

6.5.1.1 Simulation Results for different values of 'λ'

For IMC-PID tuning, simulations were done with different values of 'λ' for system-1. The simulation response in Figure 6.2 illustrates the behaviour of the final-response as the value of 'λ' increases. The fluctuation of Gain and Phase Margin with 'λ' is shown in Table 6.1, since Gain and Phase Margins describe the controller's robustness.

Table 6.1: PID Parameters of System for Different Values of 'λ'

'λ' >2.24	K_c	T_i	$K_i = \frac{K_c}{T_i}$	T_d	$K_d = K_c \cdot T_d$	T_f	$N = \frac{1}{T_f}$	Gain Margin	Phase Margin
3	0.756	4.387	0.172	0.96	0.7258	0.724	1.381	9.85 dB	76.5 deg
4	0.645	4.387	0.147	0.96	0.6192	0.823	1.215	11.1 dB	78.4 deg
6	0.499	4.387	0.114	0.96	0.4790	0.955	1.047	13.2 dB	80.9 deg
8	0.406	4.387	0.092	0.96	0.3897	1.037	0.964	15 dB	82.8 deg

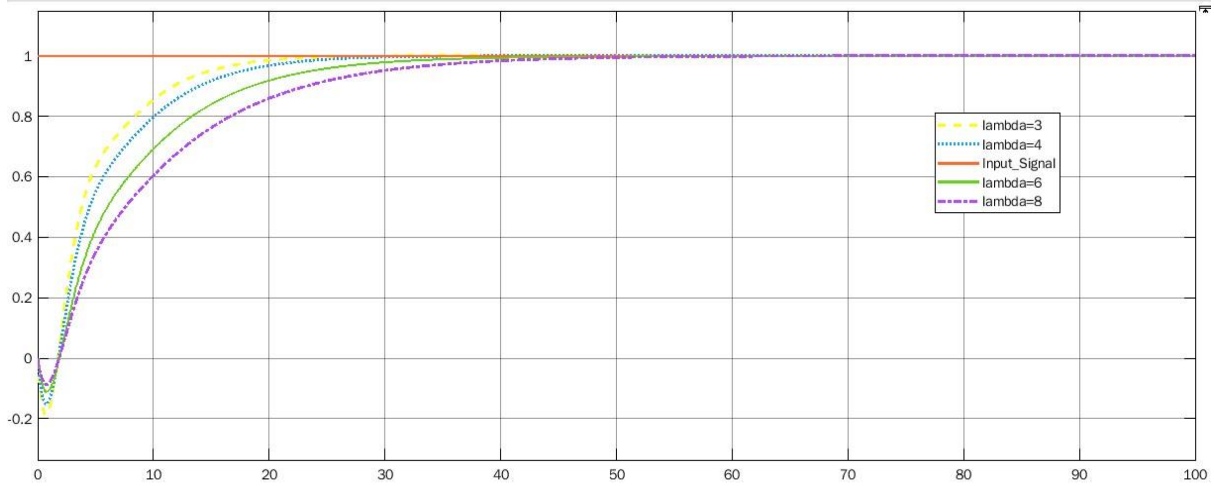

Figure 6.2: Unit Step Response for various λ values

6.5.1.2 Simulation Results for Performance

The performance of the controller is analysed by obtaining performance indices such as IAE, ISE, and ITAE, along with evaluating time response characteristics such as settling time (t_s), rise time (t_r), and peak overshoot using closed-loop simulation in MATLAB. For several NMP Systems, simulations are performed for variations in set-point and disturbance at t=100 sec.

The IMC-PID controller's set-point tracking and disturbance rejection capabilities are demonstrated in simulation responses.

Table 6.2: Performance results of IMC-PID for System with different $\boldsymbol{\lambda}$

Specifications	$\boldsymbol{\lambda = 3}$	$\boldsymbol{\lambda = 4}$	$\boldsymbol{\lambda = 6}$	$\boldsymbol{\lambda = 8}$
Rise Time (t_r in sec)	9.67	11.8	16	20.6
Settling Time (sec)	18.8	22.5	30	38.4
Peak Overshoot (%)	0	0	0	0
Peak value	0.999	0.998	1	1
IAE (Integral Absolute Error)	5.814	6.803	8.772	10.87
ISE (Integral Square Error)	3.908	4.346	5.266	6.259

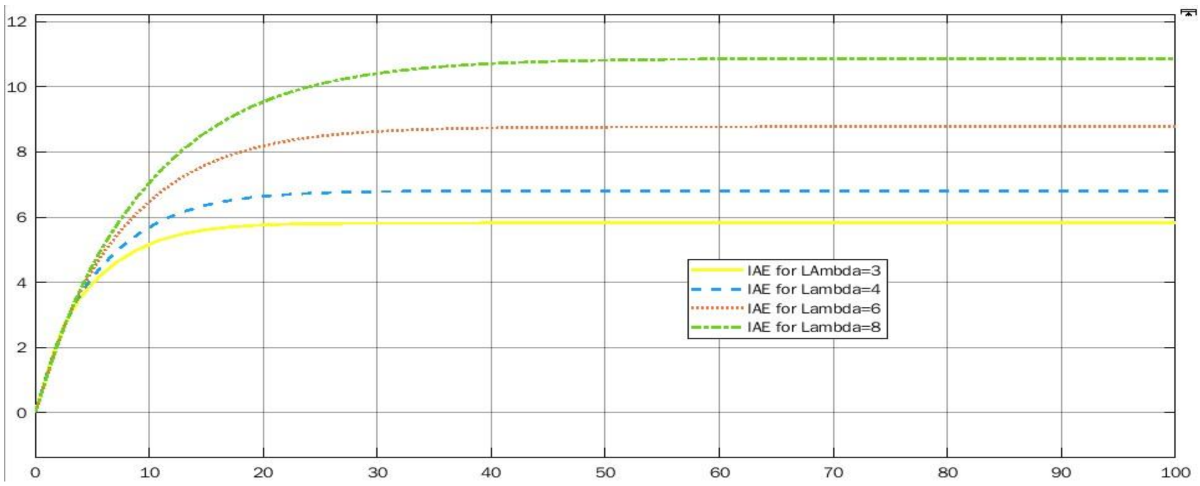

Figure 6.3: Simulation response of Integral of Absolute value of error (IAE) for different tuning parameter values ($\boldsymbol{\lambda}$)

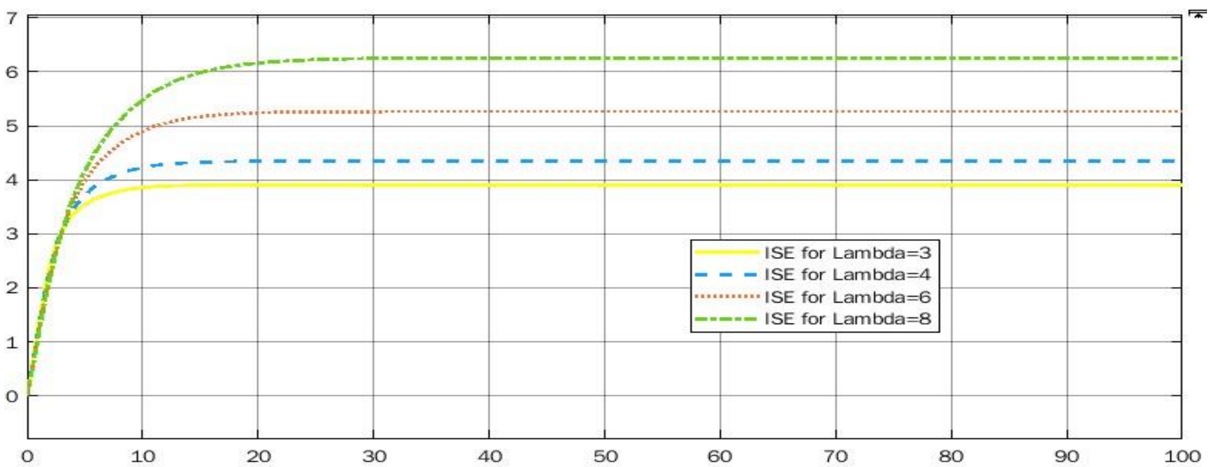

Figure 6.4: Simulation response of Integral of Square value of error (ISE) for different tuning parameter values (λ)

6.6 Summary

IMC is used to minimize the effect of load disturbances and parameter change. But the IMC-based PID also exhibits better tracking, load disturbance, and transient responses. The fundamental purpose of an IMC-based PID controller is to regulate system speed. It is one type of automatic way to control the external or internal disturbances. It works well for integrating system, time-delay system and for both minimum and non-minimum phase type systems. Here in case of non-minimum phase system as explain by using system-1(stable NMP), simulation results show that by using only one control parameter (lambda), controller is able to control the NMP process up to a significant level. With little undershoot, the controller can achieve a fairly quick response from the system having optimised rise and settling time. Performance outcomes were optimised to a decent extent (Figure 6.3 -6.4).

