

Ex/ECO/B/C1.2(OLD)/2025(OLD)

BACHELOR OF ARTS EXAMINATION, 2025

(1st Year, 1st Semester)

ECONOMICS

Mathematical Methods in Economics BI

Time : Two Hours

Full Marks : 30

Answer Question no. 1 and *any two* from the rest.

1. Check the correctness of the following statements and give reasons for your answer in each case. 2×5

(a) The function $f(x) = \frac{1}{2}x^2$ is concave everywhere over its domain.

(b) If a function is continuous and concave over its domain, it will always attain an absolute maximum over its domain.

(c) (i) The statement $x > 4 \rightarrow x > 2$ is contrapositive to the statement $x > 2 \rightarrow x > 4$

(ii) The proposition $\forall x, x > 10 \rightarrow \forall y(y < x \rightarrow y < 9)$ is true $\forall x \in R, y \in R$.

(d) $f(x) = \frac{1}{3}x^3 + 2x, x \in R^+$ is a quasi-convex function.

(e) The function $f(x) = \frac{x^2 - xy + y^2}{x + y}$ is homogeneous of degree 2.

ECO-1137

[Turn Over]

(2)

2. (a) Find out the supremum and infimum of the set $S = \left\{ x \in \mathbb{R}; x < \frac{1}{x} \right\}$.
- (b) Identify the local extrema in \mathbb{R}^2 of $f(x, y) = x^{0.5}y^{0.5}$ subject to $2x + 4y = 16$.
- (c) Find out the critical points of the function $f(x, y) = 2x^2 - x^3 - y^2$ and classify them (relative maximum, relative minimum, saddle point). Is it possible to identify absolute maximum and/or absolute minimum?
2+4+4
3. (a) Define absolute maximum and relative maximum, absolute minimum and relative minimum for a function $f : D^2 \rightarrow \mathbb{R}$.
- (b) Consider the function : $\mathbb{R} \rightarrow \mathbb{R} f(x) = x^2 + x + 1$
Find out the critical points of the function : $g(x) = f(f(x)) - \{f(x)\}^2$ and classify them into relative maximum and/or relative minimum. Are there any absolute maxima and/or minima?
- (c) Find out the maximum and the minimum values of the function $f(x, y, z) = x + y + z^2$ subject to the constraints $x^2 + y^2 + z^2 = 1$ and $y = 0$.
3+3+4
4. (a) State mean value theorem. Consider a function f , which is continuous over $[a, b]$ and has continuous derivatives over (a, b) . Applying mean value theorem, prove that $f'(x) > 0 \forall x \in (a, b)$ implies f is strictly increasing and $f'(x) < 0 \forall x \in (a, b)$ implies $f(x)$ is strictly decreasing.

(3)

- (b) Let $f : D^2 \rightarrow \mathbb{R}$. Explain in detail under what conditions f will have a global maximum. Under what conditions the global maximum will be unique?
- (c) Determine the degree of homogeneity of the function $\phi(x, y) = x \log \frac{y}{x}$. Verify the Euler's theorem for ϕ .

3+4+(1+2)

★ ★ ★