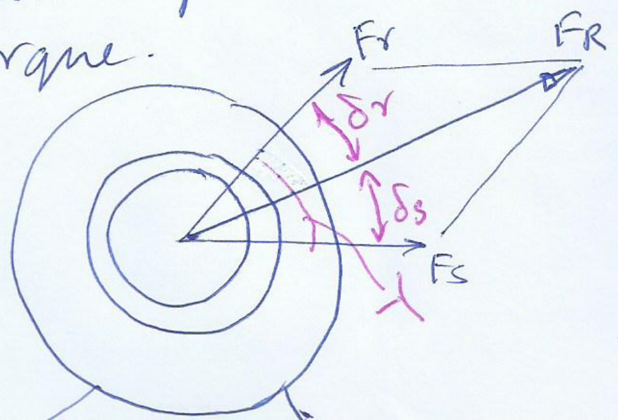
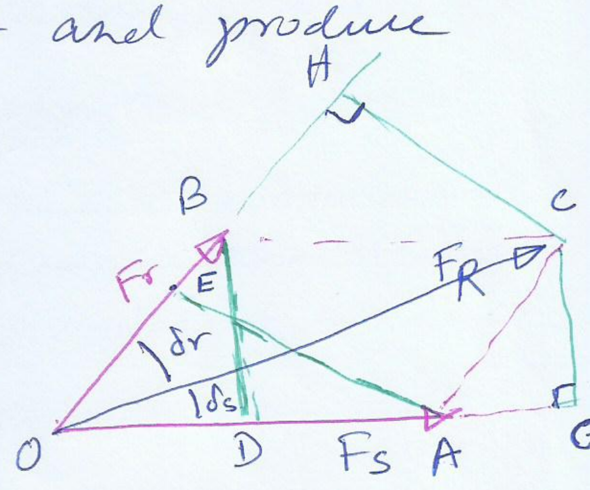
Production of Torque in Non Salient Pole Machines

In a non salient pole Synchronous machine we have two magnetic fields. One is the field created by the field windings. The magnitude is constant and it is rotated at Synchronous speed by the prime mover. The other field is the armature reaction field, created by the three phase current flowing through the three phase stator winding. We have seen that three phase distributed winding carrying three phase current will creat a rotating magnetic field rotating at synchronous speed. This to fields rotating at the same speed will interact and produce torque.





current. in the stator winding produce stator mmf F_s directed along the magnetic axis of stator. Similarly rotating rotor winding produce mmf F_r directed along the rotor magnetic axis. F_s and F_r are the peak values of the stator and rotor mmfs per pole (or per air gap crossing) respectively. F_s and F_r in effect cause the appearance of stator and rotor poles along their respective magnetic axis. The poles so produced have a tendency to align their magnetic axes and electromagnetic torque is developed.

The length of the air gap is 'g' and the average radius is r. The effective axial length is l. To derive the torque expression the following assumptions are made:

(i) The stator and rotor iron have infinite permeability. (i.e. saturation & hysteresis in neglected)

(ii) The length of the air gap 'g' is very small compared to the average radius 'r'.

(iii) Only the fundamental component of the mmf wave is considered.

As both F_s & F_r are sinusoidaly distributed in space and rotating a same speed. They can be represented by space, phasors $\bar{F}_s$ and $\bar{F}_r$ directed along their respective magnetic axis.

Phasor sum of $\bar{F}_s$ and $\bar{F}_r$ give the peak value of the resultant mmf wave $\bar{F}_R$

$$F_R = \sqrt{F_s^2 + F_r^2 + 2F_sF_r\cos\lambda}$$

if ϕ_R is the peak value of flux per pole

then $$\phi_R = \frac{F_R}{\text{One air gap reluctance}}$$

$$= \frac{F_R \mu_0 A}{g}$$

A = area under one pole.

ϕ_R is the resultant flux or mutual flux which links both the stator and rotor. There will be some stator leakage flux and rotor leakage flux. But the leakage flux do not take part in torque production. The effect of leakage flux is accounted for by means of leakage reactances.

