STATE VARIABLE FILTER

A state variable filter is a type of active filter. It consists of one or more integrators. A state variable filter realizes the state-space model directly. The instantaneous output voltage of one of the integrators corresponds to one of the state variables of the state-space model.

The useful property of the filter is that depending upon where the circuit is tapped as an output, the filter can generate a low-pass filter, high-pass filter, band-pass and band-stop filters all simultaneously.

In comparison to other topologies such as Sallen-Key filter, the main draw-back is that they require several op-amps in their configuration.

Practical realizations of analog filters are usually based on factoring the transfer function into cascaded second-order sections, each based on complex conjugate pole pair or a real poles, and a first order section if the order is odd. Each first and second order section is then implemented by an active filter and connected in series.

The state-variable filter design method is based on the block diagram representation that uses the outputs of a chain of integrators as state-variables.

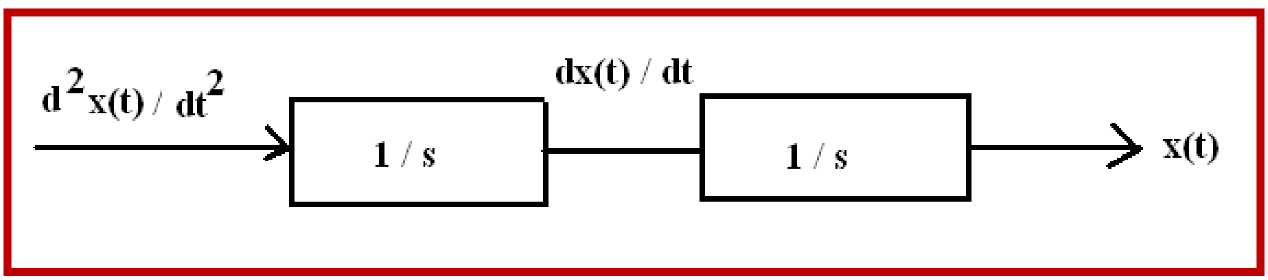


Fig. Cascaded Integrators with output x(t)

The second order transfer function can be represented by the following equation

$$H(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

The above transfer function can also be represented a

$$H(s) = \frac{K_1 s^2 - K_1K_2 s + K_1K_2K_3}{s^2 + K_2K_4 s + K_2K_3}$$

$$= \frac{K_1 s^2}{s^2+K_2K_4s+K_2K_3} - \frac{K_1K_2 s}{s^2+K_2K_4s+K_2K_3} + \frac{K_1K_2K_3}{s^2+K_2K_4s+K_2K_3}$$

= H_{HP}(s) - H_{BP}(s) +H_{LP}(s)

Where $K_1 = a_{2,}$ $a_1 = K_1K_2$ and $a_o = K_1K_2K_3$

Now

$$H_{HP}(s) = \frac{K_1 s^2}{s^2 + K_2K_4 s + K_2K_3}$$

Also $H_{BP}(s)$ may be derived from $H_{HP}(s)$ by multiplying $H_{HP}(s)$ by $(-\frac{K_2}{s})$ i.e. an inverting integrator with gain K_2.