7 NMP Systems Performance with Feed-Forward Compensator

7.1 Introduction

Designing a feedback control system for a plant with positive real parts and zeros is always a difficult task, as these systems are known to be unstable when controller gains are large. A small controller gains, result in poor closed loop system transient performance. A traditional solution entails the creation of a sophisticated control law. This chapter will describe an extra parallel feedforward compensator (PFC) that can be used to maintain closed loop stability, resulting in an enhanced system with redefined output. The parallel connectivity of the plant and the compensator allows the system zeros to be assigned in the left half plane as a first step. To stabilise the plant, a typical feedback controller with no gain constraints can be used in a second stage [43]. In this chapter, a straightforward design strategy for stable NMP system and a class of unstable systems with unstable zero dynamics is provided.

Non-minimum phase zeros have a number of constraints that may be addressed using a variety of ways. Feedforward compensation [44] [45], which involves designing a modified transfer function by cancelling the plant response, is applicable if the system's poles are stable [46] [47], but if the poles are unstable, this results in cancelling unstable plant behaviour by designing an unstable compensator [48]. This chapter explains the designing of feedforward compensators for both stable and unstable NMP systems.

7.2 Designing of Feedforward Compensators

The transfer function of SISO NMP systems is decomposed into two sub-systems as the first phase. One should be stable including NMP or MP subsystems, while the other will adopt unstable part with MP subsystems. The non-minimum phase zeros may then be relocated to the left side of the S plane using inverse root locus procedures for system, making the system minimum phase. Finally, a large gain can be used to get the desired performance. A simple structure of feedforward compensator is shown below.

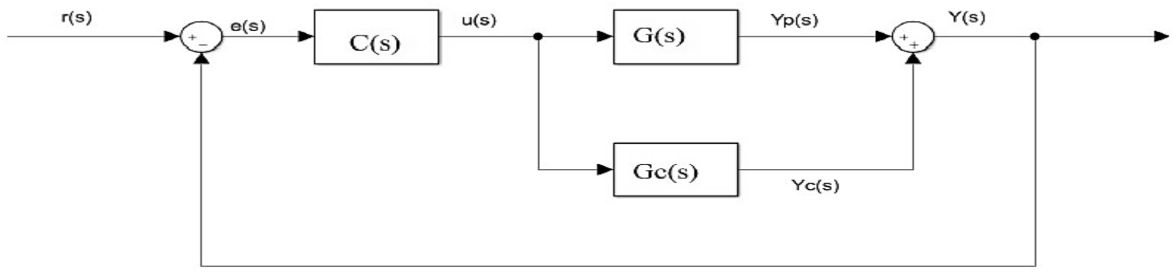

Figure 7.1: Block Diagram for system with parallel compensator and feedback controller

Here in Figure 7.1,

$\mathrm{G(s)}, \mathrm{G_c(s)}$ is plant and parallel copmensator transfer function respectively *and*

$Y_p(\mathrm{s}), \mathrm{Y_c(s)}$ is plant and parallel copmensator output and $\mathrm{u(s)}$is control signal and

$\mathrm{C(s)}$ is controller and $\mathrm{Y(s)}$is the resultant output of compensated plant.

Let's consider $G_1(s)$ is the TF which is selected as the desired one. Then,

$$\mathrm{G(s)} = \frac{Y_p(\mathrm{s})}{u(s)} \; and \; \mathrm{G_c(s)} = \frac{\mathrm{Y_c(s)}}{u(s)} = G_1(s) - G(s) \tag{7.1}$$

$$\frac{\mathrm{Y(s)}}{u(s)} = G(s) + \mathrm{G_c(s)} = G(s) + G_1(s) - G(s) = G_1(s) \tag{7.2}$$

Here, it is assuring that,

$$G(s = 0) = G_1(s = 0) \tag{7.3}$$

7.2.1 Stable NMP System

As in Figure 7.1, $G(s)$ is plant TF. Assuming that the order of denominator is greater than or equal to the order of numerator. The feedforward compensator is designed by decomposing the plant's transfer function G(s) into two subsystems, G1(s) and G2(s), as illustrated in Figure 7.2. Here, G2(s) contains all NMP zeros and G1(s) will have relative degree at least 1(one) to make the overall augmented plant strictly proper, which help to achieve the steady-state value. The feedforward gain, L is designed for G2(s) using inverse root locus. After proper selection of L-value, the whole augmented plant becomes minimum-phase, which removes the restriction upon range of proportional gain "K" and system will be able to track the reference signal.

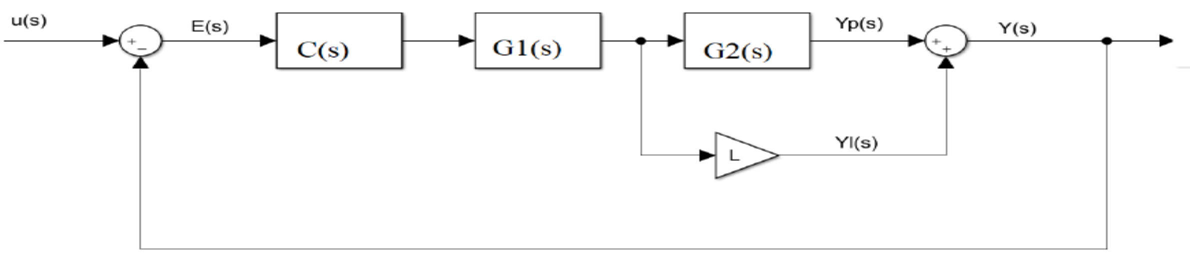

Figure 7.2: Decomposed Transfer Function with Feedforward Gain L [44]

7.2.2 Unstable NMP System

The sole requirement that must be met in order to use the suggested decomposition method is that the number of non-minimum phase zeros must be less than or equal to the number of stable poles. In case of unstable system, in Figure 7.2, G2(s) contains all NMP zeros and stable poles and G1(s) contains the remaining part of plant TF with relative degree at least 1 (one).

Feedforward gain, L is designed in such a way that G2(s) will become minimum-phase by using inverse root-locus. But system is still remains unstable so, the poles may be relocated to the left half of the S plane with the correct choice of feedback gain.

7.3 Simulation

Unit step signal is used to simulate the response of stable and Unstable NMP systems by using the feed-forward compensators and analysing the root-locus of plant alone and augmented plant. All simulation experiments were carried out in the MATLAB environment.

7.3.1 Simulation for Stable NMP System

Let's assume the stable NMP plant (System-1) TF is given as:

$$G_p(s) = \frac{(-1.4s + 1)}{(4.2s^2 + 4.4s + 1)} \tag{7.4}$$

Here, plant TF can be decomposed in G1(s) and G2(s) as given below,

$$G1(s) = \frac{1}{(3s+1)} \, and \; G2(s) = \frac{(-1.4s+1)}{(1.4s+1)} \tag{7.5}$$

And, the augmented plant (plant with compensator) with feedforward gain, L will be:

$$G(s) = \frac{1}{(3s+1)}\left\{\frac{(-1.4s+1)}{(1.4s+1)} + L\right\} = \frac{(-1.4s+1) + L(1.4s+1)}{(3s+1)(1.4s+1)} \tag{7.6}$$

The estimated value of L can be found out from inverse root-locus of G2(s) TF to relocate the RHP zeros to LHP. By selecting $L = 3$, the RHP zeros at $s = 0.714$ is shifted to left-half of s-plane at $s = -1.428$. Figure 7.3 shows root-locus of original plant and Figure 7.4 shows root-locus of augmented plant model.

