BACHELOR OF ARTS EXAMINATION, 2025

2nd Year, 2nd Semester

DEPARTMENT OF ECONOMICS

Subject Code : ARTS/ECOM/UG/MAJOR/TH/22/203

ADVANCED STATISTICS

Time : Two Hours **Full Marks : 30**

*Answer question(s) from **each CO** as per instruction.*

[**Note**: All notations carry their usual meaning]

CO 1

1. The probabilities of solving a problem by three students A, B and C are 3/7, 3/8 and 1/3, respectively. If all of them try independently, find the probability that the problem could be solved by one person only. Find also the probability that the problem is not solved. (2+1=3)

Or

Show that $f(x)$ defined by

$$f(x) = \begin{cases} x & for\ 0 \leq x \leq 1 \\ k - x & for\ 1 \leq x \leq 2 \\ 0 & elsewhere \end{cases}$$

is a probability density function for a suitable value of the constant k. (3)

CO 2

2. Show that for a normal distribution with mean μ and variance σ^2, all odd order central moments are equal to zero. (3)

Or

If $X \sim N(2, 16)$, evaluate $P(|X + 2| \geq 3)$. (3)

CO 3

*Answer **Question no. 3** and **any two** from the rest* (3×3=9)

3. Let $x_1, x_2, \dots\dots, x_n$ be a random sample from $N(0,1)$. If $\bar{x}_k = \frac{1}{k}\sum_{i=1}^{k} x_i$ and $\bar{x}_{n-k} = \frac{1}{n-k}\sum_{i=k+1}^{n} x_i$, find the distribution of

   (i) $k\bar{x}_k^2 + (n-k)\bar{x}_{n-k}^2$

   (ii) $\frac{x_i^2}{x_j^2}, i \neq j$ (2+1=3)

4. Derive the expectation and standard error of sample proportion in SRSWOR from a finite population. (1+2=3)

5. From a normal population with variance 25, a random sample of size 20 is taken. What is the probability that the sample mean will not differ from the population mean by more than 2 in absolute value? (3)
6. Given the joint probability density function

$$f(x,y) = \begin{cases} \frac{3}{5}\, x(x+y), & 0 < x < 1;\ 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

of two random variables X and Y, find $P[(X,Y) \in A]$, where A is the region $\{(x,y)\ /\ 0 < x < \frac{1}{2}, 1 < y < 2\}$. (3)

CO 4

Answer ***Question no. 7, 8*** *and* ***any three*** *from the rest* (3×5=15)

7. Argue whether the following statements are true or false:
   (i) A biased estimator can not be consistent.
   (ii) If H_0 is accepted at $\alpha_1\%$ level of significance, it will definitely be accepted at $\alpha_2\%$ level of significance, where $\alpha_1 < \alpha_2$.
   (1.5+1.5=3)
8. A random sample $(x_1, x_2, x_3, x_4, x_5)$ of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators of μ.

$$T_1 = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$T_2 = \frac{x_1 + x_2}{2} + x_3$$

$$T_3 = \frac{2x_1 + x_2 + \lambda x_3}{3}$$

   where λ is such that T_3 is an unbiased estimator of μ.
   (i) Find λ.
   (ii) Which estimator will you select among T_1, T_2 and T_3? Give reasons. (1+2=3)
9. A random sample of 500 oranges was taken from a large consignment and 65 were found to be bad. Can we conclude that the proportion of bad oranges in the entire consignment is 15%? (3)
10. Suppose the standard deviation of a random sample of size 20 from a normal population is 12. Find a 95% confidence interval for the population variance. (3)
11. A random sample of size 40 from $N(\mu, \sigma^2)$ gives a sample mean 22 with standard deviation 4. Test the hypothesis that $\mu = 25$ against $H_1\colon \mu \neq 25$ at 99% level. (3)
12. Let p denote the probability of getting a head when a given coin is tossed once. Suppose that the hypothesis $H_0\colon p = 0.5$ is rejected in favour of $H_1\colon p = 0.6$ if 10 trials result in 7 or more heads. Calculate the probability of type-I and type-II errors. (3)

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