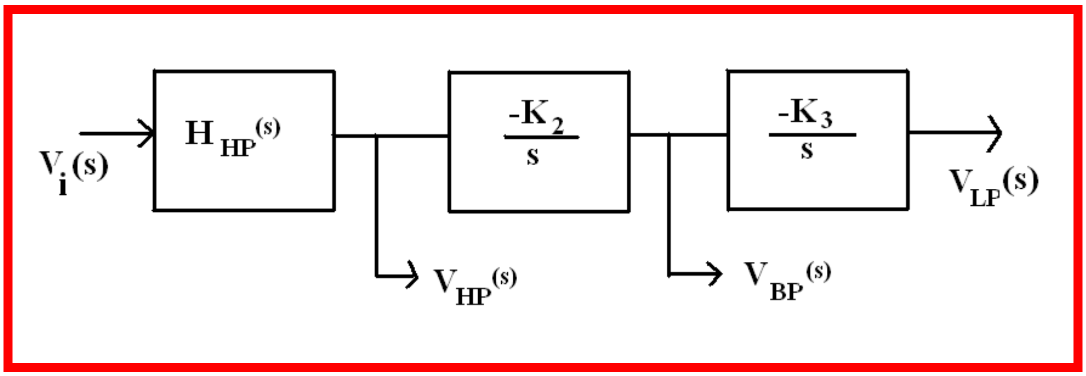
Therefore

$$H_{BP}(s) = H_{HP}(s)\left(-\frac{K_2}{s}\right) = \frac{K_1K_2 s}{s^2 + K_2K_4 s + K_2K_3}$$

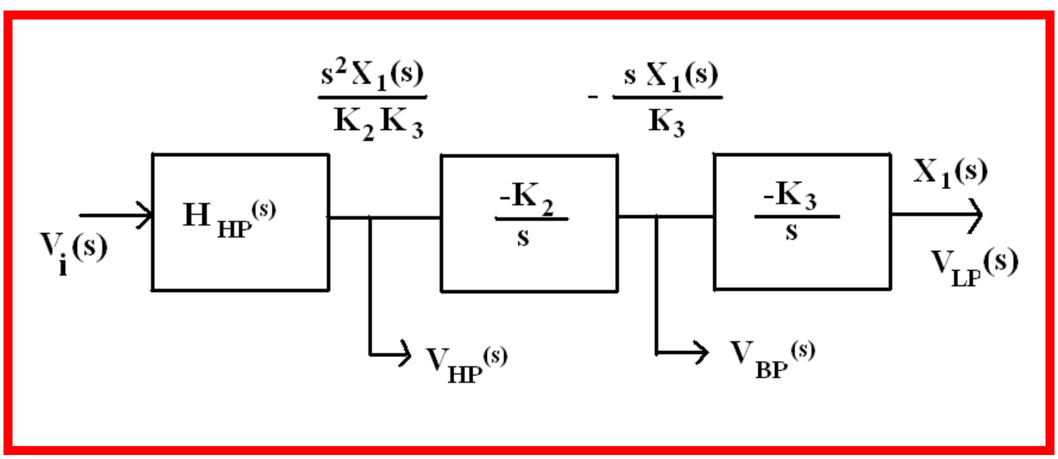
H_{LP}(s) again be derived from $H_{BP}(s)$ by multiplying $H_{BP}(s)$ by $(-\frac{K_3}{s})$ i.e. an inverting integrator with gain K_3.

$$H_{LP}(s) = H_{BP}(s)\left(-\frac{K_3}{s}\right) = \frac{K_1K_2K_3}{s^2+K_2K_4s+K_2K_3}$$

The above three filter can be realized with an integrated filter called as an universal filter as shown in the following block diagram



Let $X_1(s) = V_{LP}(s)$, **then the above block diagram will be**



$$\frac{X_1(s)}{V_i(s)} = \frac{K_1K_2K_3}{s^2 + K_2K_4s + K_2K_3}$$

Or, $s^2X_1(s) + K_2K_4sX_1(s) + K_2K_3X_1(s) = K_1K_2K_3V_i(s)$

In state variable approach two steps have to be considered:

- ✓ **The set of differential equation can be re-written so that highest derivatives are on L.H.S. of equations.**
- ✓ **The equations are then reduced to sets of first order equation by choosing the derivatives as new variables as necessary.**

Therefore,

Step 1 :

$\ddot{X}_1 = -K_2K_4\dot{X}_1 - K_2K_3X_1 + K_1K_2K_3V_i$ **… in time domain**

Step 2 :