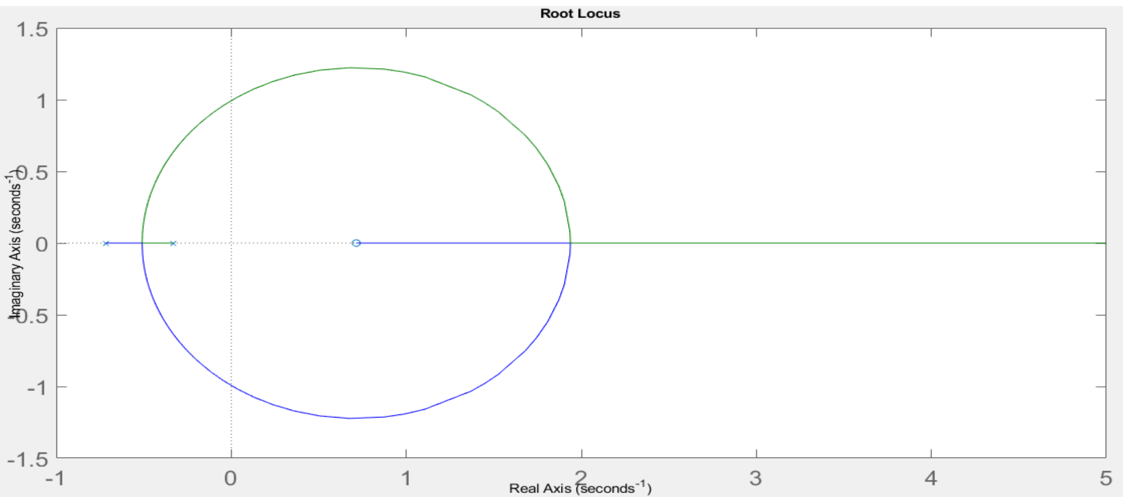

Figure 7.3: Root-Locus of Actual Plant without FeedForward Gain

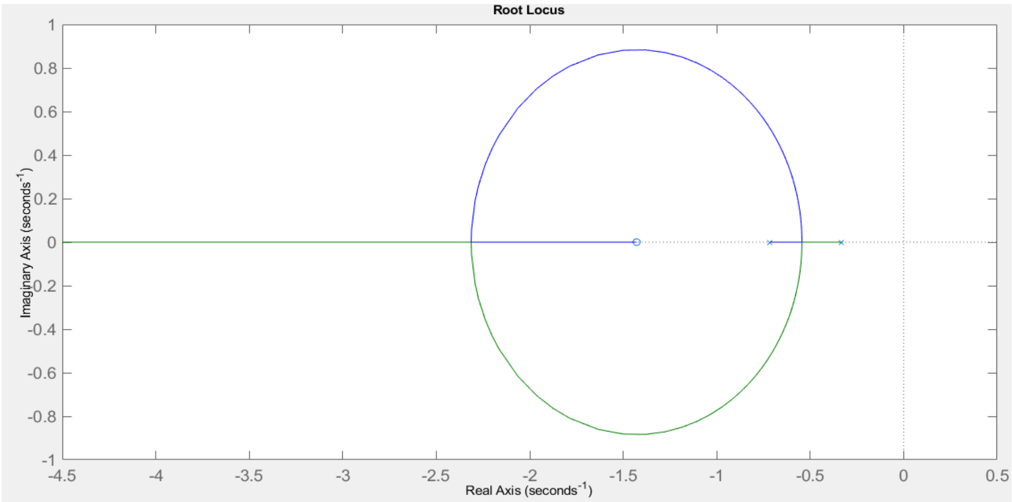

Figure 7.4: Root-Locus Plot of Plant with FeedForward Gain

Now, with feedback gain “K” is used to obtain the desired performance of the augmented plant by varying the locations of the closed-loop system. With $K = 20$ the closed loop poles of Eq. (7.6) are -12.88 and -1.5. The augmented plant response is able to track the reference step signal properly using the specified feedback controller K, as shown in Figure 7.6.

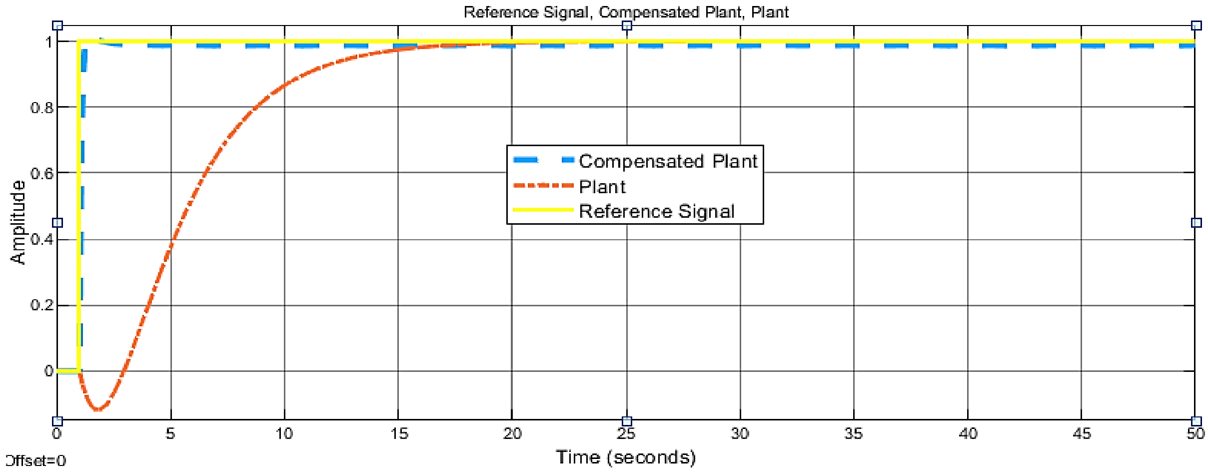

Figure 7.6: Unit Step Response of Feed-Forward Controlled NMP System (K=20, L=3)

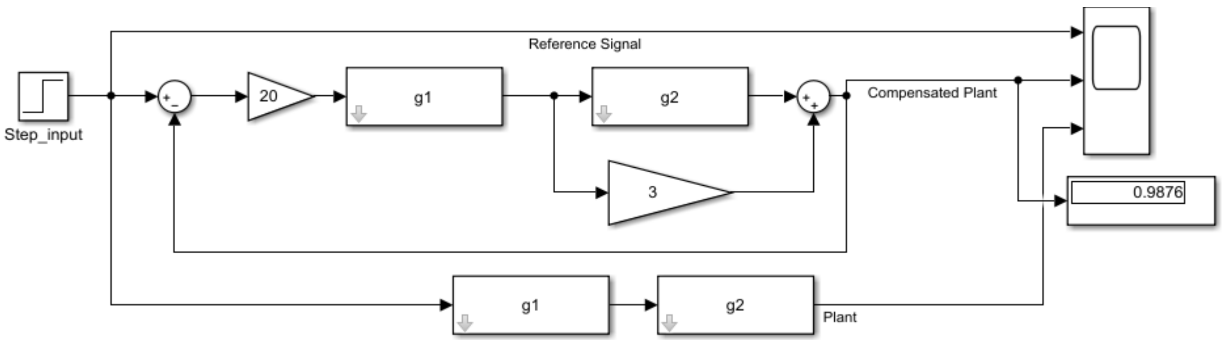

Figure 7.5: Simulink Block diagram for Augmented Plant

7.3.2 Simulation for Unstable NMP system

Let’s assume the unstable NMP plant (System-2) TF is given as:

$$G_p(s) = \frac{5(s-5)}{(s-2)(s+7)} \tag{7.7}$$

Here, plant TF Eq. (7.6) can be decomposed in G1(s) and G2(s) as given below,

$$G1(s) = \frac{5}{(s-2)} \; and \; G2(s) = \frac{(s-5)}{(s+7)} \tag{7.8}$$

And, the augmented plant (plant with compensator) with feedforward gain, L will be:

$$G(s) = \frac{5}{(s-2)}\left\{\frac{(s-5)}{(s+7)} + L\right\} = \frac{5\{(s-5) + L(s+7)\}}{(s-2)(s+7)} \tag{7.9}$$