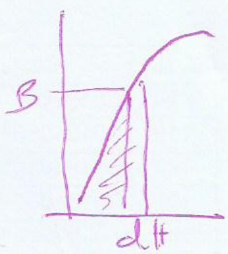
The resultant peak flux density is given by

$$B_R = \frac{\phi_R}{\text{Area}} = \frac{\phi_R}{A} = \frac{F_R \mu_0}{g}$$

Corresponding to B_R, the peak energy density in the air gap is given by

$$\frac{1}{2}\frac{B_R^2}{\mu_0} = \frac{1}{2}\frac{\mu_0^2}{\mu_0}\cdot\frac{F_R^2}{g^2}$$

$$= \frac{1}{2}\frac{\mu_0 F_R^2}{g^2}$$



energy density

$$= \int_0^B B \cdot dH$$

$$= \int_0^B B \cdot \frac{dB}{\mu_0}$$

$$= \frac{1}{\mu_0}\int_0^B B\, dB$$

$$= \frac{1}{\mu_0}\frac{B^2}{2}$$

F_S and F_r are sine waves, so F_R is also a sine wave.

Average value of the square of a sine wave is equal to half of the peak value square. Since F_R is a sine wave, so the average value of square of F_R is equal to $\frac{1}{2}F_R^2$

Average energy density in the air gap

$$= \frac{1}{2}\frac{\mu_0}{g^2}\left(\text{average value of } F_R^2\right)$$

$$= \frac{1}{2}\frac{\mu_0}{g^2}\left(\frac{1}{2}F_R^2\right)$$

$$= \frac{1}{4}\frac{\mu_0}{g^2}F_R^2$$

Total magnetic energy stored in the air gap

$$W_{fld} = \frac{1}{4}\frac{\mu_0}{g^2}F_R^2\left(\text{air gap volume}\right)$$

$$= \frac{1}{4} \frac{\mu_0}{g^2} F_R^2 (2\pi r l g)$$

$$= \frac{1}{2} \frac{\mu_0 \pi r l}{g} F_R^2 \text{ Joules}$$

$$= \frac{1}{2} \frac{\mu_0 \pi r l}{g} (F_s^2 + F_r^2 + 2 F_s F_r \cos \lambda)$$

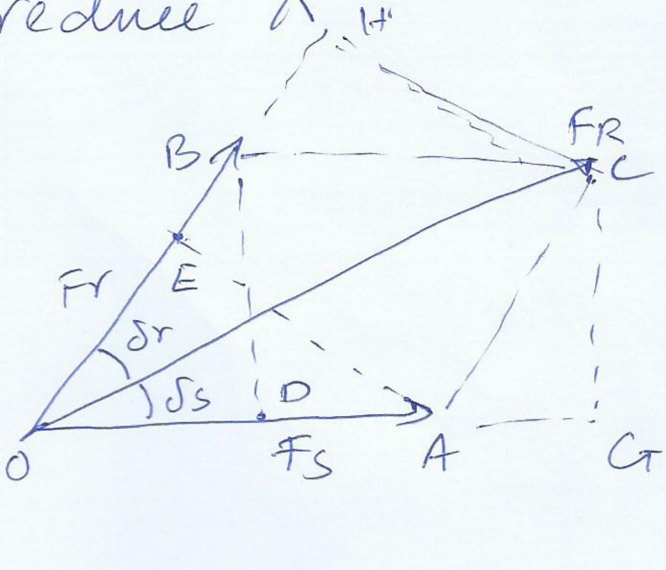
For two pole machine, electrical degree is equal to mechanical degree. Thus the electromagnetic torque is given by

$$T_e = \frac{\partial W_{fld}}{\partial \lambda}$$

$$= \frac{1}{2} \frac{\mu_0 \pi r l}{g} \frac{\partial}{\partial \lambda} (F_s^2 + F_r^2 + 2 F_s F_r \cos \lambda)$$

$$= - \frac{\mu_0 \pi r l}{g} F_s F_r \sin \lambda$$

'–' sign indicate that the direction of the torque is such that it tries to reduce 'λ'