Let chose state variable

$$\dot{X}_1 = X_2$$

$\dot{X}_2 = \frac{dX_2}{dt} = \ddot{X}_1 = -K_2K_4X_2 - K_2K_3X_1 + K_1K_2K_3V_i$ **…(1)**

The output equations are

$$V_{LP} = X_1,$$

$$V_{BP} = \frac{X_2}{K_3} = \frac{\dot{X}_1}{K_3},$$

$$V_{HP} = \frac{\dot{X}_2}{K_2 K_3} = \frac{\ddot{X}_1}{K_2 K_3}$$

From the state equation (1)

$$\frac{\dot{X}_2}{K_2 K_3} = -\left[\frac{K_4}{K_3} X_2 + X_1\right] + K_1 V_i$$

$$= K_4 \left[-\frac{X_2}{K_3}\right] - X_1 + K_1 V_i$$

Or

$$\frac{\ddot{X}_1}{K_2 K_3} = K_4 \left[-\frac{X_2}{K_3}\right] - X_1 + K_1 V_i$$

From the above relation, the general structure of the filter is as follows

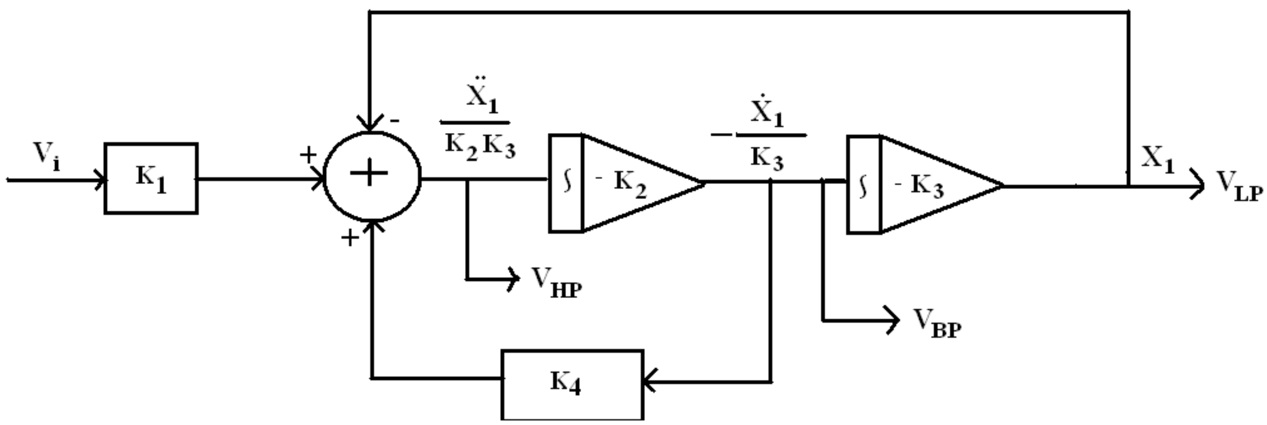
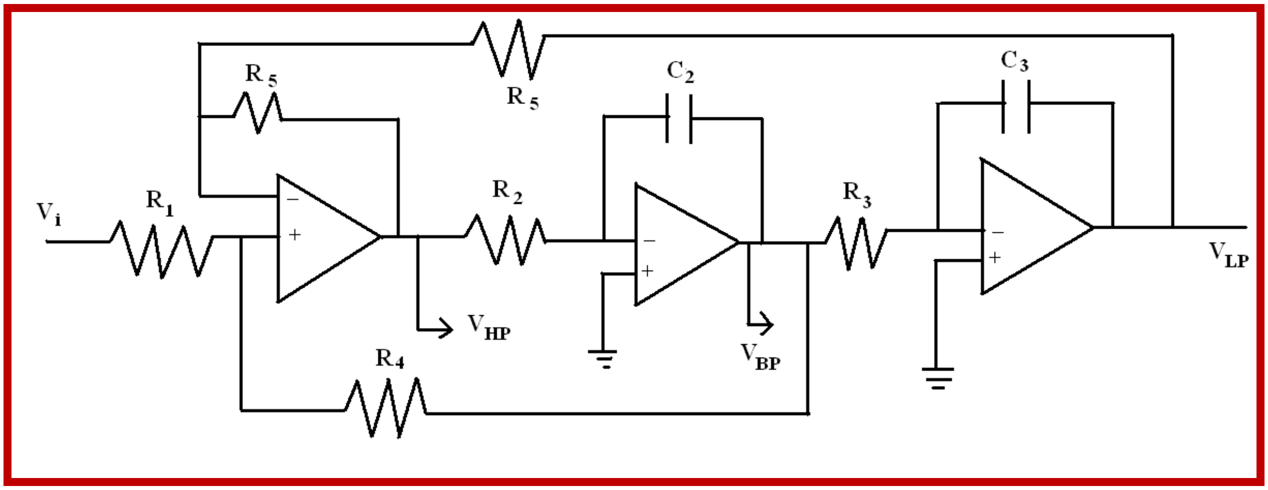