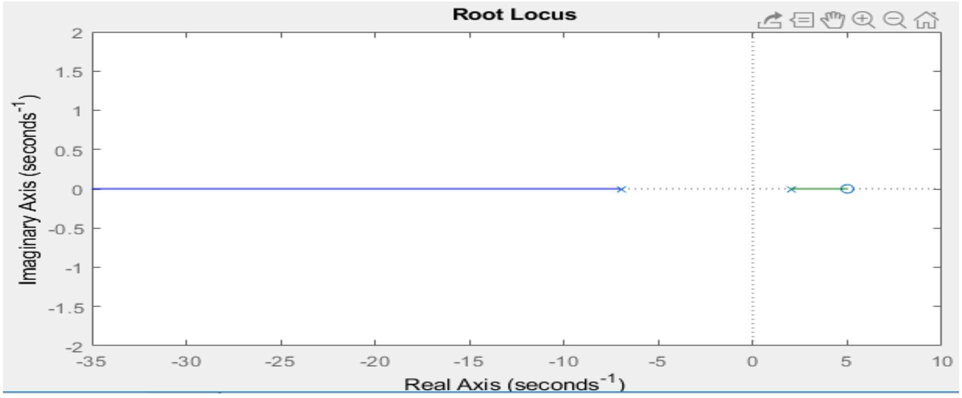

Figure 7.7: Root-Locus of Actual Plant without FeedForward Gain (System-2, Unstable)

The estimated value of L can be found out from inverse root-locus of G2(s) TF to relocate the RHP zeros to LHP. By selecting $L = 3$, the RHP zeros at $s = 5$ is shifted to left-half of s-plane at $s = -4.0$. Figure 7.7 shows root-locus of original unstable plant and Figure 7.8 shows root-locus of augmented plant model.

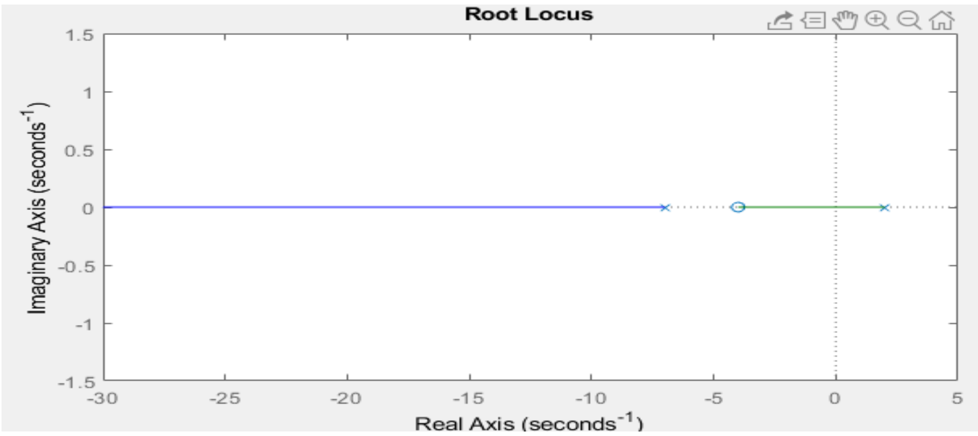

Figure 7.8: Root-Locus of Plant with FeedForward Gain (Unstable System)

Now, with feedback gain "K" is used to obtain the desired performance of the augmented plant by varying the locations of the closed-loop system. With $K = 10$ the closed loop poles of Eq. (7.9) are -201.1 and -3.91. The augmented plant response is able to track the reference step signal properly using the specified feedback controller K, as shown in Figure 7.9.

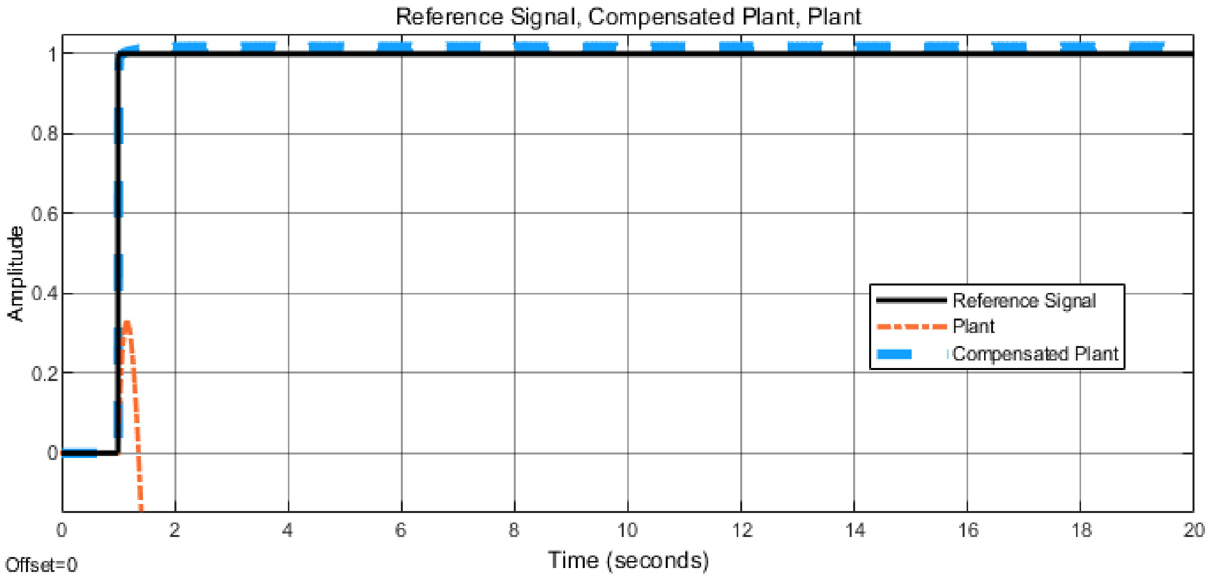

Figure 7.9: Unit Step Input Response of Controlled Unstable Plant (K=10, L=3)

7.4 Conclusion

Here, in the case of an NMP system with both stable and unstable conditions, a feed-forward compensator is used. A feedforward gain with the NMP component of plant makes that part of TF a minimum phase and rest part of plant-TF makes the whole augmented plant strictly proper. Most importantly, it works well in case of an unstable NMP system by turning the NMP component of the TF into minimum phase and shifting the unstable poles to the left-half of the s-plane by selecting a correct proportional gain “K”. The PFC and the plant are connected in parallel, resulting in a unique system with asymptotically stable zero dynamics, allowing for the application of traditional feedback control techniques without gain restrictions. For a number of NMP systems, this technique is simple to implement. The augmented plant with proper feed-forward gain "L" and exact selection of feedback gain, both types of system are able to track the reference signal in a well-mannered manner with a high accuracy and response time is also very short to achieve the negligible steady-state error value, as shown in the simulation response.

8 NMP Systems Performance with Model Reference Controller (MRC)

8.1 Introduction

Model reference control refers to obtaining the actual plant output same as of the reference model. Non-minimum phase systems cannot be controlled using control strategies like model reference control [49] [50]. The control signal is constructed in such a manner that it causes the system to act in accordance with the reference model chosen. As a result, the plant's zeros and poles must be cancelled in order to achieve the reference model behaviour. If the plant contains zeros in the right half of the s-plane, the compensator will have to introduce an unstable pole, which is not advised for robustness. To produce the enhanced plant's minimal phase, we apply a feedforward compensation approach. Then, using the current control schemes, controllers for the augmented plant (plant with feedforward compensator) may be designed.

We may design feedforward compensators for systems that are non-minimum phase and stable or non-minimum phase and unstable by using the decomposition technique. The process model's transfer function is decomposed into two sub-systems, one of which is stable but not in the minimum phase, and the other of which is unstable but in the minimum phase. The non-minimum phase zeros may then be shifted to the left side of the s-plane using root-locus procedures, bringing the system into minimal phase. Model Reference Control is not possible to implement on non-minimum phase systems, now can be implemented on the resulting feedforward compensated system.

8.2 Model Reference Control

Let us consider the reference model and plant model is defined as:

$$Y_m(s) = W(s) \cdot R(s) \tag{8.1}$$

$$W(s) = \frac{K_m \cdot Z(s)}{R(s)} \tag{8.2}$$

And,

$$Y_p(s) = G(s) \cdot U(s) \tag{8.3}$$

$$G(s) = \frac{K_p \cdot N(s)}{D(s)} \tag{8.4}$$

Where, K_p and K_m are the gains of plant and reference model respectively.

It is also assumed that R(s), Z(s), D(s) and N(s) are monic polynomials.

