EX/PROD/MATH/T/114/2017(OLD) (S)

Bachelor of Production Engineering Examination, 2017

(1st Year, 1st Semester, Supplementary)

Mathematics - IS (Old)

FULL MARKS : 100 TIME : 3 HOURS

ANSWER ANY 10 QUESTIONS:

1. (a) Find y_n when $y = x^3\sin 2x$.

   (b) If $y = (\sin^{-1}x)^2$, then show that

   $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2)\, y_n = 0$ 4+6

2. (a) State and prove Lagrange's Mean Value theorem on successive differentiation. Give geometrical interpretation of the result.

   (b) If $f(x) = tanx, then\ f(0) = 0\ and\ f(\pi) = 0$. Is Rolle's theorem is applicable to $f(x)$ in $(0, \pi)$? 6+4

3. (a) Show that $cos\ x > 1 - \frac{1}{2}x^2$, if $0 < x < \frac{1}{2}\pi$.

   (b) Expand the function cos^3x in power of x in a finite form with Lagrange's form of remainder. 5+5

4. (a) Show that $sin^3x\ cosx$ is a maximum when $x = \frac{1}{3}\pi$.

   (b) Evaluate $\lim_{x\to 0}\frac{(e^x-1)tan^2x}{x^3}$. 5+5

5. (a) Expand sinh x in an infinite series in powers of x.

   (b) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$. 5+5

6. (a) If $u = f(x^2 + 2yz, y^2 + 2zx)$, prove that

   $$(y^2 - zx)\frac{\partial u}{\partial x} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z} = 0.$$

   (b) If $u = e^{(x^2+y^2)}$, show that

   $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u\ logu.$ 5+5

[Turn over

7. (a) Show that

$$\begin{vmatrix} a^2 & bc & c^2 + ca \\ a^2 + ab & b^2 & ca \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

(b) Find the value of x, which satisfy the equation

$$\begin{vmatrix} x^3 - a^3 & x^2 & x \\ b^3 - a^3 & b^2 & b \\ c^3 - a^3 & c^2 & c \end{vmatrix} = 0.$$ 5+5

8. (a) Verify that $A = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$ is an orthogonal matrix.

(b) Find the solution of the following system of equation by matrix method

$$x + y + z = 4$$

$$2x - y + 3z = 1$$

$$3x + 2y - z = 1.$$ 5+5

9. (a) Find the point of inflexion of the curve $y^2 = x(x+1)^2$.

(b) Show that the curve $y^3 = 8x^2$ is concave to the foot of the ordinate everywhere except at the origin. 5+5

10. (a) Obtain the point of inflexion of the curve $x = a(2\Theta - \sin\Theta)$, $y = a(2 - \cos\Theta)$.

(b) Evaluate $\int \frac{dt}{(t^2-1)(t^2+2)}$. 5+5

11. (a) Find at which points on the curve $y = 2x^3 - 15x^2 + 34x - 20$ where the tangents are parallel to the straight line $y + 2x = 0$.

(b) Find the equation of the tangent at the point Θ to the curve $x = a(\Theta + \sin\Theta)$, $y = a(1 - \cos\Theta)$. 5+5

12. (a) Find the maximum and minimum values of $x^4 + x^2y + y^2$.

(b) Find y_n, when $y = \cos^3 x \sin^2 x$. 5+5