Master of Arts Examination, 2025

(1st Year, 1st Semester)

Subject : Economics

Paper : Econometrics I

Time : *Two Hours* *Full Marks* : 30

Answer *any* **five** of the following questions. 6×5=30

1. Suppose $E(y \mid x_1, x_2) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$, where $E(x_1) = E(x_2) = 0$ and x_1 and x_2 are independent of each other. Show that $L(y \mid 1, x_1, x_2) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$.

2. Let $\{\mathbf{Z}_N : N = 1, 2, \ldots\}$ be a sequence of random $K \times K$ matrices and let $\mathbf{A}$ be a non-random $K \times K$ invertible matrix. If $\mathbf{Z}_N \xrightarrow{p} \mathbf{A}$ prove that

   (a) $\mathbf{Z}_N^{-1}$ exists w.p.a. 1, and

   (b) $\mathbf{Z}_N^{-1} \xrightarrow{p} \mathbf{A}^{-1}$ in an appropriate sense.

3. Consider the model $y_i = \beta + u_i$, where y_i and u_i are random scalar variables and β is a scalar unknown parameter. u_i are iid with $E(u_i) = 0$, $E(u_i^2) = \beta^2$, $E(u_i^3) = 0$ and $E(u_i^4) = m$. What is the limiting distribution of the vector $\left(\frac{1}{N}\sum_{i=1}^{N} y_i^2, \frac{1}{N}\sum_{i=1}^{N} y_i\right)'$?

4. Consider the linear regression model $y_i = \boldsymbol{x}_i\beta + u_i$ where $\boldsymbol{x}_i$ is non-stochastic and $E(u_i u_j) = \sigma^2$ if $i = j$, $E(u_i u_j) = \rho\sigma^2$ if $|i - j| = 1$ and $E(u_i u_j) = 0$ if $|i - j| > 1$.

   (a) What is a consistent estimator of $\text{var}(\hat{\beta})$?

   (b) Is White's heteroskedasticity consistent robust estimate of $\text{var}(\hat{\beta})$ consistent here?

5. Consider the model $y_1 = \mathbf{z}_1\delta_1 + \alpha_1 y_2 + u$, where y_2 is endogenous. Consider the reduced form for $y_2 : y_2 = \mathbf{z}_2\pi_2 + v_2$, where $\mathbf{z}_2$ has at least one exogenous element more than $\mathbf{z}_1$. Estimate the reduced form by OLS and save the residuals in $\hat{v}_2$. Estimate the following by OLS: $y_1 = \mathbf{z}_1\delta_1 + \alpha_1 y_2 + \rho_1\hat{v}_2 + \text{error}$. Show that OLS estimates of δ_1 and α_1 from this equation are identical to the 2SLS estimates of the same parameters.

6. Let the N-vector $\mathbf{y}$ be a vector of mutually independent realizations from the uniform distribution on the interval $[\beta_1, \beta_2]$. Let $\hat{\beta}_1$ be the maximum likelihood estimator of β_1 given by $\hat{\beta}_1 = \min(y_t)$, $t = 1, \ldots, N$ and the true values of β_1 and β_2 are 0 and 1, respectively. Find the cdf of the sampling distribution of $\hat{\beta}_1$.

7. Let $y_i = F(\mathbf{x}_i\beta) + v_i$ represent a binary choice model where $F(\mathbf{x}_i\beta) = E(y_i \mid \mathbf{x}_i)$ is the cumulative density function of a standardized random variable. Find the asymptotic variance of $\hat{\beta}$.

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