The basic goal of model reference control is to determine the plant input u(s) or controlled variable, ensuring that all signals are constrained and that the plant's output closely resembles the output of the reference model. On the plant and reference model, the aforementioned assumptions should be assumed. [50].

Plant:

P1. N(s) is a monic Hurwitz (negative roots) polynomials of degree m.
P2. An upper bound n, of degree n_p of D(s) is known.
P3. The relative degree, j=n-m is known.
P4. The sign of the high frequency gain K_p is also known.

Reference Model:

M1. Z(s) and R(s) are monic Hurwitz (negative roots) of degree p and q respectively.
M2. The relative degree s=p-q of W(s) is same as that of G(s).

The block diagram for this control scheme with closed-loop can be represented as:

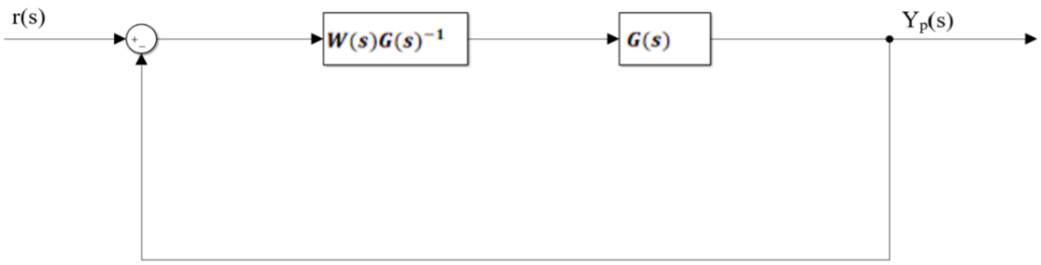

Figure 8.1: Basic Block Diagram of Plant with Controller [50]

So, the input signal to the plant will be:

$$u(s) = C(s) \cdot r(s) \tag{8.5}$$

$$C(s) = W(s) \cdot G(s)^{-1} = \frac{K_m}{K_p} \cdot \frac{Z(s)}{R(s)} \cdot \frac{D(s)}{N(s)} \tag{8.6}$$

$$\frac{Y_p(s)}{r(s)} = \frac{K_m}{K_p} \cdot \frac{Z(s)}{R(s)} \cdot \frac{D(s)}{N(s)} \cdot \frac{N(s)}{D(s)} \cdot K_p = W(s) \tag{8.7}$$

As here it is clear that the pole-zero cancellation occurs here. This control law is acceptable only when the plant is stable. Pole-zero cancellation happens beyond the left half of S-palne if the plant's poles are unstable, resulting in unbounded internal states with non-zero initial conditions.

As a result, the control law should be changed to handle such cases. The control law in equation (8.5) has been changed to equation (8.8) as follows:

$$u(s) = \theta_1^T \cdot \frac{\alpha(s)u(s)}{\Lambda(s)} + \theta_2^T \cdot \frac{\alpha(s)Y_p(s)}{\Lambda(s)} + \theta_3^T \cdot Y_p + C_0 r(s) \tag{8.8}$$

Here,

$$K_1 = \theta_1^T \cdot \frac{\alpha(s)u(s)}{\Lambda(s)} \tag{8.9}$$

$$K_2 = \theta_2^T \cdot \frac{\alpha(s)Y_p(s)}{\Lambda(s)} \tag{8.10}$$

The complete derivation of the above mentioned steps and for selection of the values for θ_1, θ_2, θ_3, $\Lambda(s)$, $\alpha(s)$, C_0 are given in pages (333-342) of [50].

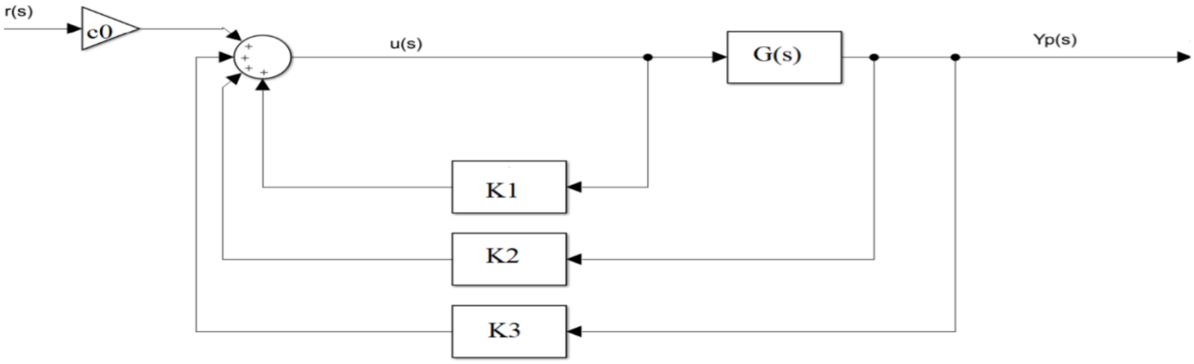

Figure 8.2: Modified Block Diagram for Model Reference control [50]

I apply this design procedure for model refrence control to solve the problem of non-minimum phase system which is unstable too.

8.3 Simulation

Unit step signal, Square-wave signal and Sin-wave signal are used to simulate the response of stable and Unstable NMP systems by using model reference controller technique. All simulation experiments were carried out in the MATLAB environment.

8.3.1 Simulation for Stable NMP System

Let's assume the unstable NMP plant (System-1) TF is given as:

$$G(s) = \frac{(-1.4s + 1)}{(4.2s^2 + 4.4s + 1)} \tag{8.11}$$

And, Reference model is:

$$W(s) = \frac{3}{(s + 3)} \tag{8.12}$$

This plant has a zero in right half plane and poles are in left half of s-plane. As a result, it is evident that the transfer function of the aforementioned system is non-minimum phase and stable. As a result, model reference control will be impossible since pole-zero cancellations would be unstable. The system is subjected to a compensation approach before the control scheme is implemented.

Here, the augmented plant (plant with compensator) with feedforward gain, L will be:

$$\mathrm{G(s)} = \frac{1}{(3\mathrm{s}+1)}\left\{\frac{(-1.4s+1)}{(1.4s+1)} + L\right\} = \frac{(-1.4s+1) + L(1.4s+1)}{(3s+1)(1.4s+1)} \quad (8.13)$$

The estimated value of L can be found out from inverse root-locus of G2(s) TF to relocate the RHP zeros to LHP. By selecting $L = 3$, the RHP zeros at $s = 0.714$ is shifted to left-half of s-plane at $s = -1.428$. Figure 7.3 shows root-locus of original plant and Figure 7.4 shows root-locus of augmented plant model.

Then, the plant transfer function becomes:

$$\mathrm{G(s)} = \frac{0.233(s+0.714)}{\left(s+\frac{1}{3}\right)(s+0.25)} \quad (8.14)$$

Here, order of the plant, $n_p = 2$ and its relative degree, $n^* = 1$ which is same as the relaive degree of the refrence model. We choose the polynomial:

$$\Lambda(s) = s + 1 \ and \ \alpha(s) = 1 \quad (8.15)$$

And the control input is

$$u(s) = \theta_1^T \cdot \frac{1}{s+1} u(s) + \theta_2^T \cdot \frac{1}{s+1} Y_p(s) + \theta_3^T \cdot Y_p + C_0 r(s) \quad (8.16)$$

Which gives the closed-loop transfer function as:

$$\frac{Y_p}{r} = \frac{0.233C_0(s+0.174)(s+1)}{(s+1-\theta_1)\left(s+\frac{1}{3}\right)(s+0.25) - 0.233(s+0.174)(\theta_2 + \theta_3(s+1))} = G_c(s) \quad (8.17)$$

It should be equivalent to reference model. So, forcing to $G_c(s) = \frac{3}{s+3}$, we have

$$C_0 = \frac{3}{0.233} = 12.85 \quad (8.18)$$

And by comparing the coeffeicients with respect to the power of “s”:we get,

$$\begin{bmatrix} 1 & 0 & 0.233 \\ 0.4167 & 0.233 & 0.2735 \\ 0.0833 & 0.0405 & 0.0405 \end{bmatrix} . \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} -2.7573 \\ -3.196 \\ -0.4387 \end{bmatrix} \quad (8.19)$$

$$\theta_1 = 0.8159 \quad (8.20)$$

$$\theta_2 = 2.8255 \quad (8.21)$$

$$\theta_3 = -15.3358 \quad (8.22)$$

Thus, the control input is given by:

$$u(s) = 0.8159 \cdot \frac{1}{s+1} u(s) + 2.8255 \cdot \frac{1}{s+1} Y_p(s) - 15.3358 \cdot Y_p + 12.85 r(s) \qquad (8.23)$$

This control structure is now used to regulate a variety of reference signals, including step, square wave, and sin wave.