Fig. General structure of the state-variable Filter

Thus the state variable configuration uses two op-amp integrators and one op-amp summer to provide simultaneous second order low-pass, band-pass and high-pass filter responses.

The state variable filter circuit is shown the figure below



For the above circuit

$$K_1 = \frac{2R_4}{R_1 + R_4}, \quad K_2 = \frac{1}{R_2 C_2}, \quad K_3 = \frac{1}{R_3 C_3}, \quad K_4 = \frac{2R_1}{R_1 + R_4}$$

Center frequency of the pass band= $\omega_0 = \sqrt{K_2 K_3} = \frac{1}{\sqrt{R_2 C_2 R_3 C_3}}$

Bandwidth = $K_2 K_4$

Thus the centre/cut-off frequency of operation and the Bandwidth can be varied independently. So, state-variable filters are widely used in analog synthesizers.

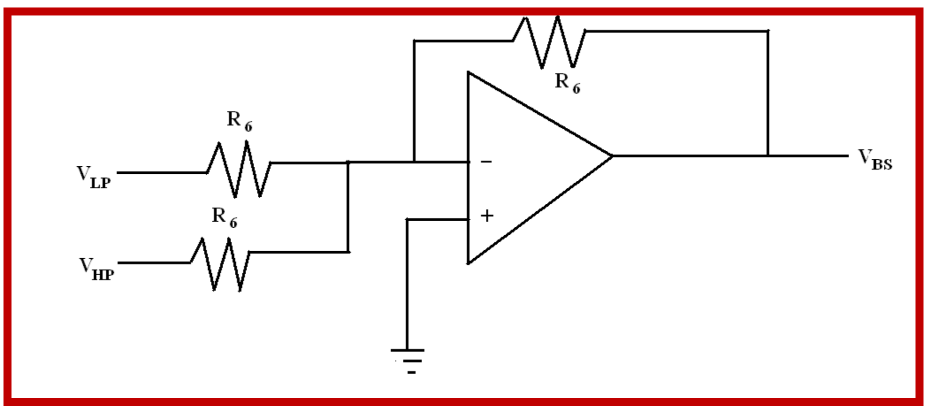
The circuit above can be modified where a fourth op-amp has been used to get a Band stop or *notch filter* response. The op-amp provides the notch filter response by combining the low-pass and high-pass output. The notch filter output can be written as

$$V_{BS}(s)$$

$$= -V_{HP} - V_{LP}$$

Putting the values of V_{HP} and V_{LP}

$$H_{BS}(s) = \frac{K_1(s^2 + K_2K_3)}{s^2 + K_2K_4s + K_2K_3}$$



Thus it is possible to obtain LP, HP,BP and BS outputs from a state-variable filter and therefore second order state variable filter are also known as *universal active filter*-i.e. all filters may be realized from a single circuit.