4. CO4 (Answer *any* **one** question from the following)

A. *(i)* Find out the maximum and the minimum of the function $f(x, y, z) = x + y + z^2$ subject to the constraints $x^2 + y^2 + z^2 = 1$ and $y = z$.

(ii) State the condition for global maximum. 5+2½

B. *(i)* Identify the local extrema in R^2 of $f(x, y) = x + 2y$ subject to $x^2 + y^2 = 5$.

(ii) Show that in the linear constraint case, if the bordered determinants satisfy the conditions of quasi-concavity, the sufficient condition for constrained maximization will also be satisfied. 5+2½

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BACHELOR OF ARTS EXAMINATION, 2025

(1st Year, 1st Semester)

ECONOMICS

Course : ECO/UG/SEC/11/101

(Mathematical Methods in Economics–I)(SEC)

Time : *Two Hours* *Full Marks* : 30

1. CO1 (Answer Question A and *any* **one** from B)

A. Define the limit of a function $f : D \to R$ and $f : D^2 \to R$.

Define the total derivative and partial derivatives for the function $f : D^2 \to R$. 5

B. *(i)* Find out the supremum and infimum of the set $S = \left\{x \in R; x < \frac{1}{x}\right\}$.

(ii) Find out the supremum and infimum of the set $A = \left\{x \in Q : 0 < \left(\sqrt{2} - 1\right)x < \sqrt{2} + 1\right\}$. 2½

2. **CO2** (Answer *any* **one** question from the following)

A. *(i)* Check the validity of the statements :

(a) If a function is continuous and concave over its domain it will always attain an absolute maximum over its domain.

(b) A linear function is both concave and convex over its domain.

(ii) Find out the critical points of the function : $g(x) = f\left(f(x)\right) - \{f(x)\}^2$ and classify them into relative maximum and/or relative minimum. Are there any absolute maxima and/or minima?

3+4½

B. *(i)* Check the validity of the statements :

(a) The statement $x > 4 \rightarrow x > 2$ is contrapositive to the statement $x > 2 \rightarrow x > 4$.

(b) The proposition

$$\forall x, x > 10 \rightarrow \forall y\left(y < x \rightarrow y < 9\right)$$

is true $\forall x \in R, y \in R$.

(ii) Find out the critical points of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$ and classify them (relative maximum, relative minimum, saddle point). Is it possible to identify absolute maximum and/or absolute minimum? 3+4½

3. **CO3** (Answer *any* **one** question from the following)

A. *(i)* Check the validity of the following statements :

(a) $f(x) = \frac{1}{3}x^3 + 2x,\ x \in R^+$ is a quasi-convex function.

(b) The function $f(x) = \dfrac{x^{\beta+1} - x^{\beta}y + y^{\beta+1}}{x+y}$ is homogeneous of degree β.

(ii) Prove that the equation $x^2 - xy^3 + y^5 = 17$ defines an implicit function $y = \phi(x)$ around the point (5,2). What properties will this implicit function satisfy? Will the implicit function be defined around the point $(\sqrt{17}, 0)$? 3+4½

B. *(i)* Justify the following statements :

(a) The function $f(x,y) = x^4 + y^4 (x,y) \in R^2$ is locally convex at (1, 2) but is not globally convex.

(b) The function $\phi(x,y) = x \log \frac{y}{x}$ is homogeneous of degree 0.

(ii) Let $f : D^3 \rightarrow R$. Explain in detail under what conditions f will have a global maximum. Under what conditions the global maximum will be unique? 3+4½