EMF Polygon

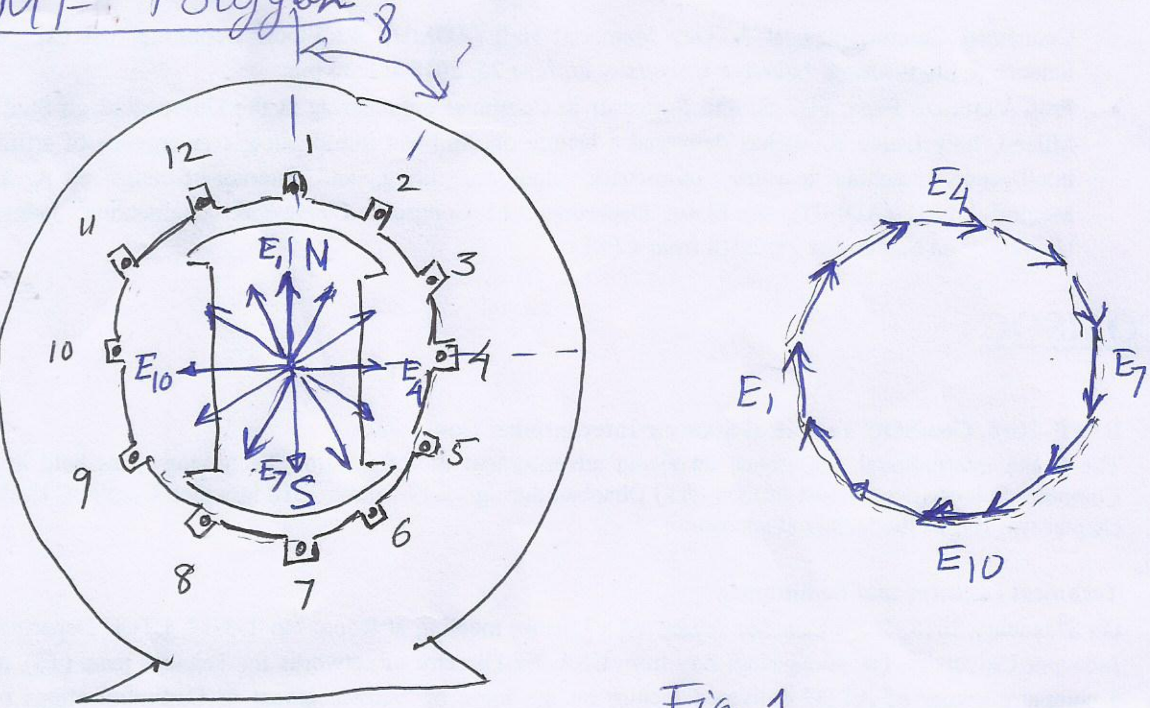


Fig 1

Let us consider an elementary machine with twelve conductors housed in its stator slots as shown in the above figure. The machine has two poles and for the position shown conductor 1 will ~~no~~ have maximum emf induced it it. (Say Em) This emf is represented by a phasor E_1 whose vertical component is equal to the rms value of the emf. generated in conductor 1 ($\therefore E_1 = Em/\sqrt{2}$). The conductor 2 is at an angle γ with ~~the~~ field axis. So, considering sinusoidal flux distribution the emf in conductor 2 will be $Em\cos\gamma$. This emf can be represented by a phasor E_2 ($|\bar{E_2}| = |\bar{E_1}|$) such that the vertical projection is $E_2\cos\gamma$ each to the rms value of the generated emf. Angle between E_1 & E_2 is γ. Similarly emf generated in all the conductors can be represented by phasors so that the vertical component is equal to the rms value of the emf induced in them for this particular position.

As the construction is symetric and all slots have equal number of turns magnitude of the induced emfs are equal and it is observed that the summation of these emf.s is zero. So, emfs $E_1, E_2 \cdots E_{12}$ forms a closed polygon, whose vertices lie on a circle. Angle subtented by each phasor at the centre of the circle is γ. γ is called the slot pitch angle

$$\gamma = \frac{\pi P}{\text{total no. of slots}} \text{ elec. radians.}$$

<u>Distribution (or breadth) factor:</u>

Emf generated in a full pitch armature coil of N turn is given by

$$e = N \omega_r \phi . \sin \omega_r t$$

But in practice all the N turns are not placed in the same slot, but distributed uniformly along the air gap periphery. The advantages of a distributed

(i) reduction of harmonics in the generated emf. making the emf approximately sinusoidal.