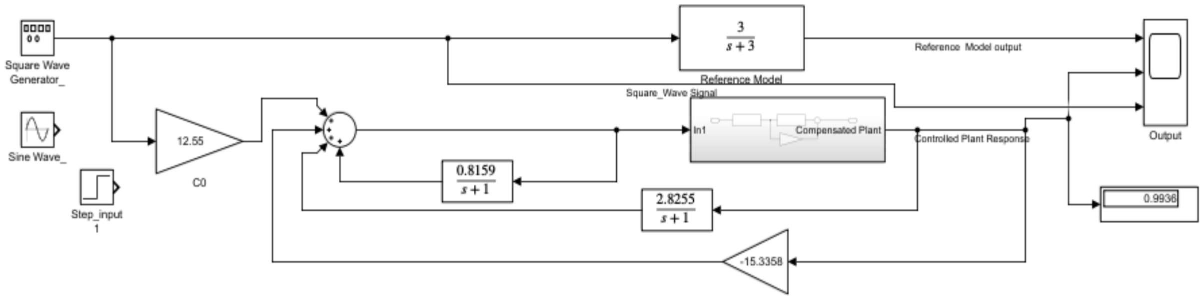

Figure 8.3: Block Diagram of Simulink Model (MRC for Stable NMP System)

Now, responses of reference model and plant with compensator for different types of refernce signals are:

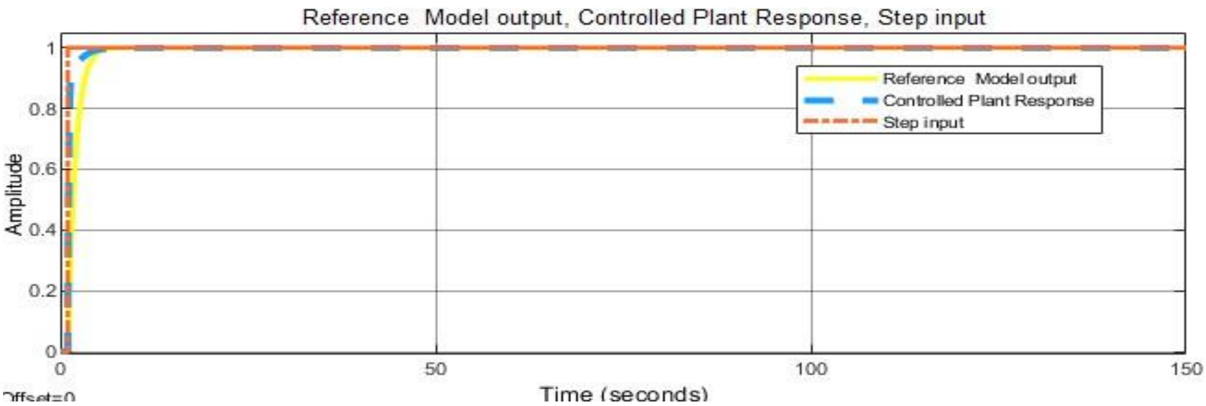

Figure 8.4: Unit Step Response of Plant with Model Reference Controller

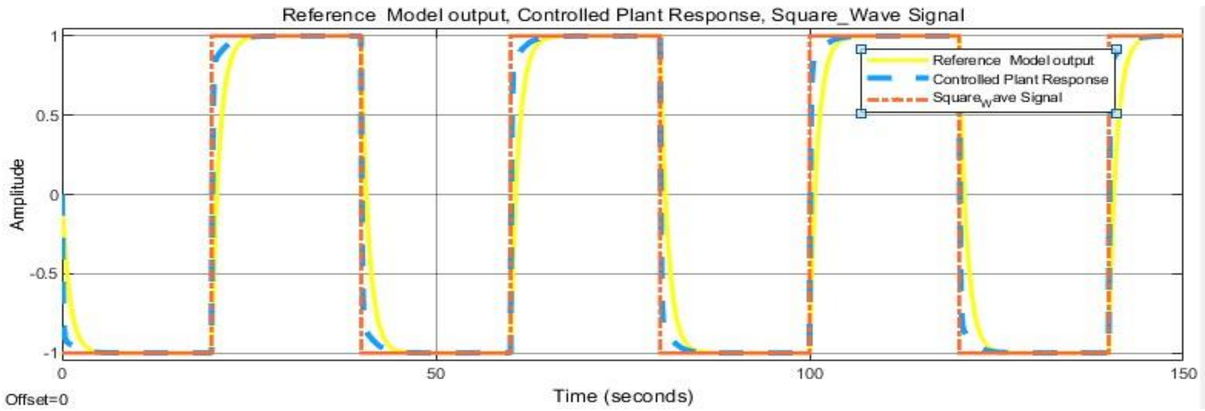

Figure 8.5: Square Wave Response of Plant with Model Reference Controller

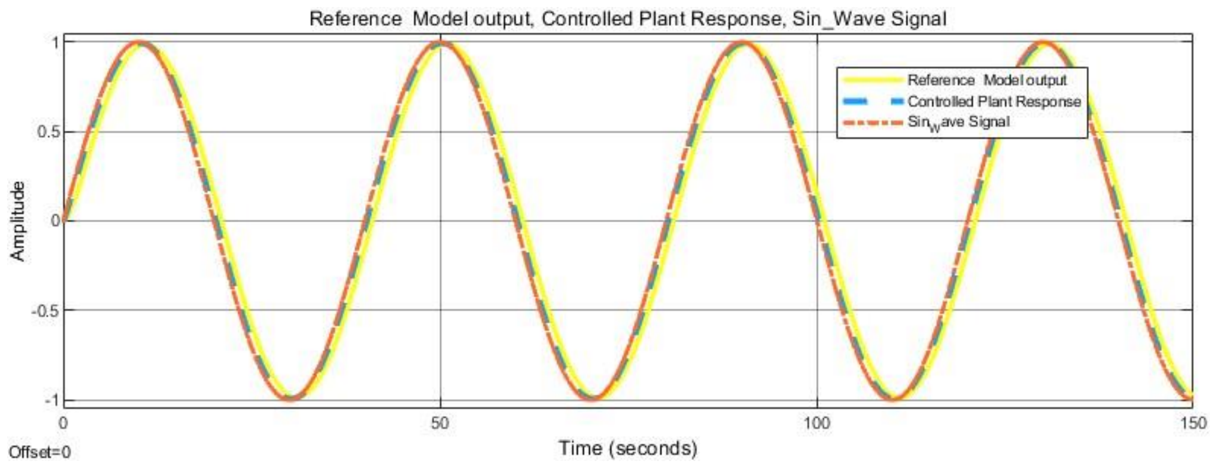

Figure 8.6: Sin-Wave Response of Plant with Model Reference Controller

8.3.2 Simulation for Unstable NMP System

Let's assume the unstable NMP plant (System-2) TF is given as:

$$G(s) = \frac{-5(s-5)}{(s-2)(s+7)} \tag{8.24}$$

And, Reference model is:

$$W(s) = \frac{3}{(s+3)} \tag{8.25}$$

This plant has a zero in right half plane at +5 and one pole in left half plane and one pole in right half plane at +2 and -7. As a result, it is evident that the transfer function of the aforementioned system is non-minimum phase and unstable. As a result, model reference control will be impossible since pole-zero cancellations would be unstable. The system is subjected to a compensation approach before the control scheme is implemented.

The following are the stages to take while constructing a compensating technique:

Step-1: Decompose the system G(s) into two sub-systems: G1(s) is minimum phase and stable/unstable part of transfer function G(s) and G2(s) is non-minimum phase and stable part of transfer function G(s).

$$G_1(s) = \frac{-5}{(s-2)} \tag{8.26}$$

And,

$$G_2(s) = \frac{(s-5)}{(s+7)} \tag{8.27}$$

Step-2: Design of feedforward gain L:

$$\hat{G_2}(s) = \frac{(s-5)}{(s+7)} + L \tag{8.28}$$

Now, I have to find the appropriate value of 'L' so that the right half zero will shift to left half plane. Using inversee root-locus of $G_2(s)$, With L=10, the zero shifts to -5.9.

