6. Consider the problem:

Minimize $\psi = (x_1 - 1)^2 + (x_2 - 2)^2$ subject to $2x_1 - x_2 = 0$, $x_1 \le 5$.

Find the solution of the problem using interior penalty function method with the calculus method of unconstrained minimization. 4

Ex/SC/MATH/PG/CORE/TH/12/2023

M. Sc. Mathematics Examination, 2023

(2nd Year, 2nd Semester)

Mathematics

Paper – Core-12

[Optimization and Control Theory]

Time : 2 hours Full Marks : 40

The figures in the margin indicate full marks.

(Symbols and notations have their usual meanings)

(Use a separate Answer-Script for each Part)

Part – I (Marks: 24)

Answer ***any three*** from the following four questions.

1. a) Describe briefly the Moon-lander problem.

   b) Explain how the Pontryagain Maximum Principle can be applied to this problem to land the moon-lander safely while *maximizing* the remaining fuel $m(\tau)$ and equivalently, *minimizing* the total applied thrust before landing. 2+6=8

2. a) The $n \times mn$ controllability matrix is defined as $G = G(M, N) := \left[N, MN, M^2N, \ldots. M^{n-1}N\right]$. Prove that $Rank\ G < n$ implies that $C^0 = \phi$ and also conversely, $0 \notin C^0$ implies that $Rank\ G < n$.

   b) Consider the rocket railroad car problem with $n = 2$, $m = 1$, $A = [1,1]$ and $\dot{x} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \alpha$.

Justify whether this problem satisfies the hypotheses concerning the criterion for controllability. 6+2=8

3. a) State Pontryagin's Maximum Principle - I in detail which provides the necessary conditions for achieving an admissible pair (x^*, u^*) whenever solving a control problem.

b) Solve the following control problem given as:

$\max \int_0^1 x(t)\,dt$, $\dot{x} = x + u$, $x(0) = 0$, $x(1) \geq 1$,
$u(t) \in [-1,1] = U \ \forall\ x \in [0,1]$.

c) Using First version of Mangasarian's Theorem, solve the following control problem for optimal pairs:

$\max \int_0^T \left[1 - tx(t) - u(t)^2\right] dt$, $\dot{x} = u(t)$, $x(0) = x_0$,
$x(T)$ free and x_0, T are positive constants.

2+3+3=8

4. Consider the following continuous-time system (when the sampling period T=1):

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s(s+2)}.$$

a) Obtain the continuous-time state-space representation of the above system. Discretize the state and output equations and hence obtain the discrete-time state-space representation.

b) Evaluate also the pulse transfer function of the given system. 4+4=8

Part – II (Marks: 16)

Answer ***any four*** questions.

All questions carry equal marks.

1. Prove that the gradient vector represents the direction of the steepest ascent. 4

2. Use Nelder-Mead method to find the minimum value of $g(x,y) = \left|\sin x - y^3 + 1\right| + x^2 + \frac{y^4}{10}$. Compute upto third iteration defining the initial simplex as $v_1 = (1.5, 0)$, $v_2 = (2, 0)$ and $v_3 = (2, 0.5)$. 4

3. Minimize $f = x_1^2 + 3x_2^2 + 6x_3^2$ by Hooke and Jeeves pattern search method by taking $\Delta x_1 = \Delta x_2 = \Delta x_3 = 0.5$ and the starting point as (2, –1, 1). Perform two iterations. 4

4. Using Davidon Fletcher Powell method, Minimize $f(x_1, x_2) = 2x_1^2 + 4x_2^2 - 12x_1 + 16x_2 + 41$ with $X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as starting point. 4

5. Using Steepest Descent method, minimize $\phi(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ starting from the point $X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. 4

[Turn over