7. (a) Let $\vec{F} = -3x^2\,\hat{i} + 5xy\,\hat{j}$ and let C be the curve y = zx^2 in the xy plane. Evaluate the line integral $\int_C \vec{F}.d\vec{r}$ from (0,0) to (1,2). 5

(b) Let $\vec{V} = x^2z^2\hat{i} - 2y^2z^2\,\hat{j} + xy^2z\,\hat{k}$. Find curl $\left(\vec{V}\right)$ at the point (1,–1,1). 5

(c) State Green's theorem in a plane. Verify Green's theorem for $\int_c \left[\left(xy + y^2\right)dx + x^2dy\right]$ where C is the closed curve of the region bounded by y = x and y = x^2. 2+8

8. (a) Define the following :

(i) Random experiment (ii) Outcome (iii) sample space (iv) Mutually exclusive events (v) Equally likely events. 2x5=10

(b) In a group of 20 males and 5 females, 10 males and 3 females are service holders. What is the probability that a person selected at random from the group is a service holder, given that the selected person is a male. 5

(c) Find the probability that in the throw of two unbiased dice, the sum of points will be even or less than 5. 5

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Ex/IEBE/MATH/T/124/2019(OLD)

BACHELOR OF (I.E.E.) ENGINEERING EXAMINATION, 2019

(1st Year, 2nd Semester, Old Syllabus)

Mathematics - IV J

Time : Three hours Full Marks : 100

Notations/Symbols have their usual meanings.

Answer any ***five*** questions.

1. (a) Prove that a necessary and sufficient condition that w = f(z) = u(x,y) + iv(x,y) tends to a limit $\ell = m + in$ as $z = x + iy \to \alpha + i\beta = a$ is that

$$\lim_{\substack{x\to\alpha \\ y\to\beta}} u(x,y) = m\ ,\ \lim_{\substack{x\to\alpha \\ y\to\beta}} v(x,y) = n$$ 10

(b) When a function f(z) is said to be continuous ? Test the function $f(z) = \dfrac{z}{|z|}$ for continuity. 5

(c) Prove that a continuous function f(z) defined on a closed bounded rectangular region must be bounded. 5

2. (a) Let $f(z) = \dfrac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}\ ,\ z = x + iy \neq 0$

$= 0\ ,\ z = 0$

(Turn Over)

Prove that f(z) is continuous at 0 and satisfied Cauchy-Riemann conditions but is not differentiable at 0. 8

(b) If f(z) is differentiable at z=a then prove that it is also continuous at a. 4

(c) Define a harmonic function. Find analytic function f(z) of which the real part is given by $u = 3x^2 + xy - 3y^2$. 2+6

3. (a) If f(z) is analytic everywhere on the region bounded by two simple closed curves C_1 and C_2 where C_1 is inside C_2 then prove that

$$\oint_{C_1} f(z)dz = \oint_{C_2} f(z)dz$$ 6

(b) If C denotes the circle |z|=1, consider the integral $\int_c \frac{dz}{Z+z}$ and hence deduce the value of $\int_0^{\pi} \frac{1+z\cos\theta}{5+4\cos\theta}d\theta$.

6

(c) State and prove Cauchy's Integral Formula. 8

4. (a) Show that $f(z) = \sin\frac{1}{z}$ has an essential singularity at z=0. 4

(b) Find the following Laplace transforms.

(i) $L(\sin\sqrt{t})$ (ii) $L\left(e^{-3t}(\cos 4t + 3\sin 4t)\right)$

(iii) $L\left(e^{-t}\sin^2 t\right)$ 6+5+5

5. (a) Find $L^{-1}\left(\frac{p-1}{(p+3)(p^2+2p+2)}\right)$

(b) $L^{-1}\left(\frac{p}{(p+1)5}\right)$

(c) Using Laplace transform, solve the differential equation

$(D^2 - D - 2)y = 20 \sin zt$, y = –1, Dy = z at t=0.

7+4+9

6. (a) Let $\phi(x,y,z) = x^2+y^2+xz$. Find the directional derivative of ϕ at the point P(2,–1,3) in the direction of the vector $\hat{i}+2\hat{j}+\hat{k}$. 8

(b) State Gauss's Divergence Theorem. Hence evaluate $\iint_S \vec{F}.d\vec{s}$ where $\vec{F} = 4xz\hat{i} - y2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by x=0, x=1, y=0, y=1, z=0, z=1. 2+10

(Turn Over)