8. (a) Suppose the utility function is given by $u(w) = aw - bw^2$ (with $a$ and $b$ both positive). Does the function exhibit increasing or decreasing risk aversion?

(b) If the rate of return on risky assets is a random variable $R$ with mean $\bar{R} > 0$ and variance $\sigma_R^2$ and if the individual's initial wealth is $W$, what is the optimal amount of investment in risky assets?

(c) Show that the optimal amount of risky investment is decreasing function of wealth.

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# Master of Arts Examination, 2025

## (1st Year, 1st Semester)

## Subject : Economics

## Paper Code : Microeconomics-I

*Time* : *Two Hours* *Full Marks* : 30

*Each question carries equal marks.*

Answer *any* **six** questions. 5×6=30

1. The matrix below records the (Walrasian) demand substitution effects for a consumer endowed with rational preferences and consuming three goods at price $p_1 = 1, p_2 = 2$ and $p_3 = 6$ :

$$\begin{vmatrix} -10 & ? & ? \\ ? & -4 & ? \\ 3 & ? & ? \end{vmatrix}$$

Supply the missing numbers. Does the resulting matrix possess all the properties of a substitution matrix?

2. A consumer in a three-good economy (goods denoted $x_1, x_2$ and $x_3$; prices denoted $p_1$, $p_2$, $p_3$) with wealth level $w > 0$ has demand functions for the commodities 1 and 2 given by

$$x_1 = 100 - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} + \delta\frac{w}{p_3}$$

$$x_2 = \alpha + \beta\frac{p_1}{p_3} + \gamma\frac{p_2}{p_3} + \delta\frac{w}{p_3}$$

Where Greek letters are non-zero constants.

(*a*) Indicate how to calculate the demand for good 3 (but do not actually do it.)

(*b*) Are the demand functions for $x_1$ and $x_2$ appropriately homogeneous?

(*c*) Calculate the restrictions on the numerical values of $\alpha$, $\beta$, $\gamma$ and $\delta$ implied by utility maximization.

**3.** State and prove the Roy's Identity for utility maximization.

**4.** State and prove the Hotelling's lemma for profit maximization.

**5.** A price-taking firm produces output $q$ from inputs $z_1$ and $z_2$ according to a differentiable concave production function $f(z_1, z_2)$. The price of its output is $p > 0$ and the prices of its inputs are $(w_1, w_2) \gg 0$. However, there are two unusual things about this firm. First, rather than maximizing profit, the firm maximizes revenue (the manager wants her firm to have bigger dollar sales than any other). Second, the firm is cash constrained. In particular, it has only $C$ dollars on hand before production and as a reseult, its total expenditures on inputs cannot exceed $C$.

Suppose one of your econometrician friends tells you that she has used repeated observations of the firm's revenues under various output prices, input prices and levels of the financial constraint and has determined that the firm's revenue level $R$ can be expressed as the following function of the variables $(p, w_1, w_2, C)$:

$$R(p, w_1, w_2, C) = p[\gamma + \ln C - \alpha \ln w_1 - (1-\alpha)\ln w_2].$$





($\gamma$ and $\alpha$ are scalars whose values she tells you.) What is the firm's use of input $z_1$ when prices are $(p, w_1, w_2)$ and it has $C$ dollars of cash on hand?

**6.** Suppose a decision maker is an expected utility maximizer with a Bernoulli utility function $u(.)$ on amounts of money. Show that the following properties are equivalent :

(*a*) The decision maker is risk averse.

(*b*) $u(.)$ is concave.

(*c*) $C(F, u) \leq \int x dF(x)$ for all $F(.)$ where $C(.,.)$ is the certainty equivalent income and $F(.)$ any probability distribution over income.

(*d*) $\pi(x, \varepsilon, u) \geq 0$ for all $x$, $\varepsilon$.

**7.** Suppose each decision maker is an expected utility maximizer with a Bernoulli utility function of individual $i$ is $u_i(\cdot)$ on amount of money for $i = 1, 2$. Show that the following properties are equivalent :

(*i*) $r_A(x, u_2) \geq r_A(x, u_1)$ for every $x$

where $r_A(x, u_i) = -\dfrac{u_i''(x)}{u_i'(x)}$.

(*ii*) There exists an increasing concave function $\phi(.)$ such that $u_2(x) = \phi(u_1(x))$ at all $x$; that is, $u_2(.)$ is a concave transformation of $u_1(.)$.