Ex/Prod/Math/T/113/2017(S)

BACHELOR OF ENGINEERING IN PRODUCTION ENGINEERING EXAMINATION, 2017

(1st Year, 1st Semester, Supplementary)

MATHEMATICS - I

Time : Three hours | Full Marks : 100

Answer ***any 10*** questions

1. a) A function is defined in the following way :

$$f(x) = x \sin\frac{1}{x} \quad \text{for} \quad x \neq 0$$
$$f(0) = 0 \quad \text{for} \quad x = 0$$

Show that $f'(0)$ does not exist.

b) State and prove Lagrange's Mean value theorem and give geometrical interpretation of it. 4+6

2. a) If $y = \sin 3x \cos 2x$, find y_n.

b) If $y = \sin\left(m \sin^{-1} x\right)$, then show that

$$\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} + \left(m^2 - n^2\right)y_n = 0$$ 5+5

3. a) If $y = x^{n-1}\log x$, show that $y_n = \frac{(n-1)!}{x}$.

b) Prove that $\log(1+x) > x - \frac{1}{2}x^2$, if $x > 0$. 5+5

4. a) It f(x) = tan x, then f(0) = 0 and f(π) = 0.

Is Rolle's theorem applicable to f(x) in (0, π).

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11. a) Determine the value of α, β, γ, when

$$\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$$ is an orthogonal matrix.

b) Solve the equations by Matrix method

$$x + y + z = 4$$
$$2x - y + 3z = 1$$
$$3x + 2y - z = 1$$ 5+5

12. a) If $lx + my = 1$ be a normal to the parabola $y^2 = 4ax$, then prove that $al^3 + 2alm^2 = m^2$

b) Find the point of inflexion of the curve

$$y = x^4 - 6x^3 + 12x^2 + 5x + 7$$ 5+5

b) Expand $\cos^3 x$ in a finite series with Lagrange's form of remainder. 5+5

5. a) Show that

$$(x+h)^{3/2} = x^{3/2} + \frac{3}{2}x^{1/2}h + \frac{3.1}{2.2}\cdot\frac{h^2}{2!}\frac{1}{\sqrt{x+\theta h}}, 0<\theta<r$$

Find θ, when $x=0$.

b) Expand coshx in power of x in an infinite series. 4+6

6. a) Show that $\sec x + \log\cos^2 x$ is a maximum for $x=0$ and a minimum for $x = \pi/2$.

b) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$ 5+5

7. a) If $u = \log\left(x^3+y^3+z^3-3xyz\right)$, show that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = -\frac{3}{(x+y+z)^2}$$

b) If $u = \log r$ and $r^2 = x^2+y^2+z^2$, prove that

$$r^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) = 1.$$ 5+5

8. a) If $u = f\left(x^2+y^2+z^2\right) f\left(xy+yz+zx\right)$, prove that

$$(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z} = 0.$$

b) Show that $f(x,y) = 4x^2y - y^2 - 8x^4$ is a maximum at (0, 0). 5+5

9. a) Prove that

$$\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \left(a^3+b^3+c^3-3abc\right)^2$$

b) Solve the equations by Cramer's rule :

$$-x_1 + 3x_3 + 1 = 0$$
$$2x_1 - x_2 - 4x_3 - 2 = 0$$
$$x_1 + 2x_3 - 4 = 0$$

5+5

10. a) Determine the rank of the matrix

$$A\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 7 & 8 \\ 3 & 6 & 7 & 12 & 15 \\ 4 & 8 & 12 & 14 & 16 \end{bmatrix}$$

b) If $A\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, prove that $A^3 = A^{-1}$ 5+5

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