Then, the plant transfer function becomes:

$$G(s) = \frac{-55(s+5.9)}{(s-2)(s+7)} \tag{8.29}$$

Here, order of the plant, $n_p = 2$ and its relative degree, $n^* = 1$ which is same as the relaive degree of the refrence model. . We choose the polynomial:

$$\Lambda(s) = s + 1 \; and \; \alpha(s) \; = \; 1 \tag{8.30}$$

$$C_0 \; = \; \frac{K_m}{K_p} = \frac{-3}{55} \tag{8.31}$$

$$\theta_1 = -4.9 \tag{8.32}$$

$$\theta_2 = 0.327 \tag{8.33}$$

$$\theta_3 = -0.018 \tag{8.34}$$

And thus the control input is given as:

$$u(s) = -4.9\frac{1}{s+1}u(s) + 0.327\frac{1}{s+1}Y_p(s) - 0.018Y_p - \frac{3}{55}r(s) \tag{8.35}$$

This control structure is now used to regulate a variety of reference signals, including step, square wave, and sin wave.

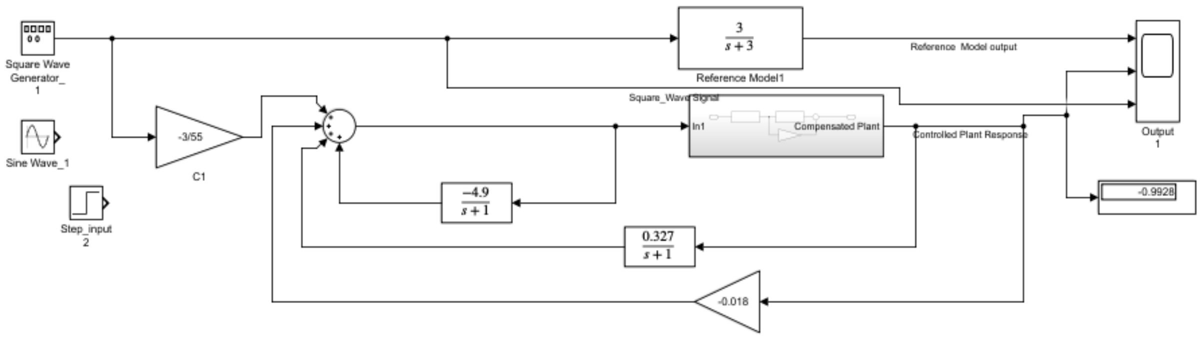

Figure 8.7: Block Diagram of Simulink Model for Unstable NMP system

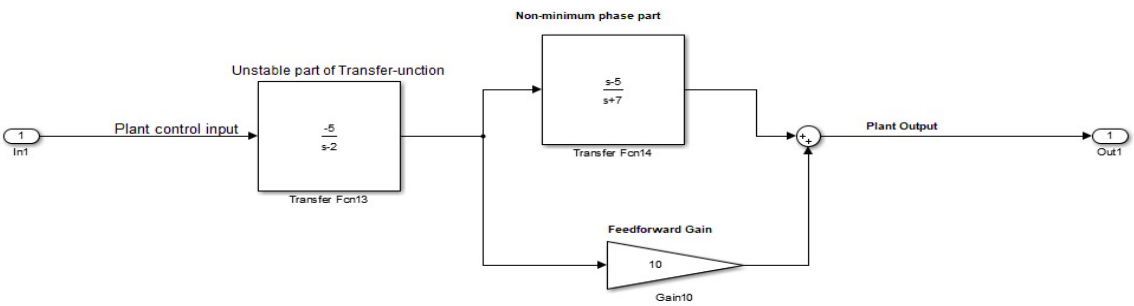

Figure 8.8: Block Diagram of Plant with Compensator

Now, Responses of Reference Model and Plant with compensator are given below:

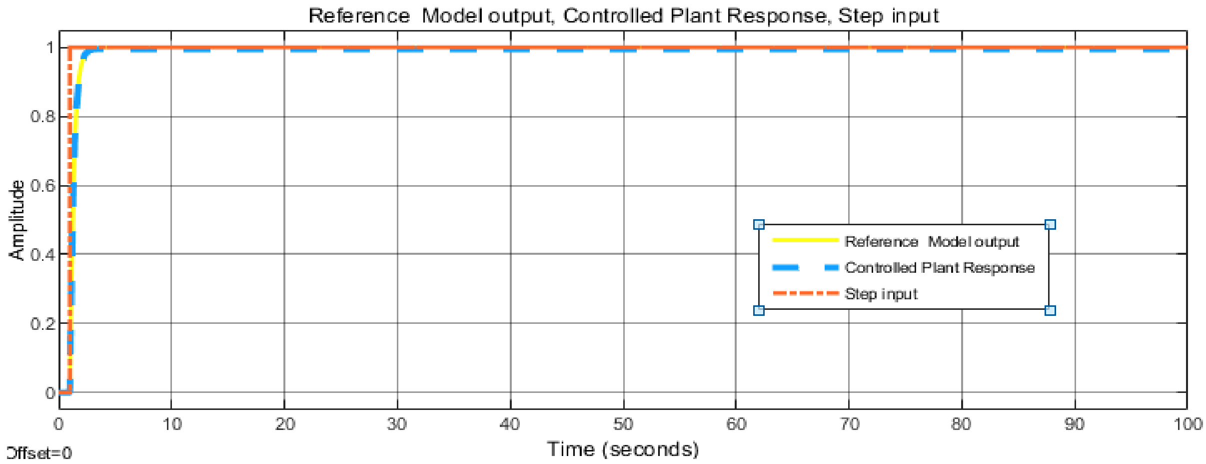

Figure 8.9: Response of Reference and Plant with Compensator for Step input

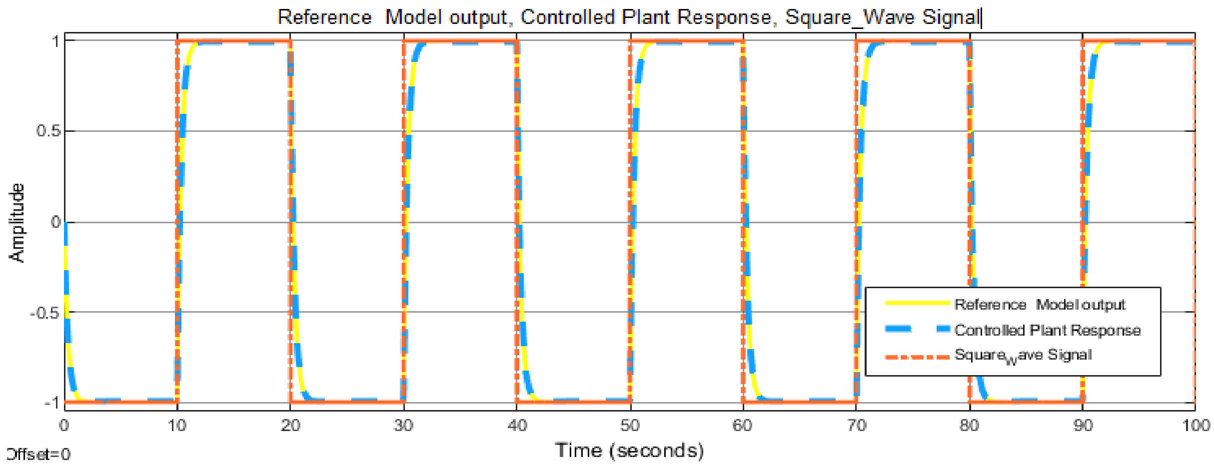

Figure 8.10: Response of Reference and Plant with Compensator for Square-wave input

