

MA 1ST YEAR 1ST SEM 2017

Econometrics I (OLD)

(Ref.: EX/PG/ECO/14/6/2017(OLD))

Time: 2 Hours

Full Marks: 30

Answer any five of the following questions.

6 × 5 = 30

1. Consider the least square estimates of the model: $\mathbf{y}_{N \times 1} = \mathbf{X}_{N \times K} \boldsymbol{\beta}_{K \times 1} + \mathbf{u}_{N \times 1}$, where $E(\mathbf{u}|\mathbf{X}) = \mathbf{0}$ and $E(\mathbf{u}\mathbf{u}'|\mathbf{X}) = \boldsymbol{\Sigma} = \sigma^2(\mathbf{I} + \mathbf{A}\mathbf{A}')$, where \mathbf{A} is an $N \times m$ matrix with $K < m < N$. Assume, for simplicity, that σ^2 and \mathbf{A} are known.
 - (a) Obtain the variance of the OLS estimator of $\boldsymbol{\beta}$.
 - (b) Compare your answer in (a) with default OLS variance $\sigma^2(\mathbf{X}'\mathbf{X})^{-1}$. Are the default OLS standard errors biased/inconsistent in any particular direction?
 - (c) Determine the variance of the GLS estimator of $\boldsymbol{\beta}$, using the result $(\mathbf{I} + \mathbf{A}\mathbf{A}')^{-1} = \mathbf{I}_N - \mathbf{A}(\mathbf{I}_m + \mathbf{A}'\mathbf{A})\mathbf{A}'$.
 - (d) Compare the default variance $\sigma^2(\mathbf{X}'\mathbf{X})^{-1}$ of OLS with the true variance of GLS. Does your finding violate that fact that GLS must be BLUE when disturbances are non-spherical?
2. Let $y = \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_K x_K + u$, where $E(u) = 0$, $\text{cov}(x_j, u) = 0 \forall j = 1, 2, \dots, K-1$, but x_K is correlated with u . Let $x_1 = 1$ to admit an intercept in the model. Let $\mathbf{x} = (x_1, x_2, \dots, x_K)$ and $\mathbf{z} = (x_1, x_2, \dots, x_{K-1}, z_1, z_2, \dots, z_M)$. Assume $E(\mathbf{z}'\mathbf{z})$ is non-singular. Let the linear projection of x_K on \mathbf{z} be $x_K^* = \delta_1 x_1 + \dots + \delta_{K-1} x_{K-1} + \theta_1 z_1 + \dots + \theta_M z_M$. Prove that $\text{rank } E(\mathbf{z}'\mathbf{x}) = K$, iff at least one θ_j is different from zero.
3. Derive attenuation bias clearly stating assumptions of the classical error in variables model.
4. Let the model be

$$y_t = \mathbf{x}_t \boldsymbol{\beta} + u_t,$$

where \mathbf{x} is $1 \times K$ and $u_t = \rho u_{t-1} + \varepsilon_t$, $|\rho| < 1$ and $\varepsilon_t \stackrel{iid}{\sim} (0, \sigma_\varepsilon^2)$. What is the autocorrelation of u in this model? Now consider the model

$$y_t - y_{t-1} = (\mathbf{x}_t - \mathbf{x}_{t-1}) \boldsymbol{\beta} + v_t,$$

where $v_t = u_t - u_{t-1}$. Compare autocorrelation of v with that of u .

5. Consider the model $y_i = \beta + u_i$, where y_i and u_i are random scalar variables and β is a scalar unknown parameter. u_i are iid with $E(u_i) = 0$, $E(u_i^2) = \beta^2$, $E(u_i^3) = 0$ and $E(u_i^4) = m$. What is the limiting distribution of the vector $\begin{pmatrix} \frac{1}{N} \sum_{i=1}^N y_i^2 \\ \frac{1}{N} \sum_{i=1}^N y_i \end{pmatrix}$?

[Turn over

6. Let $\hat{\boldsymbol{\theta}} = (\hat{\theta}_1, \hat{\theta}_2)'$ be \sqrt{N} -asymptotically normal estimator of $\boldsymbol{\theta} = (\theta_1, \theta_2)'$, with $\theta_2 \neq 0$. Let $\hat{\gamma} = \hat{\theta}_1/\hat{\theta}_2$ be an estimator of $\gamma = \theta_1/\theta_2$.

(a) Find $Avar(\hat{\gamma})$ in terms of $\boldsymbol{\theta}$ and $Avar(\hat{\boldsymbol{\theta}})$.

(b) If, for a sample of data, $\widehat{Avar}(\hat{\boldsymbol{\theta}}) = \begin{pmatrix} 1 & -0.4 \\ -0.4 & 2 \end{pmatrix}$ and $\hat{\boldsymbol{\theta}} = (-1.5, 0.5)'$, find the asymptotic standard error of $\hat{\gamma}$.

7. Suppose family i chooses annual consumption c_i (in dollars) and annual contribution to a charitable fund q_i (in dollars) to solve the problem

$$\max_{c, q} c + a_i \log(1 + q)$$

subject to the constraint $c + p_i q \leq m_i$; $c, q \geq 0$, where m_i is the annual income of family i , p_i is the price of one dollar of charitable fund (where $p_i < 1$ because of tax-deductibility of charitable contributions) and this price differs across families because of different marginal tax rates and different state tax codes, $a_i \geq 0$ determines marginal utility of charitable contributions. Consider m_i and p_i to be exogenous to the family in this problem.

(a) What is the optimal solution for q_i ?

(b) Define $y_i = 0$ if $q_i = 0$ and $y_i = 1$ if $q_i > 0$. Suppose $a_i = \exp(\mathbf{z}_i \boldsymbol{\gamma} + v_i)$, where \mathbf{z}_i is a $J \times 1$ vector of observable family traits and v_i is unobservable. Assume that v_i is independent of (\mathbf{z}_i, m_i, p_i) and v_i/σ has symmetric distribution function $G(\cdot)$, where $\text{var}(v_i) = \sigma^2$. Show that,

$$P(y_i = 1 | \mathbf{z}_i, m_i, p_i) = G[(\mathbf{z}_i \boldsymbol{\gamma} - \log p_i)/\sigma].$$

8. Consider the following two way random effects model:

$$y_{it} = \alpha + \mathbf{x}_{it} \boldsymbol{\beta} + \varepsilon_{it} + u_i + v_t, \quad i = 1 \dots n, \quad t = 1 \dots T,$$

where

$$\begin{aligned} E[\varepsilon_{it}] &= E[u_i] = E[v_t] = 0, \quad \forall i, t, \\ E[\varepsilon_{it} u_j] &= E[\varepsilon_{it} v_s] = E[u_i v_t] = 0 \quad \forall i, j, t, s, \\ \text{Var}[\varepsilon_{it}^2] &= \sigma^2, \quad \text{Cov}[\varepsilon_{it}, \varepsilon_{js}] = 0 \quad \forall i, j, t, s, \\ \text{Var}[u_i] &= \sigma_u^2, \quad \text{Cov}[u_i, u_j] = 0 \quad \forall i, j, \\ \text{Var}[v_t] &= \sigma_v^2, \quad \text{Cov}[v_t, v_s] = 0 \quad \forall t, s. \end{aligned}$$

Write the full error variance-covariance matrix for a data set with $n = 2$ and $T = 2$.