(ii) better utilization of armature iron and copper

(iii) Adding rigidity and mechanical strength to the winding.

Consider the machine shown in fig 1 we have 12 slots. If we want to make a two pole 3 phase winding, then

slots / pole / phase $= \frac{12}{2 \times 3} = 2$ (normally indicated by q).

slot pitch $\gamma = \frac{2 \times 180}{12} =$

coil spain $= \frac{\text{Number of slot}}{\text{no. of pole}} = \frac{12}{2} = 6$

So, for a full pitch coil, if one coil side is in slot 1 the other side is in slot 7. The coils are connected such that the total emf is $E_1 + E_7$. Similarly we connect conductor 2 with conductor 8. The resultant emf of $E_1 + E_7$ and $E_2 + E_8$ are shifted by slot pitch angle γ.

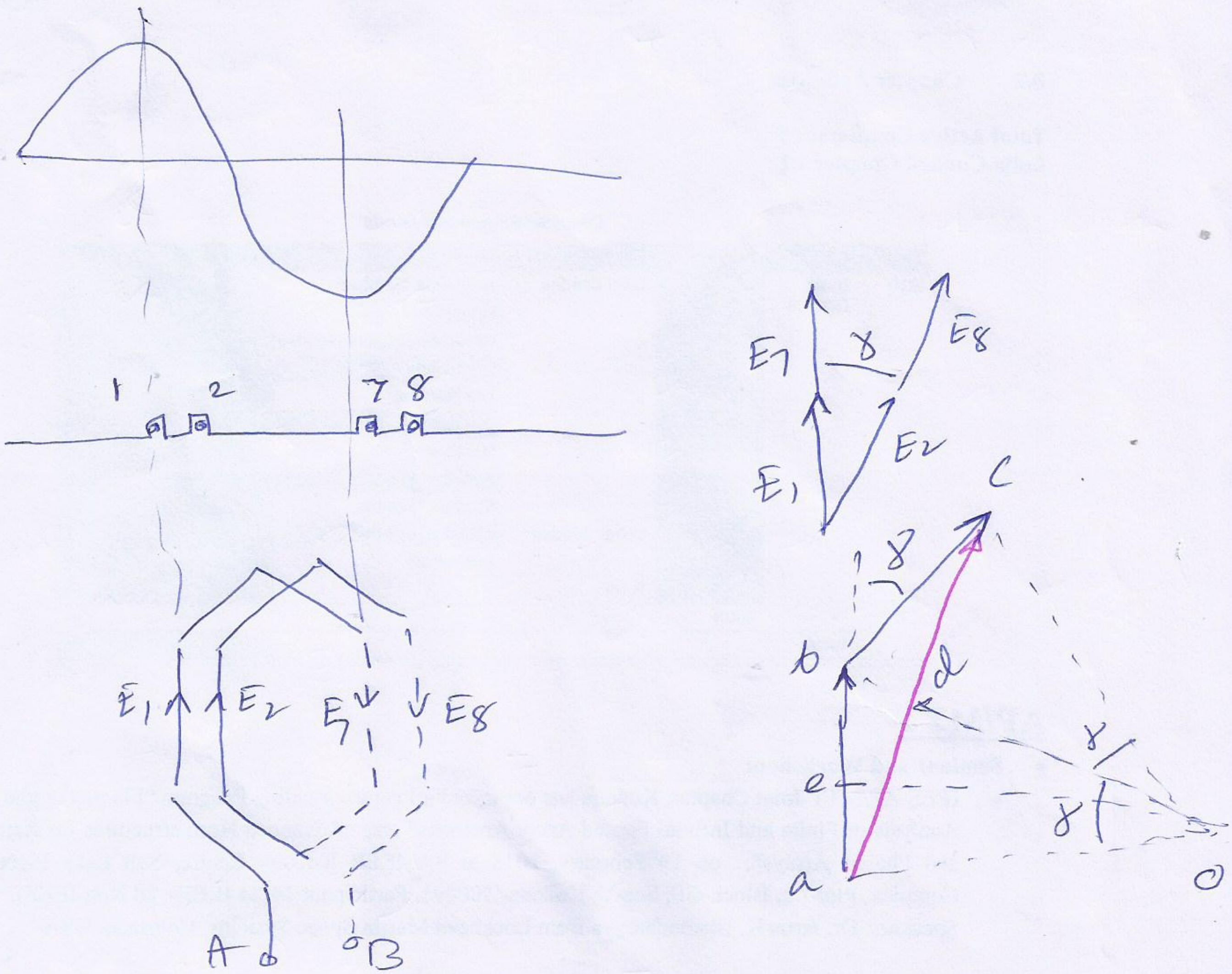


Fig 2

In fig 2 the phasor ab represent the resultant emf of conductor 1 and 7. Similarly bc represents resultant emf of conductor 2 and 8. ~~Angle between~~ As the phasors are part of a closed polygon, ~~the~~ bisector of ab and bc passes through the centre of the circle.