$F_s \sin \lambda = AE = CH$

$= F_R \sin \delta_r$

& $F_r \sin \lambda = BD = CG$

$= F_R \sin \delta_s$

$$\therefore T_e = -\frac{\mu_0 \pi r l}{g} F_s F_R \sin\delta_s$$

$$T_e = -\frac{\mu_0 \pi r l}{g} F_r F_R \sin\delta_r$$

again $F_R = \dfrac{g B_R}{\mu_0}$

So, $T_e = -\pi r l B_R F_s \sin\delta_s$

$\quad = -\pi r l B_R F_r \sin\delta_r$

As F_R is a sine wave B_R sould also be a sine wave as the air gap is uniform with peak value of B_R

So, for a two pole machine, flux per pole

$$\phi = 2 B_R l r$$

$$\therefore B_R = \frac{\phi}{2 l r}$$

$dx = r d\theta$

$d\phi = B_R \sin\theta \, l . dx$

$d\phi = B_R \sin\theta . l r d\theta$

$\phi = \int_0^{\pi} B_R \sin\theta \, l r d\theta$

$= 2 B_R l r.$

$$\therefore T_e = -\frac{\pi}{2} \phi F_s \sin\delta_s$$

$$= -\frac{\pi}{2} \phi F_r \sin\delta_r$$

For a P-pole machine:

mechanical angle $\lambda_m = \frac{2}{P} \lambda$

$$\therefore d\lambda_m = \frac{2}{P} d\lambda$$

$$\frac{d\lambda}{d\lambda_m} = \frac{P}{2}$$

$$T_e = \frac{\partial W_{fld}}{\partial \lambda_n}$$

$$= \frac{\delta W_{fld}}{\delta \lambda} \cdot \frac{\delta \lambda}{\delta \lambda_{m}}$$

$$= \frac{P}{2} \frac{\delta W_{fld}}{\delta \lambda}.$$

$$= \frac{P}{2} \cdot \frac{1}{2} \frac{\mu_0 \pi r l}{g} \frac{\delta}{\delta \lambda}\left(F_s^2 + F_r^2 + 2 F_r F_s \cos \lambda\right)$$

$$= -\frac{P}{2} \cdot \frac{\mu_0 \pi r l}{g} F_s F_r \sin \lambda$$

In terms of B_R

$$T_e = -\frac{P}{2} \pi r l \, B_R F_s \sin \delta_s$$

$$= -\frac{P}{2} \pi r l \, B_R F_r \sin \delta_r$$

Flux per pole for P pole machine

$\theta_m = \frac{2}{p}\theta.$

$$\phi = \frac{4}{P} B_R l r.$$

$$B_R = \frac{P \phi.}{4 l r.}$$

$$d\phi = B_R \sin\theta \, l r \, d\theta_m$$

$$= \frac{2}{P} \int_0^{\pi} B_R \sin\theta \, l r \, d\theta$$

$$= \frac{4}{P} B_r l r$$

$$T_e = -\frac{P^2}{8} \pi \phi F_s \sin \delta_s \ \text{Nm}$$

$$= -\frac{\pi}{8} P^2 \phi F_r \sin \delta_r \ \text{Nm}$$

if δ_r is varying with time then the average torque over a complete cycle is when F_s and F_r are stationary w.r.t one another. This is true only at synchronous speed. So, this torque is zero at alother speed.

now magnitude of T_e is

$$T_e = \frac{\pi}{8} p^2 \phi F_r \sin\delta_r$$

now mmf $F_r = m \frac{2\sqrt{2}}{\pi} K_w \frac{N_{ph} I}{p}$.

$$\phi = \frac{E_{ph}}{\sqrt{2}\pi f N_{ph} K_w}$$

$$T_e = \frac{\pi}{8} p^2 \left(\frac{E_{ph}}{\sqrt{2}\pi f N_{ph} K_w}\right) \cdot \left(m \frac{2\sqrt{2}}{\pi} K_w \frac{N_{ph} I}{p}\right) \sin\delta_r$$

$$= \frac{p}{4\pi f} m E_{ph} I \sin\delta_r$$

it can be shown that $\delta_r \simeq 90 + \theta$

again

$$\omega_m = \frac{2}{p} \omega.$$

$$= \frac{2}{p} \cdot 2\pi f = \frac{4\pi f}{p}.$$

$$T_e = \frac{1}{\omega_m} m E_{ph} I \cos\theta$$

mechanical power = $T_e \omega_m = m E_{ph} I \cos\theta$ = electrical developed power