10. (a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$$

(b) Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$. 5+5

11. (a) Show that $\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}\,.\,\Gamma(2n+1)}{2^{2n}\,\Gamma(n+1)}$.

(b) Show that $B(m,n) = B(m+1, n) + B(m, n+1)$. 6+4

12. (a) Prove that $\int_0^1 \frac{x^6\,dx}{\sqrt{1-x^2}} = \frac{5}{32}\pi$.

(b) Evaluate $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{n+r}{n^2+r^2}$. 5+5

— X —

Ex./ARCH/MATH/T/115/3/2018(OLD)

BACHELOR OF ARCHITECTURE EXAMINATION, 2018

(1st Year, 1st Semester, Old Syllabus)

Mathematics - I A

Time : Three hours Full Marks : 100

The figures in the margin indicate full marks.
Notations/Symbols have their usual meanings.

Answer any ***ten*** questions.

1. (a) State and prove Lagrange's Mean Value theorem and give geometrical interpretation of it.

(b) If $y = x^2 e^{ax}$, Find y_n 7+3

2. (a) Find y_n when $y = x^3 \sin 4x$.

(b) If $y = a\cos(\ln x) + b\sin(\ln x)$, then show that

$x^2 y_{n+2} - (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. 4+6

3. (a) If $y = e^{ax}\cos bx$, find y_n.

(b) If $\log y = \tan^{-1} x$, show that

$(1+x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n+1)y_n = 0$. 5+5

(Turn over)

4. (a) $y = a\cos(\log x) + b\sin(\log x)$, then prove that

$$x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

(b) Show that $\frac{2\theta - \sin 2\theta}{\theta^2}\,(\theta > 0)$ is maximum when $x = \pi/2$. 6+4

5. (a) Determine the values of a, b, c so that

$$\lim_{x\to 0}\frac{(a + b\cos x)x - c\sin x}{x^5} = 1$$

(b) Examine the following for extreme values

$$x^4 + y^4 - 2x^2 + 4xy - y^2$$ 5+5

6. (a) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$.

(b) Find the maximum and minimum values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. 5+5

7. (a) Express the following integrals as the limit of a sun and evaluate the values $\int_a^b \cos x\,dx$.

(b) Show that

$$\lim_{n\to 0}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2\left(1+\frac{3^2}{n^2}\right)^3 \cdots\left(1+\frac{n^2}{n^2}\right)^n\right]^{1/n^2}$$

$$= \log c^{1/2}$$ 5+5

8. Show that

(a) $\int_0^{n/2} \log(\sin x)dx = \frac{\pi}{2}\log\left(\frac{\pi}{2}\right)$

(b) $\int_0^{n/2} \frac{\cos x}{1+\sin x+\cos x}dx = \frac{\pi}{4} - \frac{1}{2}\log 2$ 5+5

9. (a) Prove that $\int_0^{\infty}\frac{n^{m-1}}{(a+bx)^{m+n}}dx = \frac{1}{a^n b^m}B(m,n)$ and hence show that

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(b+cx)^{m+n}}dx = \frac{1}{b^n(b+c)^m}B(m,n)$$

(b) Find by Simpson's Rule an approximate value of $\int_{2\pi}^{4\pi}\frac{\sin x}{n}dx$, when n = 12. 5+5

(Turn over)