From the figure

$$ae = oa \sin \gamma/2$$

$$\therefore \text{emf per coil} = ab = bc = 2\, oa \sin \gamma/2$$

∴ arithmatic sum of the coil emfs

$= q(\text{emf per coil})$

$= q\, 2.oa \sin \gamma/2$

Similarly it is observed that

$ad = oa \sin \frac{2\gamma}{2} = oa \sin \frac{q\gamma}{2}$ ($q = 2$ in this case)

∴ resultant emf $= ac$

$= 2ad = 2oa \sin \frac{q\gamma}{2}$

$=$

Kd = distribution factor (or breadth factor)

$= \dfrac{\text{phasor sum of coil emfs}}{\text{Arithmatic sum of coil emfs.}}$

$= \dfrac{2oa \sin q\gamma/2}{q \cdot 2oa \sin \frac{\gamma}{2}} = \dfrac{\sin \frac{q\gamma}{2}}{q \sin \gamma/2}$

Kd → defined as above is the distribution for the fundamental.

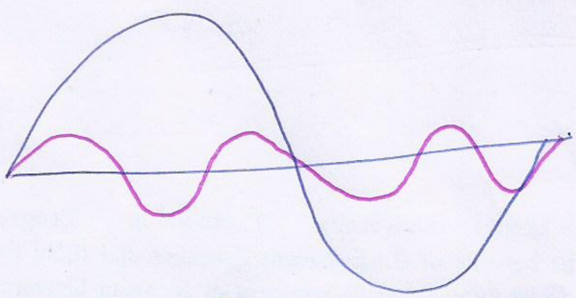


Fig 3

IF we consider the third harmonics then we see as the fundamental emf wave travels through 180°, the third harmonics travels through 3X180°. In view of this $\gamma°$ for the fundamental will be $n\gamma$ for

$\therefore K_{dn}$ = distribution factor for the n^{th} harmonics

$$= \frac{\sin \frac{q n \gamma}{2}}{q \sin \frac{n\gamma}{2}}$$

Now, conductor 1 & 2 belong to one phase, these phase group in adjacent slots is called phase belt or phase band. Peripheral angular distance of a phase belt is cased phase spread $\sigma (= q\gamma)$

$$\therefore K_{d_1} = \frac{\sin \frac{q\gamma}{2}}{q \sin \gamma/2} = \frac{\sin \sigma/2}{q \sin \gamma/2}$$

now as γ is small, when expressed in radian

$$K_{d_1} = \frac{\sin q\gamma/2}{q \gamma/2} \quad (\text{as } \sin \gamma/2 \simeq \gamma/2 \text{ for small } \gamma)$$

$$= \frac{\sin \sigma/2}{\sigma/2}$$

Similarly $K_{d_n} = \dfrac{\sin \frac{n\sigma}{2}}{n\sigma/2}$

Pitch (or coil-span) Factor:

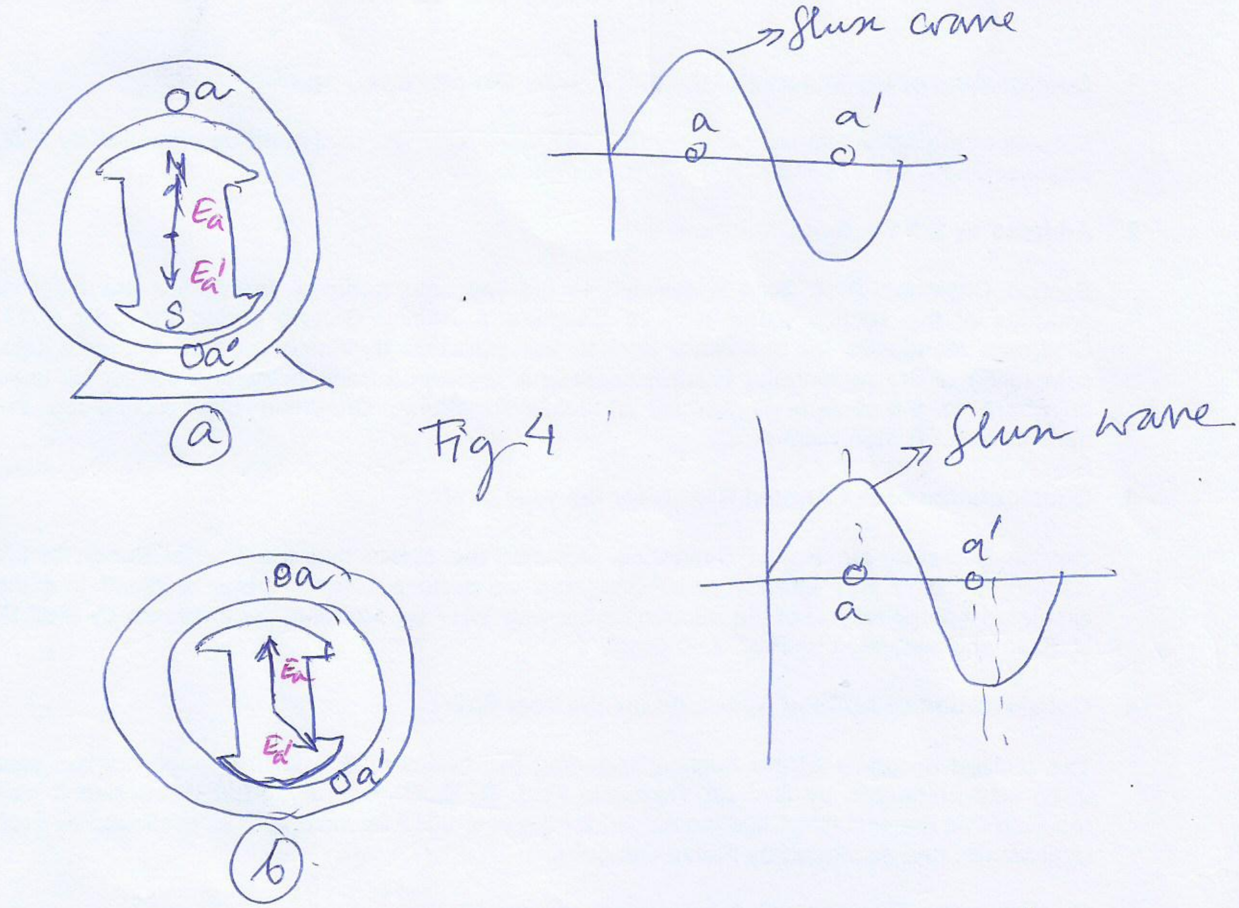
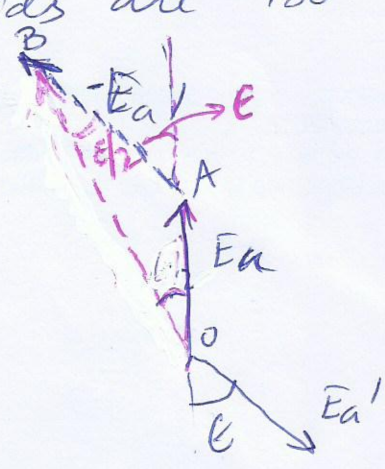


Fig 4

fig 4(a) shows a full pitch coil were the two coil sides are 180° elec. apart.

Fig 4(b) shows a short pitch coil shorted by an angle ϵ. So the two coil sids are $180^\circ - \epsilon$ degree apart.



the resultant voltage of coil aa' is OB.

$$OB = 2 \cdot OA \cos \epsilon/2$$

$$= 2 E \cos \epsilon/2$$

$$K_p = \text{pitch factor} = \frac{\text{resultant emf of a short pitched coil}}{\text{resultant emf of full pitch coil}}$$

As discussed earlier chording angle ε° for the fundamental becomes $n\varepsilon^\circ$ for the n^{th} harmonics. So,

$$K_{p_n} = \cos \frac{n\varepsilon}{2}$$

Advantages of short pitched coil:

(i) it reduces copper overhung

(ii) it reduces harmonics in the emf wave.

<u>Winding factor:</u>

The product of distribution factor K_d and pitch factor K_p is referred to as the winding factor.

$$K_W = K_d \times K_p$$

In case of three phase alternators the third and triplent frequency harmonics are taken care off by star delta connection. So during design importance is given to reduce fifth & seventh harmonics. By selecting proper chording angle these harmonics can be reduced. For example taking $\varepsilon = 30^\circ$ we get

$K_{p_1} = 0.966$, $K_{p_3} = 0.707$ $K_{p_5} = 0.259$

$K_{p_{[illegible]}} = (-)0.259$ & $K_{p_{[illegible]}} = (-)0.707$