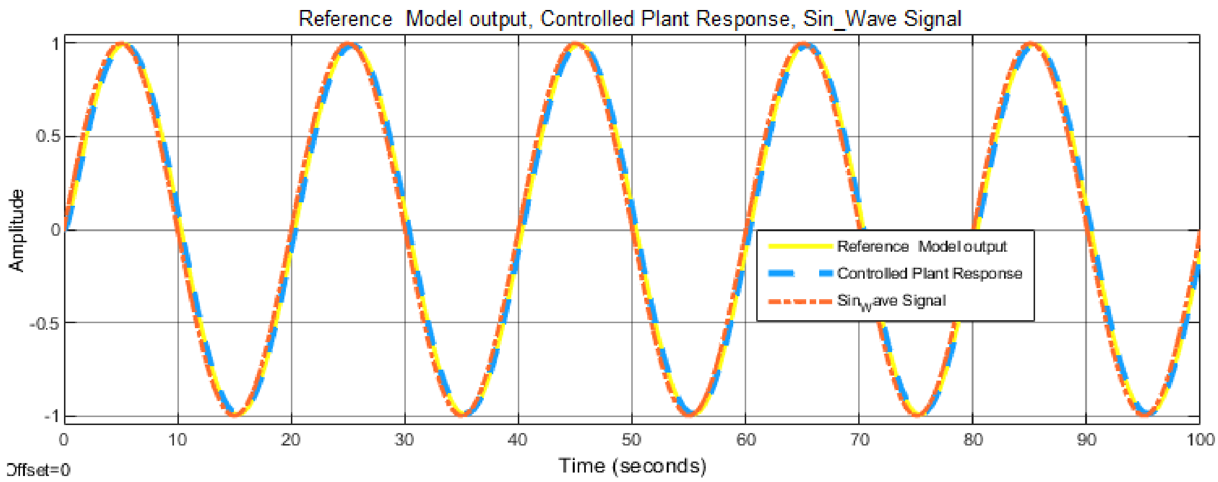

Figure 8.11: Response of Reference and Plant with Compensator for Sin-wave input

8.4 Conclusion

Here, a model reference control scheme is applied for both stable and unstable NMP systems with various types of reference signals such as step input signal, square wave and sin wave signal. But, it is clear that to satisfy the certain conditions to implement MRC, the plant and reference model should have to satisy some criterias. For this plant transfer functin should be of minimum phase, So here in case of NMP systems the plant is atfirst compensated by using feed-forward compensator techniques and then MRC is applied.The upgraded plant closely matches the output of the reference model, as seen in Figure 8.4-Figure 8.6 for stable NMP system and in Figure 8.9-Figure 8.11 for unstable NMP system.Also, the plant output signals are likewise bounded, indicating that the system is stable, according to the plots.

9 Discussion and Conclusion of Different Control Paradigms

There are several ways in the literature for dealing with the limits provided by non-minimum phase zeros in case of NMP systems. Pole-zero cancellation is one of them, in which an unstable pole cancels the right half plane zero [5]. The problem of this strategy is that it results in an unstable controller design [6] [11] to cancel out the transfer function's non-minimum phase zeros. Outside of the left half of the S plane, pole-zero cancellation occurs, resulting in unbounded internal states. In addition, if the plant's characteristics change over time, inexact pole-zero cancellations occur.

It is usually challenging to design a feedback control system for a plant with positive real parts and zeros since these systems become unstable when controller gains are high. Small controller gains, on the other hand, result in poor closed loop system transient performance.

A fast analysis of the amount of undershoots exhibited can be used to change controller parameters to lessen its influence. A traditional solution entails the creation of a sophisticated control law. It greatly increases the complexity of the control system, making practical implementation more expensive and time-consuming. Furthermore, a mathematical analysis of this scenario might produce more insightful results.

This paper deals with the concepts for designing different controllers to achieve the required response with optimal characteristics for a practical system which possess non-minimum phase characteristics. The practical system analysed here is coupled-tank system as described in Chapter-2. From Chpater-3, it is important to note that PID controllers frequently do not produce results that are desirable when the process is non-minimum phase (NMP), the poles have low damping ratios, or there is a significant amount of dead time. In PID controller technique to achieve the desired performance criteria, the gains are adjusted by varying the gain using a particular tuning method or by trial-and-error method. Ziegler-Nichol’s method of tuning is used here. This tuning method shows the output response of time delay system analysing as NMP system by using Pade’s approximation as shown in Figure 3.2 has not optimised response as it still shows significant delay to achieve the specified output with an undesired level of overshoot and settling time. However, it gives the slightly improved response by using fine-tuning method. Therefore, from a practical perspective, it is crucial to create efficient techniques to get rid of these restrictions.

Internal Model Control gives the better response with respect to the PID controller as it uses the Internal-model based approach. It uses a single tuning parameter 'lambda', so it is easier to tune to get desired response. IMC sometimes also called as Lambda tuning.

It provides a steady and resilient alternative to conventional procedures. This is clearly observed by the Simulink of the model described above that the controller is fairly resilient to uncertainty in plant parameters and hence it can be effectively used in any industrial process. This paper explains how to use IMC tuning rule to tune control loops. A clear trade-off between closed-loop performance and robustness to model inaccuracies is thus achieved with a single tuning parameter (lambda) when using the IMC-PID tuning method. Also, to manage unstable processes, the IMC design technique must be adjusted. Furthermore, because the conventional IMC structure is incapable of handling unstable processes, the controller for an unstable process must be developed using regular PID feedback.

If comparing the IMC and IMC-based PID controller, the results can be concluded from the simulation results is, for stable process with no time-delay the response of IMC-based-PID is same as that of the feedback performance as IMC. But, when for a stable process there is a time-delay unit present then, the response of IMC-based PID is not exactly same as that of the IMC controller because the IMC-based PID controller uses the Pade` approximation for the time-delay element in the controller design.

As described the major effect of this IMC-based PID controller is that it provides the efficient compensation for time delay with extremely high precision, and it can also provide offset-free response. Both input tracking and disturbance rejection responses may be shaped using the controller. An IMC-based PID controller might readily overcome some model imperfections, which is not possible with a traditional PID system. Simulation results suggest that IMC-based PID is superior. It can be observed easily by comparing the table of results as obtained. The goal is to implement it in real time in the future.

Finally, it is observed that the controller may be used to shape both the input tracking and disturbance rejection responses, as well as offer time delay compensation, and it will deliver offset free response (perfect control at steady state) at steady state. Table 9.1: Performance Comparison Table for Different Controllers for Stable NMP System (Coupled-Tank System considered as System-1, $G1(s)$)Table 9.1 provides a concise summary of the performance study of several controllers for a stable NMP system (system-1).

Using feed-forward compensator, the non-minimum phase plant can be converted to minimum phase to apply the existing controlling techniques which results effective response characteristics. In case of stable non-minimum phase, the system transfer function can be easily compensated by using parallel compensation techniques without doing much effort. But in case of Unstable NMP system as described in chapter-6, the compensation techniques need extra effort. The feedforward compensation is applied to the non-minimum phase part of the system by using suitable value of feedforward gain (L) results in minimum phase system. So, the choice on the range of feedback gain to control the overall system response has no longer any restrictions. This technique is capable enough to remove the limitations of gain in the design of feedback control scheme possessed by the system having right half plane zeros. Finally, the model reference control (MRC), which previously could not be used to regulate non-minimum phase systems, may now be utilised to do so. This Model Reference Control provide a good amount of acceptable response for different reference signals such as step, square-wave and sin-wave signal.

Here in this thesis only SISO system is used for which feed-forward compensation techniques is designed. Multi-input multi-output (MIMO) system is a topic for future research. For IMC and IMC-based-PID Controller only stable NMP system is analysed. Unstable NMP system using IMC and IMC based PID is remaining for a future work. Also, to make this model-reference controller adaptive may be considered as a future work.

Table 9.1: Performance Comparison Table for Different Controllers for Stable NMP System (Coupled-Tank System considered as System-1, $G1(s)$)

$$G1(s) = \frac{(-1.4s + 1)}{(4.2s^2 + 4.4s + 1)} \tag{9.1}$$

Performance Characteristics	**Open Loop Response**	**PID+f Controller**	**2-DoF PID Controller**	**IMC-PID Controller**
Rise Time (sec)	7.125	19.5	12	9.67
Settling Time (sec)	16	33.5	26.3	18.8
Overshoot (%)	0	0.428	0	0
Minimum amplitude (Undershoot)	-0.118	-0.0102	-0.03	0
Stability	Stable	Stable	Stable	Stable
IAE (Integral Absolute Error)	--	10.940	9.472	5.814
ISE (Integral Square Error)	--	10.801	6.817	3.908

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